Worksheet 2 – Memo

Straight Lines

1. a) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m = \frac{4-2}{1-(-2)}$

$\therefore m = \frac{2}{3}$

$\therefore y = mx + c$

$\therefore y = \frac{2}{3}x + c$ Substitute A (1; 4)

$\therefore 4 = \frac{2}{3}(1) + c$

$\therefore c = 3\frac{1}{3}$

$\therefore y = \frac{2}{3}x + 3\frac{1}{3}$

b) $y = \frac{2}{3}x + 3\frac{1}{3}$ x-intercept $\therefore y = 0$

$\therefore 0 = \frac{2}{3}x + 3\frac{1}{3}$

$\therefore -3\frac{1}{3} = \frac{2}{3}x$

$\therefore -10 = 2x$

$\therefore x = -5$

2. a) $f(x) = \frac{2}{3}x + 5$ and $h(x) = -\frac{3}{2}x - 6$

$\therefore \frac{2}{3}x + 5 = -\frac{3}{2}x - 6$

$\therefore 4x + 30 = -9x - 36$

$\therefore 13x = -66$

$\therefore x = -5\frac{1}{13}$

$\therefore y = -\frac{3}{2}\left(-5\frac{1}{13}\right) - 6$

$= 1\frac{8}{13}$

$\therefore \left(-5\frac{1}{13}; 1\frac{8}{13}\right)$

b) If gradient 1 times gradient 2 is equal to -1 then the lines are perpendicular. The lines are not parallel as the gradients are not equal.

$\therefore \frac{2}{3} \times -\frac{3}{2}$

$= -1$

$\therefore$ The lines are perpendicular.

3. a) $y = a(x-p)^2 + q$

$y = 1\left(x + \frac{9}{4}\right)^2 - 14\frac{1}{16}$

$y = x^2 + 4\frac{1}{2}x + 5\frac{1}{16} - 14\frac{1}{16}$

$y = x^2 + 4\frac{1}{2}x - 9$

$\therefore b = 4\frac{1}{2}$

$c = -9$

b) $\therefore y = (-x)^2 + 4\frac{1}{2}(-x) - 9$

$\therefore y = x^2 - 4\frac{1}{2}x - 9$

c) $y = \left(x + \frac{9}{4} - 2\frac{1}{4}\right)^2 - 14\frac{1}{16} + \frac{49}{16}$

$\therefore y = (x-0)^2 - 11$

$\therefore y = x^2 - 11$

d) $y = x^2 - 11 = 0$

$$x^2 = 11$$

$$x = \pm\sqrt{11}$$

$$\therefore x = -3.3 \quad or \quad x = 3.3$$

4. a) TP = (3; -2)

