SHARP

Worksheet 8 Memo – Analytical Geometry

1. a) $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AC} = \frac{-2-2}{0+3}$

$\therefore m_{AC} = -\frac{4}{3}$

$\therefore y = -\frac{4}{3}x + c$ A (-3; 2)

$\therefore 2 = -\frac{4}{3}(-3) + c$

$\therefore c = -2$

$\therefore y = -\frac{4}{3}x - 2$

b) $r^2 = (x-a)^2 + (y-b)^2$

$r^2 = (x+3)^2 + (y-2)^2$ $r = 3$

$\therefore 9 = (x+3)^2 + (y-2)^2$

c) $\therefore 9 = (x+3)^2 + (y-2)^2$ $\therefore y = -\frac{4}{3}x - 2$

$\therefore 9 = (x+3)^2 + \left(-\frac{4}{3}x - 2 - 2\right)^2$

$\therefore 9 = x^2 + 6x + 9 + \left(-\frac{4}{3}x - 4\right)^2$

$\therefore 0 = x^2 + 6x + \frac{16}{9}x^2 + \frac{32}{3}x + 16$

$\therefore 0 = 9x^2 + 54x + 16x^2 + 96x + 144$

$\therefore 0 = 25x^2 + 150x + 144$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-150 \pm \sqrt{(150)^2 - 4(25)(144)}}{2(25)}$

$\therefore x = -\frac{6}{5}$ OR $x = -4\frac{4}{5}$

$\therefore y = -\frac{4}{3}\left(-4\frac{4}{5}\right) - 2$
$\therefore y = 4\frac{2}{5}$

$\therefore D \left(-4\frac{4}{5}; 4\frac{2}{5}\right)$

d) $9 = (x+3)^2 + (y-2)^2$ $\quad$ x-intercept → make $y = 0$

$\therefore 9 = (x+3)^2 + (0-2)^2$
$\therefore 9 = (x+3)^2 + 4$
$\therefore 5 = (x+3)^2$

$\therefore x + 3 = -\sqrt{5}$ $\quad$ OR $\quad$ $x + 3 = \sqrt{5}$
$\therefore x = -3 - \sqrt{5}$ $\quad$ $x = -3 + \sqrt{5}$
$\therefore x = -0{,}76$ $\quad$ $x = -5{,}24$

$\therefore E\ (-5{,}24; 0)$

e) $m_{AC} = -\frac{4}{3}$

$m_{AE} = \frac{y_2 - y_1}{x_2 - x_1}$
$\therefore m_{AE} = \frac{2-0}{-3+5{,}24}$
$\therefore m_{AE} = \frac{2\sqrt{5}}{5}$

$\therefore m_{tang} = -\frac{\sqrt{5}}{2}$

∴ AC is not parallel to the tangent at E.

f) $y = -\frac{\sqrt{5}}{2}x + c$
$\therefore 0 = -\frac{\sqrt{5}}{2}(-5{,}24) + c$
$\therefore c = -5{,}86$

$\therefore y = -\frac{\sqrt{5}}{2}x - 5{,}86$

2. a) $M_{AE} = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$\therefore M_{AE} = \left(\frac{-1+3}{2}; \frac{5-3}{2}\right)$

$\therefore D = (1; 1)$

b) $r^2 = (x-a)^2 + (y-b)^2$

$r^2 = (x-1)^2 + (y-1)^2$ Substitute E(3; -3)

$\therefore r^2 = (3-1)^2 + (-3-1)^2$

$\therefore r^2 = 20$

$\therefore 20 = (x-1)^2 + (y-1)^2$

c) $m_{AE} = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{AE} = \frac{5+3}{-1-3}$

$\therefore m_{AE} = -2$

$\therefore y = -2x + c$ Substitute E (3; -3)

$\therefore -3 = -2(3) + c$

$\therefore c = 3$

$\therefore y = -2x + 3$

d) $m_{BD} = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{BD} = \frac{-2-1}{-3-1}$ B (-3; -2) D (1; 1)

$\therefore m_{BD} = \frac{3}{4}$

$\therefore \frac{3}{4} \times -2 = -1\frac{1}{2}$

∴ BD is not ⊥ to ADE

e) $D_{BC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ C (3; 0)

$\therefore D_{BC} = \sqrt{(-3-3)^2 + (-2-0)^2}$

$\therefore D_{BC} = 2\sqrt{10}$

$\therefore D_{BC} = 6{,}32$

f) $m_{AD} = -2$

$\therefore m_{tang} = \frac{1}{2}$

$\therefore y = \frac{1}{2}x + c$ Substitute A (-1; 5)

