SHARP

Worksheet 1 – Logs, Annuities and Finance

Exponents

1. a) $\frac{x^3y^2}{z^{-1}} \div \frac{(x^{-2}y^3)^2}{z^{-3}y^2}$

$= x^3\, y^2\, z^1 \div \frac{x^{-4}y^6}{z^{-3}y^2}$

$= x^3y^2z \div \frac{y^4z^3}{x^4}$

$= x^3y^2z \times \frac{x^4}{y^4\, z^3}$

$= \frac{x^7}{y^2z^2}$

b) $\frac{\left(\sqrt[3]{x^5} \times y^4\right)^{-\frac{1}{2}}}{x^3y^2} \times \frac{\left(\sqrt{x^4\, y^3}\right)^5}{x^{-1}}$

$= \frac{\left(x^{\frac{5}{3}} \times y^4\right)^{-\frac{1}{2}}}{x^3y^2} \times \frac{\left(x^{\frac{4}{2}} \times y^{\frac{3}{2}}\right)^5}{x^{-1}}$

$= \frac{x^{-\frac{5}{6}} \times y^{-2}}{x^3y^2} \times x^{10} \times y^{\frac{15}{2}} \times x$

$= \frac{x^{-\frac{5}{6}} \times y^{\frac{15}{2}} \times x^8}{y^4}$

$= x^{\frac{43}{6}}\, y^{\frac{7}{2}}$

$\left(= \sqrt[6]{x^{43}} \times \sqrt{y^7}\right)$ not necessary

c) $\frac{\sqrt{72}}{\sqrt{32}-\sqrt{18}}$

$= \frac{6\sqrt{2}}{4\sqrt{2}-3\sqrt{2}}$

$= \frac{6\sqrt{2}}{1\sqrt{2}}$

$= 6$

d) $\left(\frac{\left(\frac{1}{x}+\frac{1}{y}\right)^{-1}}{\frac{1}{x}}\right)^{-1}$

$= \left(\frac{\left(\frac{y+x}{xy}\right)^{-1}}{x^{-1}}\right)^{-1}$

$= \left(\frac{\frac{xy}{x+y}}{x^{-1}}\right)^{-1}$

$= \left(\frac{xy}{x+y} \times \frac{x}{1}\right)^{-1}$

$= \left(\frac{x^2y}{x+y}\right)^{-1}$

$= \frac{x+y}{x^2y}$

e) $\frac{1}{x^2y} \div \frac{1}{x^{-2}y^3} \times \frac{(x^3y^{-2})^{-1}}{\frac{1}{x^3y^{-2}}}$

$= \frac{1}{x^2y} \times \frac{x^{-2}y^3}{1} \times \frac{\frac{1}{x^3y^{-2}}}{\frac{y^2}{x^3}}$

$= \frac{1}{x^2\,y} \times \frac{y^3}{x^2} \times \frac{\frac{y^2}{x^3}}{\frac{y^2}{x^3}}$