y-intercept: $y = -2(0-3)^2 - 2$

$$y = -20$$

x-intercept: $0 = -2(x-3)^2 - 2$

$$0 = -2(x^2 - 6x + 9) - 2$$

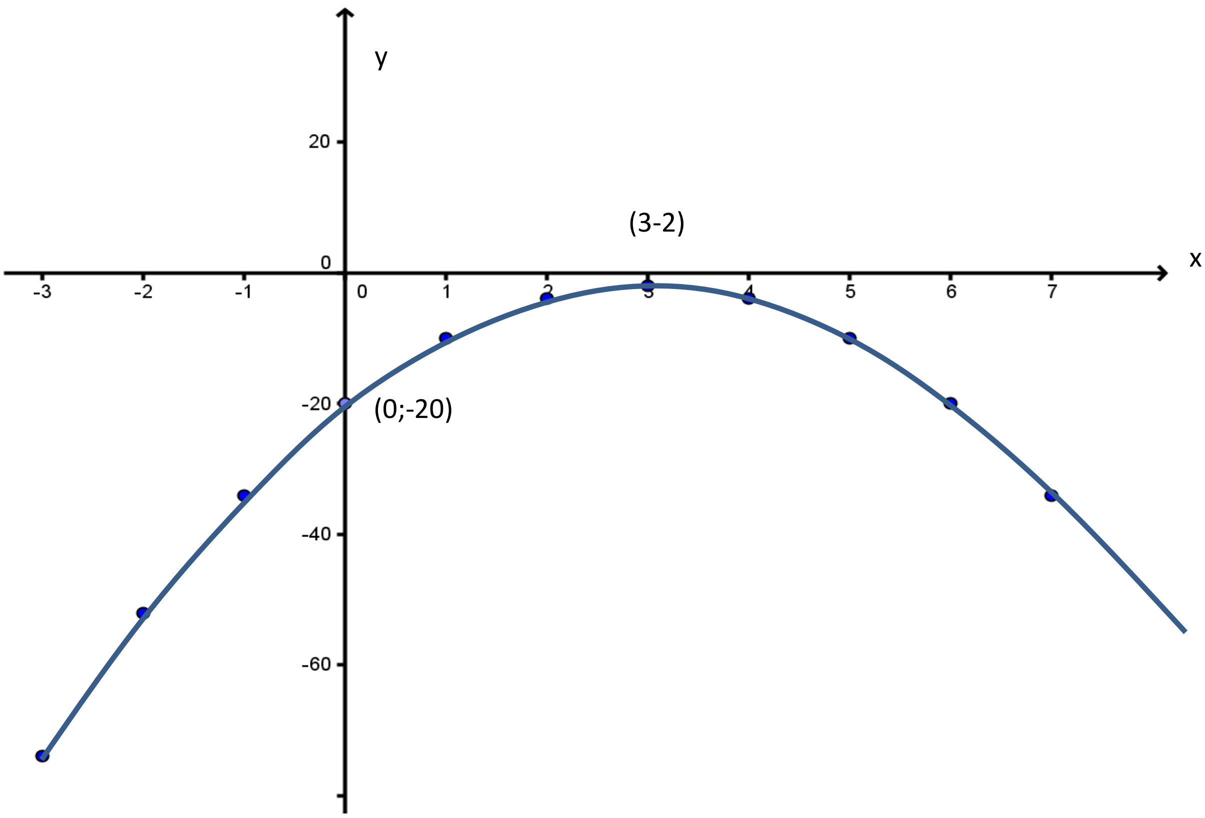
$$0 = -2x^2 + 12x - 18 - 2$$

$$0 = x^2 - 6x + 10$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

Syntax error – which means there are no x-intercepts.



b) range $y \in (-\infty;\ -2]$

c) $x \in R$

d) $x = -2(y-3)^2 - 2$

$\therefore x + 2 = -2(y-3)^2$

$\therefore -\frac{1}{2}x - 1 = (y-3)^2$

$\therefore y - 3 = \sqrt{-\frac{1}{2}x - 1}$

$\therefore y = 3 \pm \sqrt{-\frac{1}{2}x - 1}$

e) $f(x)$ is not a function as there is one x-value to many y-values.

$\therefore$ domain restriction: $x \leq 3 \quad or \quad x \geq 3$

5. a) $x = 2$ and $y = 1$

b) $y = \frac{a}{x-2} + 1$ (-1; 2)