$\therefore 5 = \frac{1}{2}(-1) + c$

$\therefore c = 5\frac{1}{2}$

$\therefore y = \frac{1}{2}x + 5\frac{1}{2}$

g) $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{BC} = \frac{0+2}{3+3}$

$\therefore m_{BC} = \frac{1}{3}$

$\therefore \tan\theta = \frac{1}{3}$

$\therefore \theta = 18{,}43°$

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AC} = \frac{0-5}{3+1}$

$\therefore m_{AC} = -\frac{5}{4}$

$\therefore \tan\theta = \frac{5}{4}$

$\therefore \theta = 51{,}34°$

$\therefore \theta = 180° - 51{,}34°$

$\therefore \theta = 128{,}66°$

$\therefore B\hat{C}A = 128{,}66° - 18{,}43°$

$\therefore B\hat{C}A = 110{,}23°$

h) x-intercepts → make $y = 0$

$\therefore 20 = (x-1)^2 + (0-1)^2$
$\therefore 20 = (x-1)^2 + 1$
$\therefore 19 = (x-1)^2$

$\therefore x - 1 = -\sqrt{19}$ OR $x - 1 = \sqrt{19}$
$\therefore x = 1 - \sqrt{19}$ $x = 1 + \sqrt{19}$
$\therefore x = -3{,}36$ $x = 5{,}36$

y-intercepts → make $x = 0$

$\therefore 20 = (0-1)^2 + (y-1)^2$
$\therefore 20 = 1 + (y-1)^2$
$\therefore 19 = (y-1)^2$

You can skip the working out because the equation is the same as the -intercepts answer.

$\therefore y = -3{,}36$ OR $y = 5{,}36$

3. a) $r^2 = (x-a)^2 + (y-b)^2$
$\therefore r^2 = (x-2)^2 + (y-1)^2$ D (5; 0)
$\therefore r^2 = (5-2)^2 + (0-1)^2$
$\therefore r^2 = 10$

$\therefore 10 = (x-2)^2 + (y-1)^2$

b) Isosceles → show that only two sides are equal.

$\therefore D_{AB} = 5$ (straight line (same x-coordinates) → so we don't need to use the formula.)
$\therefore D_{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\therefore D_{BC} = \sqrt{(-2-2)^2 + (-2-1)^2}$
$\therefore D_{BC} = 5$

$\therefore D_{AC} = \sqrt{(2+2)^2 + (1-3)^2}$
$\therefore D_{AC} = 2\sqrt{5}$
$\therefore \Delta$ ABC is an isosceles triangle as AB = BC

c) $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AC} = \frac{3-1}{-2-2}$

$\therefore m_{AC} = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}x + c$

$\therefore 3 = -\frac{1}{2}(-2) + c$

$\therefore c = 2$

$\therefore y = -\frac{1}{2}x + 2$

$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{BC} = \frac{1+2}{2+2}$

$\therefore m_{BC} = \frac{3}{4}$

$\therefore y = \frac{3}{4}x + c$ Substitute B (-2; -2)

$\therefore -2 = \frac{3}{4}(-2) + c$

$\therefore c = -\frac{1}{2}$

$\therefore y = \frac{3}{4}x - \frac{1}{2}$

d) $m_{BC} \neq m_{AC} \quad \therefore not \ \parallel$

$-\frac{1}{2} \times \frac{3}{4} = -\frac{3}{8}$

$\therefore BC \ is \ not \ \perp \ to \ AC$

∴ the lines are not ∥ nor ⊥ .

e) Line CG is ⊥ to the x-axis.

$\therefore m_{CG} = undefined$

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AB} = \frac{3+2}{-2+2}$

$\therefore m_{AB} = undefined$

$\therefore CG \parallel AB$

f) $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AC} = \frac{3-1}{-2-2}$

$\therefore m_{AC} = -\frac{1}{2}$

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{CD} = \frac{0-1}{5-2}$

$\therefore m_{CD} = -\frac{1}{3}$

∴ Points A, C and D are not collinear.

g) $y_{BC} = \frac{3}{4}x - \frac{1}{2}$ $\qquad$ $y_{AC} = -\frac{1}{2}x + 2$

$10 = (x-2)^2 + (y-1)^2$

E → $10 = (x-2)^2 + \left(-\frac{1}{2}x + 2 - 1\right)^2$

$\therefore 10 = x^2 - 4x + 4 + \left(-\frac{1}{2}x + 1\right)^2$

$\therefore 0 = x^2 - 4x - 6 + \frac{1}{4}x^2 - 1x + 1$

$\therefore 0 = 4x^2 - 16x - 24 + x^2 - 4x + 4$

$\therefore 0 = 5x^2 - 20x - 20$

$\therefore 0 = x^2 - 4x - 4$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2(1)}$