$= \frac{y^3}{x^4y} \times 1$

$= \frac{y^2}{x^4}$

2. a) $7 \cdot 3^{2x-1} = \frac{7}{243}$

$3^{2x-1} = \frac{1}{243}$

$\therefore \log_3 \frac{1}{243} = 2x - 1$

$\therefore 2x - 1 = -5$

$\therefore 2x = -4$

$\therefore x = -2$

b) $2^{2x+1} + 4^x = 48$

$2^{2x}.2^1 + 2^{2x} = 48$

$2^{2x}(2 + 1) = 48$

$2^{2x} = 16$

$\therefore \log_2 16 = 2x$

$\therefore 2x = 4$

$\therefore x = 2$

c) $\frac{5.3^{-3x} - 3^{-3x}}{5} = 64\frac{4}{5}$

$5.3^{-3x} - 3^{-3x} = 324$

$\therefore 4.3^{-3x} = 324$

$\therefore 3^{-3x} = 81$

$\therefore \log_3 81 = -3x$

$\therefore -3x = 4$

$\therefore x = -\frac{4}{3}$

d) $2.5^{2x} - \frac{3.5^{3x}}{4.5^x} = \frac{25}{4}$

$2.5^{2x} - \frac{3}{4}.5^{2x} = \frac{25}{4}$

$\therefore 5^{2x}\left(2 - \frac{3}{4}\right) = \frac{25}{4}$

$\therefore 5^{2x}\left(\frac{5}{4}\right) = \frac{25}{4}$

$\therefore 5^{2x} = 5$

$\therefore 2x = 1$

$\therefore x = \frac{1}{2}$

e) $3^x = 50$

$\therefore x = 3.56$

3. a) $\log_a 4 + \log_{\frac{1}{a}} \frac{1}{4} - \log_a 8$

$= \log_a 4 + \log_a 4 - \log_a 8$

$= \log_a \left(\frac{4 \times 4}{8}\right)$

$= \log_a 2$

b) $\log_2 16 + \log_3 9 - \log_5 1$

$= \log_2 2^4 + \log_3 3^2 - 0$

$= 4\log_2 2 + 2\log_3 3$

$= 4 + 2$

$= 6$

c) $\frac{\log_3 54 - \log_3 18}{\log_3 81 - \log_3 1}$

$= \frac{\log_3(3^3 \times 2) - \log_3(3^2 \times 2)}{\log_3(3^4) - 0}$

$= \frac{\log_3 3^3 + \log_3 2 - \log_3 3^2 - \log_3 2}{\log_3 3^4}$

$= \frac{3\log_3 3 + \log_3 2 - 2\log_3 3 - \log_3 2}{4\log_3 3}$

$= \frac{1}{4}$

d) $\frac{\log_a 3 + \log_a 8}{\log_a 9} + \frac{\log_a 16 - \log_a 9}{\log_a 3}$

$= \frac{\log_a 3 + \log_a 2^3}{\log_a 3^2} + \frac{\log_a 2^4 - \log_a 3^2}{\log_a 3}$

$= \frac{\log_a 3}{2\log_a 3} + \frac{3\log_a 2}{2\log_a 3} + \frac{4\log_a 2}{\log_a 3} - \frac{2\log_a 3}{\log_a 3}$

$= \frac{1}{2} + \frac{3}{2}\log_3 2 + 4\log_3 2 - 2$

$= -1\frac{1}{2} + 5\frac{1}{2}\log_3 2$

e) $\log_a 3 + \log_a 9 - \log_3 a$

$= \log_a 3 + \log_a 3^2 - \log_3 a$

$= \log_a 3 + 2\log_a 3 - \log_3 a$

$= 3\log_a 3 - \frac{\log_a a}{\log_a 3}$

$= \log_a 27 - \frac{1}{\log_a 3}$

$= 3(\log_a 3)^2 - 1$

4. a) $\log_x 64 = 3$

$\therefore x^3 = 64$

$\therefore x = \sqrt[3]{64}$

$\therefore x = 4$

Restrictions: $x \geq 1$

b) $\log_3(2x+1) = 5$

$\therefore \log_3(2x+1) = 5\log_3 3$

$\therefore \log_3(2x+1) = \log_3 3^5$

$\therefore 2x + 1 = 243$

$\therefore 2x = 242$

$\therefore x = 121$

c) $\log_2(x-1) + \log_2(x+3) = 4$

$\therefore \log_2(x-1)(x+3) = 4\log_2 2$

$\therefore \log_2(x-1)(x+3) = \log_2 2^4$

$\therefore x^2 + 2x - 3 = 16$

$\therefore x^2 + 2x - 19 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-19)}}{2(1)}$