$\therefore 2 = \frac{a}{-1-2} + 1$

$\therefore 1 = \frac{a}{-3}$

$\therefore a = -3$

c) y-intercept → $x = 0$

$y = \frac{-3}{0-2} + 1$

$\therefore y = \frac{-3}{-2} + 1$

$\therefore y = \frac{5}{2}$

$\left(0;\ \frac{5}{2}\right)$

x-intercept → $y = 0$

$0 = \frac{-3}{x-2} + 1$

$\therefore -1 = \frac{-3}{x-2}$

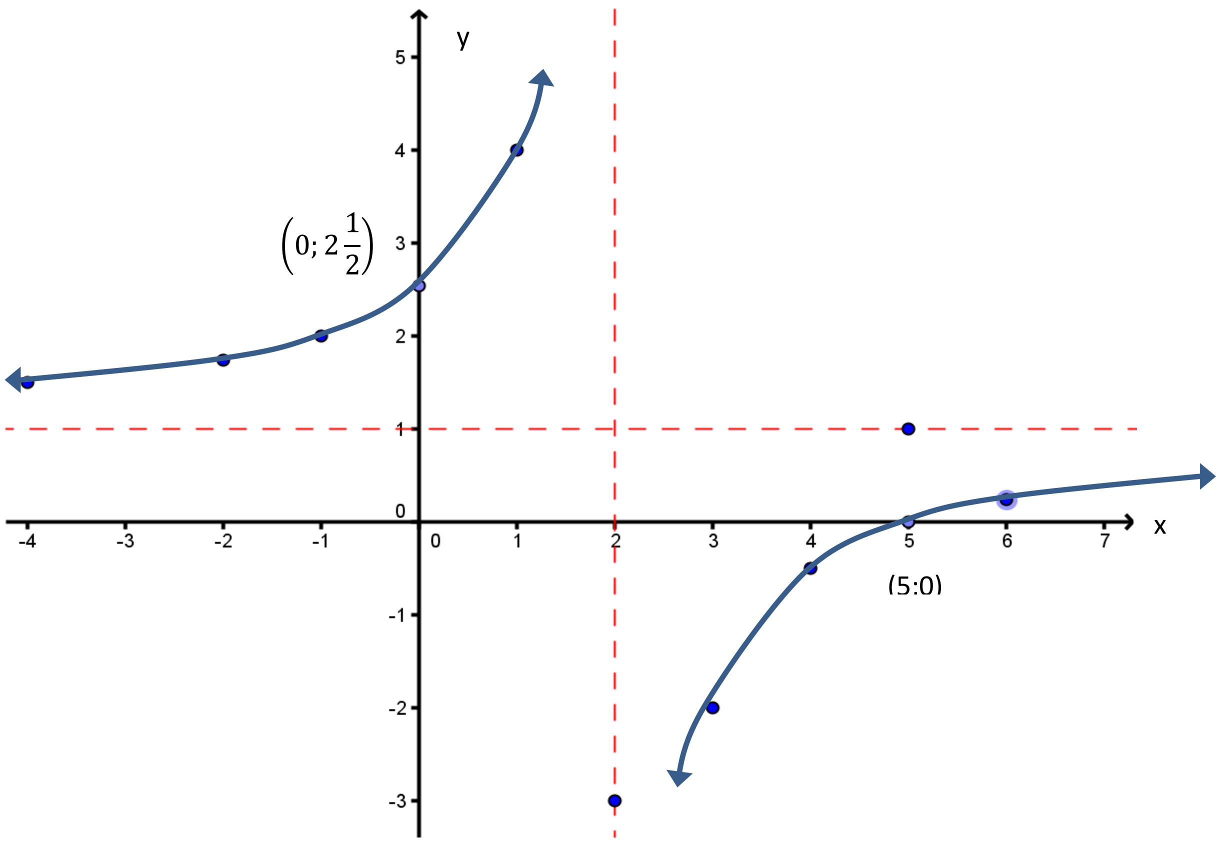
$\therefore -x + 2 = -3$

$\therefore -x = -5$

$\therefore x = 5$

$(5; 0)$

d)



e) $m = \pm 1$

(2;1) point where the asymptotes intercept

$y = 1x + c$

$1 = 1(2) + c$

$c = -1$

$\therefore y = x - 1$

$y = -x + c$

$1 = -1(2) + c$

$c = 3$

$\therefore y = -x + 3$

6. a) $x = -1$

$y = 3$

$\therefore f(x) = \frac{2}{x+1} + 3$

b) y-intercept → $x = 0$

$y = \frac{2}{0+1} + 3$

$y = 5$

$(0; 5)$

x-intercept → $y = 0$

$0 = \frac{2}{x+1} + 3$

$-3 = \frac{2}{x+1}$

$-3x - 3 = 2$

$-3x = 5$

$\therefore x = -\frac{5}{3} \quad or \; -1\frac{2}{3}$

$\left(-\frac{5}{3}; 0\right) \quad or \quad \left(-1\frac{2}{3}; 0\right)$

c) $(0; 5) \qquad \left(-\frac{5}{3}; 0\right)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{5-0}{0-\left(-\frac{5}{3}\right)}$