$\therefore x = 4{,}83$ OR $x = -0{,}83$

$\therefore x_E = -0{,}83$

$\therefore y = -\frac{1}{2}(-0{,}83) + 2$

$\therefore y = 2{,}41$

$\therefore E\ (-0{,}83; 2{,}41)$

F → $10 = (x-2)^2 + \left(\frac{3}{4}x - \frac{1}{2} - 1\right)^2$

$\therefore 10 = x^2 - 4x + 4 + \left(\frac{3}{4}x - \frac{3}{2}\right)^2$

$\therefore 0 = x^2 - 4x - 6 + \frac{9}{16}x^2 - \frac{9}{4}x + \frac{9}{4}$

$\therefore 0 = 16x^2 - 64x - 96 + 9x^2 - 36x + 36$

$\therefore 0 = 25x^2 - 100x - 60$

$\therefore 0 = 5x^2 - 20x - 12$

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\therefore x = \frac{20 \pm \sqrt{(-20)^2 - 4(5)(-12)}}{2(5)}$

$\therefore x = 4{,}53$ OR $x = -0{,}53$

$\therefore x_F = -0{,}53$

$\therefore y = \frac{3}{4}(-0{,}53) - \frac{1}{2}$

$\therefore y = -0{,}897$

$\therefore y = -0{,}9$

$\therefore F(-0{,}53;\ -0{,}9)$

4. a) $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AB} = \frac{3-b}{4-7}$

$\therefore m_{AB} = \frac{3-b}{-3}$

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AC} = \frac{3+1}{4-0}$

$\therefore m_{AC} = \frac{4}{4}$

$\therefore m_{AC} = 1$

$\therefore m_{AB} \times m_{AC} = -1$

$\therefore \frac{3-b}{-3} \times 1 = -1$

$\therefore 3 - b = 3$

$\therefore b = 0$

b) $m_{AB} = \frac{3-b}{-3}$

$m_{BC} = \frac{0-b}{-1-7}$

$\therefore m_{AB} = m_{BC}$

$\therefore \frac{3-b}{-3} = \frac{b}{8}$

$\therefore 24 - 8b = -3b$

$\therefore 24 = 5b$

$\therefore b = \frac{24}{5}$ or $4\frac{4}{5}$

5. a) $m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{CD} = \frac{4+1}{3-6}$

$\therefore m_{CD} = -\frac{5}{3} = m_{AB}$

$\therefore y = -\frac{5}{3}x + c$ Substitute B (-3; 2)

$\therefore 2 = -\frac{5}{3}(-3) + c$

$\therefore c = -3$

$\therefore y = -\frac{5}{3}x - 3$

$\therefore x$-intercept → make $y = 0$

$\therefore 0 = -\frac{5}{3}x - 3$

$\therefore 3 = -\frac{5}{3}x$

$\therefore 9 = -5x$

$\therefore x = -\frac{9}{5} \quad or \quad -1{,}8$

$\therefore A\,(-1{,}8; 0)$

b) $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{BC} = \frac{2-4}{-3-3}$

$\therefore m_{BC} = \frac{1}{3}$

$\therefore y = \frac{1}{3}x + c$ Substitute B (-3; 2)

$\therefore 2 = \frac{1}{3}(-3) + c$

$\therefore c = 3$

$\therefore y = \frac{1}{3}x + 3$

c) $M_E = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$\therefore M_E = \left(\frac{3+6}{2}; \frac{4-1}{2}\right)$

$\therefore E = \left(4\frac{1}{2}; 1\frac{1}{2}\right)$

d) $m_{AE} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{AE} = \frac{0 - 1\frac{1}{2}}{-1{,}8 - 4\frac{1}{2}}$

$\therefore m_{AE} = \frac{5}{21}$

$m_{BC} \neq m_{AE}$ $\therefore$ they are not ∥

e) $m_{BC} = \frac{1}{3}$

$\therefore y = \frac{1}{3}x + c$

$\therefore 0 = \frac{1}{3}(-1{,}8) + c$

$\therefore c = \frac{3}{5}$

$\therefore y = \frac{1}{3}x + \frac{3}{5}$

f) $m_{CD} = -\frac{5}{3}$

$\therefore y = -\frac{5}{3}x + c$ Substitute C (3; 4)