$\therefore x = 3.47 \quad or \quad x = -5.47$

N/A

d) $\log_4 15 = x$

$\therefore x = 1.95$

e) $\log_2 4x - \log_x 8 = \log_3 1$

$\therefore \log_2 4 + \log_2 x - \log_x 8 = 0$

$\therefore 2\log_2 2 + \log_2 x - \frac{\log_2 8}{\log_2 x} = 0$

$\therefore 2 + \log_2 x - \frac{3}{\log_2 x} = 0$

$(\log_2 x)^2 + 2\log_2 x - 3 = 0$

$\therefore (\log_2 x + 3)(\log_2 x - 1) = 0$

$\therefore \log_2 x = -3 \quad or \quad \log_2 x = 1$

$\therefore x = 2^{-3} \quad or \quad x = 2^1$

$\therefore x = \frac{1}{8} \quad or \quad x = 2$

5. $P = 13\,000 - 3\,000 = 10\,000$

$A = ?$

$i = 22\%$

$n = 3$ years

$A = P\,(1 + in)$

$\therefore A = 10\,000(1 + (0.22)3)$

$\therefore A = 16\,600$

$\therefore$ the monthly instalments are $= 16\,600 \div (n \times 12)$

$= 16\,600 \div 36$

$= 461.11$

6. $P = 20\,000$

$A = 30\,000$

$i = 12\% \div 4$

$n = ? \times 4$

$A = P\,(1 + i)^n$

$\therefore 30\,000 = 20\,000\left(1 + \frac{0.12}{4}\right)^{4n}$

$\therefore \frac{3}{2} = \left(\frac{103}{100}\right)^{4n}$

$\therefore \log_{\left(\frac{103}{100}\right)}\left(\frac{3}{2}\right) = 4n$

$\therefore 4n = 13.71$

$\therefore n = 3.43\ years\ or\ 3\ years\ and\ 6\ months$

7. $P = 150\,000$

$A = ?$

$i = 8.9\%\ p.a.$

$n = 4\ years$

$A = P\,(1 - i)^n$

$\therefore A = 150\,000(1 - 0.089)^n$

$\therefore A = 103\,315.33$

8. $P = ?$

$i = 11.8\% \div 12$

$x = 2\,800$

$n = 4 \times 12$

$P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$\therefore P_v = \frac{2\,800\left[1-\left(1+\frac{0.118}{12}\right)^{-n}\right]}{\frac{0.118}{12}}$

$\therefore P_v = 106\,724,63$

9. $F_v = ?$ $\quad F_v = \frac{x[(1+i)^n - 1]}{i}$

$i = 15\% \div 12$ $\quad \therefore F_v = \frac{1\,000\left[\left(1+\frac{0.15}{12}\right)^{35\times 12} - 1\right]}{\frac{0.15}{12}}$

$x = 1\,000$ $\quad \therefore F_v = 14\,677\,180.16$

$n = 35 \times 12$

10. a) $P = 15\,000$ $\quad A = P\,(1-i)^n$

$A = ?$ $\quad \therefore A = 15\,000(1-0.065)^7$

$i = 6.5\%\ p.a.$ $\quad \therefore A = 9\,370.71$

$n = 7\ years$

b) $P = 15\,000$ $\quad A = P\,(1+i)^n$

$A = ?$ $\quad \therefore A = 15\,000(1+0.078)^7$

$i = 7.8\%\ p.a.$ $\quad \therefore A = 25\,375.97$

c) $F_v = 25\,375.97 - 9\,370.71 = 16\,005.26$

$i = 9.4\% \div 12$ $\quad F_v = \frac{x[(1+i)^n - 1]}{i}$

$x = ?$ $\quad \therefore 16\,005.26 = \frac{x\left[\left(1+\frac{0.094}{12}\right)^{7\times 12} - 1\right]}{\frac{0.094}{12}}$

$n = 7 \times 12$ $\quad \therefore 125.3745367 = x\,[0.925982473]$

$\therefore x = 135.40$

11. $P_v = 1\,250\,000$ *with or without the R 30 000 deposit*

$i = 9.3\% \div 12$

$x = ?$

$n = 20 \times 12$

Without a deposit

$P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$\therefore 1\,250\,000 = \frac{x\left[1-\left(1+\frac{0.093}{12}\right)^{-20\times 12}\right]}{\frac{0.093}{12}}$

$\therefore 9\,687.5 = x\,[0.843207111]$

$\therefore x = 11\,488.87$ $\quad \therefore$ total paid $= x \times n \times 12$

$= 11\,488.87 \times 20 \times 12$

$= 2\,757\,328.80$

With a deposit

$P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$\therefore\ 1\ 220\ 000 = \frac{x\left[1-\left(1+\frac{0.093}{12}\right)^{-20\times12}\right]}{\frac{0.093}{12}}$

$\therefore 9\ 455 = x[0.843207111]$

$\therefore x = 11\ 213.14$ $\therefore$ total paid $= 11\ 213.14\ \times 20\ \times 12$

$= 2\ 691\ 153.60$

$\therefore$ Difference in total payments $= 2\ 757\ 328.80 - 2\ 691\ 153.60$

$= 66\ 175.20$

$\therefore$ Yes, it is worth it to pay the deposit because it will save them 36 175.20 in interest.