$\therefore m = 3$

$\therefore y = 3x + c$

$\therefore y = 3x + 5$

d) $m = \pm 1$

(-1; 3) where asymptotes intercept

$y = +1x + c$

$3 = +1(-1) + c$

$c = 4$

$\therefore y = x + 4$

$y = -1x + c$

$3 = -1(-1) + c$

$c = 2$

$y = -x + 2$

e) $y = -2x + c$

(-2; 1)

$1 = -2(-2) + c$

$c = -3$

$\therefore y = -2x - 3$

$-2x - 3 = \frac{2}{x+1} + 3$

$-2x - 6 = \frac{2}{x+1}$

$(-2x - 6)(x + 1) = 2$
$-2x^2 - 2x - 6x - 6 - 2 = 0$
$x^2 + 4x + 4 = 0$
$(x + 2)(x + 2) = 0$
$\therefore x = -2$
$\therefore$ there is no other point of intersection

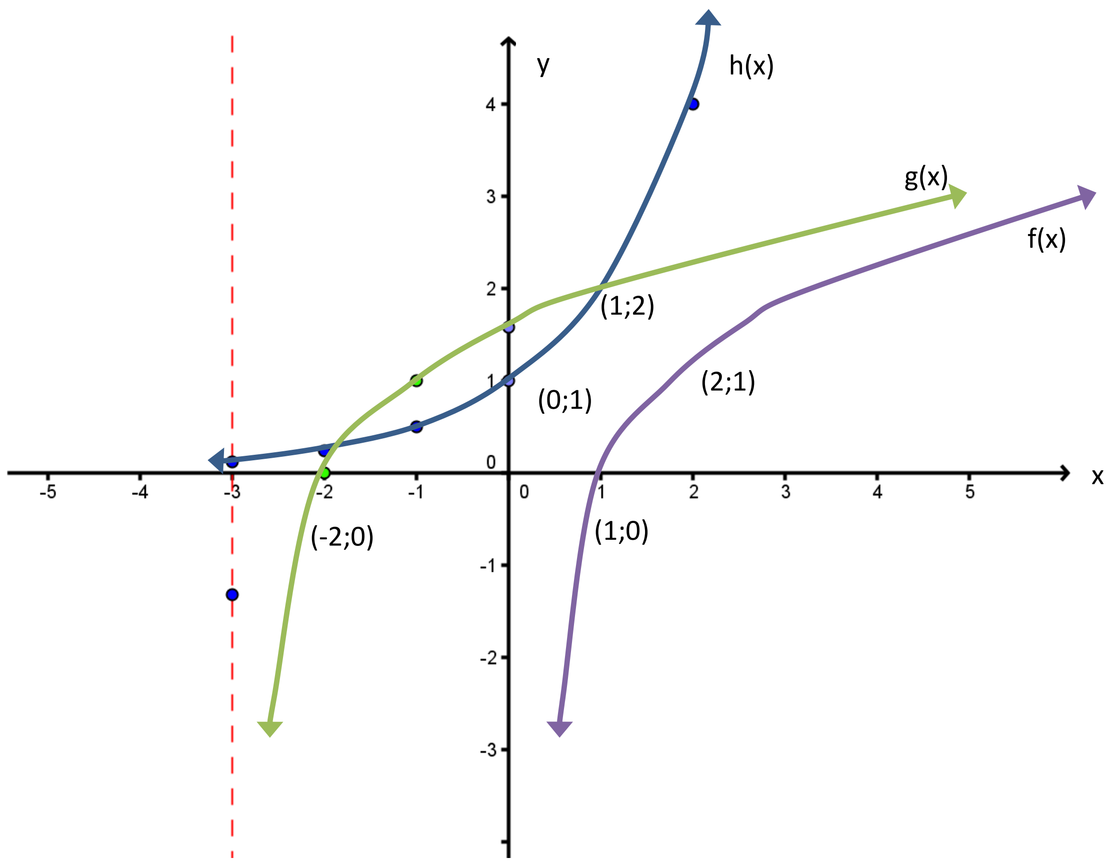
7. a) $k(x) = a^x$ (-1; 3)
$\therefore 3 = a^{-1}$
$\therefore a = \frac{1}{3}$