$\therefore 4 = -\frac{5}{3}(3) + c$

$\therefore c = 9$

$\therefore y = -\frac{5}{3}x + 9$ and $y = \frac{1}{3}x + \frac{3}{5}$

$\therefore \frac{1}{3}x + \frac{3}{5} = -\frac{5}{3}x + 9$

$\therefore x + \frac{9}{5} = -5x + 27$

$\therefore 6x = 25\frac{1}{5}$

$\therefore x = 4\frac{1}{5}$

$\therefore y = \frac{1}{3}\left(4\frac{1}{5}\right) + \frac{3}{5}$

$\therefore y = 2$

$\therefore F\ \left(4\frac{1}{5}; 2\right)$

g) $m_{CD} = -\frac{5}{3}$

$\therefore \tan\theta = \frac{5}{3}$
$\therefore \theta = 59{,}04°$

$\therefore$ angle of inclination $= 180° - 59{,}04°$
$= 120{,}96°$

h) $D_{AF} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$\therefore D_{AF} = \sqrt{\left(-1{,}8 - 4\frac{1}{5}\right)^2 + (0-2)^2}$
$\therefore D_{AF} = 2\sqrt{10} = 6{,}32$

$\therefore D_{FD} = \sqrt{\left(4\frac{1}{5} - 6\right)^2 + (2+1)^2}$
$\therefore D_{FD} = 3{,}5$

$\therefore Area\ \Delta\ AFD = \frac{1}{2}\ b\ \times h$
$= \frac{1}{2} \times 3{,}5 \times 6{,}32$
$= 11{,}06\ units^2$

6. a) $x^2 - 8x + y^2 - 6y = 0$
$\therefore (x-4)^2 + (y-3)^2 = 0 + 16 + 9$
$\therefore (x-4)^2 + (y-3)^2 = 25$

$\therefore A\ (4;3)$

b) y-intercepts → make $x = 0$

$\therefore (0-4)^2 + (y-3)^2 = 25$
$\therefore 16 + (y-3)^2 = 25$
$\therefore (y-3)^2 = 9$

$\therefore y - 3 = -3$ OR $y - 3 = 3$
$\therefore y = 0$ $y = 6$
$\therefore C(0;0)$ $\therefore B\ (0;6)$

x-intercepts → make $y = 0$

$\therefore (x-4)^2 + (0-3)^2 = 25$
$\therefore (x-4)^2 + 9 = 25$
$\therefore (x-4)^2 = 16$

$\therefore x - 4 = -4$ OR $x - 4 = 4$
$\therefore x = 0$ $x = 8$
$\therefore C\ (0; 0)$ $\therefore D\ (8; 0)$

c) A (4; 3) E $\perp$ *to the* $x - axis$ $\therefore$ E $(4; e)$

$\therefore (4-4)^2 + (e-3)^2 = 25$
$\therefore 0 + (e-3)^2 = 25$

$\therefore e - 3 = -5$ OR $e - 3 = 5$
$\therefore e = -2$ $e = 8$

$\therefore E\ (4;\ -2)$

To find this coordinate or decide which values are E look at the sketch and work out whether E is above (positive value) or below (negative value) the y-axis

d) B (0; 6)
D (8; 0)

$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$
$\therefore m_{BD} = \frac{6-0}{0-8}$
$\therefore m_{BD} = -\frac{3}{4}$

$\therefore y = -\frac{3}{4}x + c$ Substitute D (8; 0)
$\therefore 0 = -\frac{3}{4}(8) + c$
$\therefore c = 6$

$\therefore y = -\frac{3}{4}x + 6$

e) $\perp$ to BD $\therefore m_E = \frac{4}{3}$

$\therefore y = \frac{4}{3}x + c$ Substitute E (4; -2)

$\therefore \ -2 = \frac{4}{3}(4) + c$

$\therefore c = \ -7\frac{1}{3}$

$\therefore y = \frac{4}{3}x - 7\frac{1}{3}$

f) $m_{rad} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{rad} = \frac{3+2}{4-4}$

$\therefore m_{rad} = undefined$

$\therefore m_{tang} = 0$

$\therefore y = \ -2$

g) $m_{tang} = 0$

$m_E = \frac{4}{3}$

$\therefore not \ \parallel$

h) $m_{ED} = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{ED} = \frac{-2-0}{4-8}$

$\therefore m_{ED} = \frac{1}{2}$ $m_{BD} = \ -\frac{3}{4}$

$\therefore \frac{1}{2} \times -\frac{3}{4} = \ -\frac{3}{8}$

$\therefore not \ \perp$

i) $m_{BD} = \ -\frac{3}{4}$

$\therefore \tan\theta = \frac{3}{4}$

$\therefore \theta = 36,87°$

$\therefore$ angle of inclination $= 180° - 36{,}87°$

$= 143,13°$