12. $P_v = ?$ $P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$i = 14.7\% \div 12$ $\therefore\ P_v = \frac{6\ 700\left[1-\left(1+\frac{0.147}{12}\right)^{-10\times12}\right]}{\frac{0.147}{12}}$

$x = 6\ 700$ $\therefore P_v = 420\ 055.48 + 50\ 000\ deposit.$

$n = 10\ \times 12$ $\therefore\ P_v = 470\ 055.48$

13. $\therefore$ the total paid to invest-me bank would be $= 6\ 700\ \times 10\ \times 12$

$= 804\ 000$

$P_v = 470\ 055.48$ $P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$i = 11.4\% \div 12$ $\therefore\ 470\ 055.48 = \frac{x\left[1-\left(1+\frac{0.114}{12}\right)^{-15\times12}\right]}{\frac{0.114}{12}}$

$x = ?$ $\therefore\ 4\ 465.52706 = x\ [0.817668456]$

$n = 15\ \times 12$ $\therefore x = 5\ 461.29$

$\therefore$ the total paid to invest-4-you bank would be $= 5\ 461.29\ \times 15\ \times 12$

$= 983\ 032.20$

$\therefore$ He should choose invest-me bank's offer because although he is paying lower premiums to invest-4-you bank, he would pay back more money in total.

14. First find the appreciated value of the fridges:

$P = 23\,800$ $\quad A = P\,(1+i)^n$

$A = ?$ $\quad \therefore A = 23\,800\,(1+0.076)^{10}$

$i = 7.6\%\ p.a.$ $\quad \therefore A = 49\,510.77$

$n = 10\ years$

Then find the depreciated value of the fridges:

$P = 23\,800$ $\quad A = P\,(1-i)^n$

$A = ?$ $\quad \therefore A = 23\,800\,(1-0.076)^{10}$

$i = 7.6\%\ p.a.$ $\quad \therefore A = 10\,796.84$

$n = 10\ years$

$\therefore Total\ needed\ for\ Future\ value\ fund$: $49\,510.77 - 10\,796.84$

$= 38\,713.93$

Now find the monthly instalment:

$F_v = 38\,713.93$ $\quad F_v = \frac{x[(1+i)^n-1]}{i}$

$i = 13\% \div 12$ $\quad \therefore 38\,713.93 = \frac{x\left[\left(1+\frac{0.13}{12}\right)^{10\times 12}-1\right]}{\frac{0.13}{12}}$

$x = ?$ $\quad \therefore 419.4009083 = x\,[2.643733272]$

$n = 10 \times 12$ $\quad \therefore x = 158.64$

15. $P_v = 120\,000$ $\quad P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$i = 6.8\% \div 12$ $\quad \therefore 120\,000 = \frac{3\,000\left[1-\left(1+\frac{0.068}{12}\right)^{-12n}\right]}{\frac{0.068}{12}}$

$x = 3\,000$ $\quad \therefore 680 = 3\,000\left[1-\left(\frac{3\,017}{3\,000}\right)^{-12n}\right]$

$n = n \times 12$ $\quad \therefore \frac{17}{75} = 1-\left(\frac{3\,017}{3\,000}\right)^{-12n}$

$\therefore -\frac{58}{75} = -\left(\frac{3\,017}{3\,000}\right)^{-12n}$

$\therefore \log_{\left(\frac{3\,017}{3\,000}\right)}\frac{58}{75} = -12n$

$\therefore -12n = -45.48930204$

$\therefore n = 3.79\ years\ or\ 3\ years\ and\ 10\ months$

16. $1 + i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m$

$1 + i_{eff} = \left(1+\frac{0.068}{4}\right)^4$

$i_{eff} = 0.0697 \approx 7\%$