b) $y = \left(\frac{1}{3}\right)^x$
$\therefore x = \left(\frac{1}{3}\right)^y$
$\therefore y = \log_{\left(\frac{1}{3}\right)} x$ or $j(x) = \log_{\left(\frac{1}{3}\right)} x$

c) domain $x < 0$
range $y \in R$

d) anything to the power of zero is one

8. a)



b) (1;2) and approximately (-1.9; 0.30)

c) $y = 2^x$

$x = 2^y$

$\therefore f(x) = \log_2 x$

d) purple line on graph above.

e) $x \in R$

f) $m(x) = h(x+2)$

$m(x) = 2^{x+2}$

$m(x) = 2^x 2^2$

$\therefore m(x) = 4.2^x$

Mixed Questions

9. a) $f(x) = 2x^2 + bx + c$ (-4; 38) (3;25)

$\therefore 38 = 2(-4)^2 + b(-4) + c$

$\therefore 38 = 32 - 4b + c$

$\therefore 6 = -4b + c$

$\therefore c = 6 + 4b$

$-25 = 2(3)^2 + b(3) + c$

$\therefore -25 = 18 + 3b + c$

$\therefore -43 - 3b = c$

Solve Simultaneously

$\therefore 6 + 4b = -43b - 3b$

$-49 = 7b$

$\therefore b = -7$

$c = 6 + 4(-7)$

$\therefore c = -22$

b) $j(x) = 3x + c$

$f(x) = 2x^2 - 7x - 22$

$f(0) = -22$

$\therefore j(x) = 3x - 22$

c) x-intercepts → $y = 0$

$0 = 3x - 22$

$22 = 3x$

$\therefore x = 7\frac{1}{3}$

$\left(7\frac{1}{3}; 0\right)$

$0 = 2x^2 - 7x - 22$

$0 = (2x - 11)(x + 2)$

$\therefore x = \frac{11}{2} \quad or \quad x = -2$

$\left(\frac{11}{2}; 0\right) \quad or \quad (-2; 0)$

d) $3x - 22 = 2x^2 - 7x - 22$

$\therefore 0 = 2x^2 - 7x - 3x - 22 + 22$

$\therefore 0 = 2x^2 - 10x$

$\therefore 0 = x(2x - 10)$

$\therefore x = 0 \quad or \quad x = 5$

$\therefore y = 3(5) - 22$

$\therefore y = -7$

$(5; -7)$

e) Perpendicular to $j(x)$

$\therefore m_1 \times 3 = -1$

$\therefore m_2 = -\frac{1}{3}$

There are two x-intercepts $\therefore$ two possible graphs using these two intercepts

$\therefore y = -\frac{1}{3}x + c$

$\therefore 0 = -\frac{1}{3}\left(\frac{11}{2}\right) + c$ $\qquad$ $0 = -\frac{1}{3}(-2) + c$

$\therefore c = \frac{11}{6}$ $\qquad$ $\therefore c = -\frac{2}{3}$

$\therefore y = -\frac{1}{3}x + \frac{11}{6}$ OR $y = -\frac{1}{3}x - \frac{2}{3}$

10. a) $h(x) = \frac{a}{x} + q$ (-1; 3)

$\therefore 3 = \frac{a}{-1} + 2$

$\therefore 1 = \frac{a}{-1}$

$\therefore a = -1$ $\qquad$ $\therefore h(x) = \frac{-1}{x} + 2$

$g(x) = x + b$

$\therefore 3 = -1 + b$

$\therefore b = 4$ $\qquad$ $\therefore g(x) = x + 4$

b) $x = y + 4$

$\therefore y = x - 4$

$\therefore$ Yes, the inverse of $g(x)$ is a function.

c) y-intercepts → $x = 0$

$g(x)$ $\therefore y = 0 + 4$

$\therefore y = 4$

$(0; 4)$

$h(x)$ $y = \frac{-1}{0} + 2$

$\therefore y = undefined$

$\therefore x = 0$ is an asymptote.

x-intercepts → $y = 0$

$0 = x + 4$

$\therefore x = -4$

$(-4; 0)$

$0 = \frac{-1}{x} + 2$

$\therefore -2 = \frac{-1}{x}$

$\therefore -2x = -1$

$\therefore x = \frac{1}{2}$

$\left(\frac{1}{2}; 0\right)$

d) $m = \pm 1$

Asymptotes intercept at (0; 2)

$y = x + c$

$\therefore 2 = 0 + c$

$\therefore c = 2$

$\therefore y = x + 2$

$y = -x + c$

$\therefore 2 = 0 + c$

$\therefore c = 2$

$\therefore y = -x + 2$

e) $h(x) = f(x)$

$\therefore \frac{-1}{x} + 2 = 4x + 7$

$\therefore \frac{-1}{x} = 4x + 5$

$\therefore -1 = 4x^2 + 5x$

$\therefore 0 = 4x^2 + 5x + 1$

$\therefore 0 = (4x + 1)(x + 1)$

$\therefore x = -\frac{1}{4} \quad or \quad x = -1$

$\therefore y = 4\left(-\frac{1}{4}\right) + 7$ $\qquad$ $y = 4(-1) + 7$

$\therefore y = 6$ $\qquad$ $y = 3$

$\left(-\frac{1}{4}; 6\right)$ $\qquad$ $(-1; 3)$

11. a) $A(x) = 2(x + 2)(x - 7)$

$= 2(x^2 - 5x - 14)$

$= 2x^2 - 10x - 28$

$\therefore a = -10$ and $b = -28$

b) y-intercept → $x = 0$

$y = 2(0)^2 - 10(0) - 28$

$\therefore y = -28$ $\qquad$ $(0; -28)$

x-intercepts → $y = 0$

$0 = 2x^2 - 10x - 28$

$\therefore 0 = x^2 - 5x - 14$

$\therefore 0 = (x + 2)(x - 7)$

$\therefore x = -2$ or $x = 7$

$(-2; 0)$ $\qquad$ $(7; 0)$

c) $y = 2x^2 - 10x - 28$

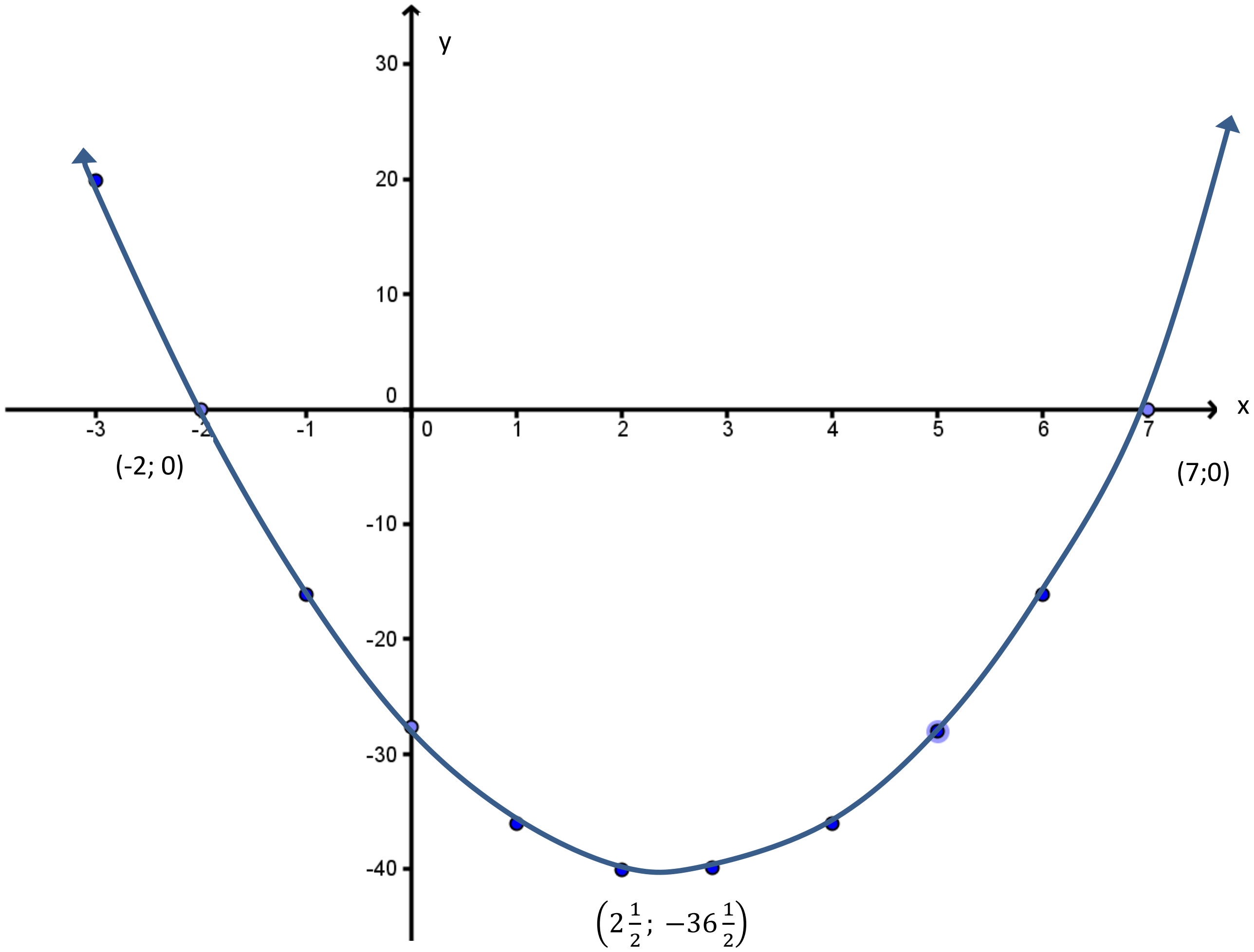
$y = 2(x^2 - 5x) - 28$

$y = 2\left(x^2 - 5x + \frac{25}{4}\right) - 28 - \frac{25}{2}$

$y = 2\left(x - \frac{5}{2}\right)^2 - \frac{81}{2}$

$\therefore$ Turning Point → $\left(\frac{5}{2}; \frac{81}{2}\right)$

d)



e) i) $A(x) = 3(x - 7)$

$\therefore A(x) = 3x - 21$

ii) $3x - 21 = 2x^2 - 10x - 28$

$\therefore 0 = 2x^2 - 10x - 3x - 28 + 21$

$\therefore 0 = 2x^2 - 13x - 7$

$\therefore 0 = (2x + 1)(x - 7)$

$\therefore x = -\frac{1}{2} \quad or \quad x = 7$

$\therefore y = 3\left(-\frac{1}{2}\right) - 21$ $\quad\quad$ $\therefore y = 3(7) - 21$

$\therefore y = -22\frac{1}{2}$ $\quad\quad$ $\therefore y = 0$

$\left(-\frac{1}{2};\ -22\frac{1}{2}\right)$ $\quad\quad$ $(7; 0)$