SHARP

Worksheet 12 – Solve for x

1. *Solve for x by factorising the following:*

a) $x^2 + 3x - 18 = 0$
$(x + 6)(x - 3) = 0$
$\therefore x = -6 \quad or \quad x = 3$

b) $x^2 - 10x - 56 = 0$
$(x + 4)(x - 14) = 0$
$\therefore x = -4 \quad or \quad x = 14$

c) $x^2 - 12x + 32 = 0$
$(x - 4)(x - 8) = 0$
$\therefore x = 4 \quad or \quad x = 8$

d) $x^2 + 22x + 85 = 0$
$(x + 5)(x + 17) = 0$
$\therefore x = -5 \quad or \quad x = -17$

e) $x^2 - 9 = 0$
$(x - 3)(x + 3) = 0$
$\therefore x = 3 \quad or \quad x = -3$

f) $x^2 + 8 = 0$
not able to factorise – be careful!

g) $2x^2 - 33x + 45 = 0$
$(2x - 3)(x - 15) = 0$
$\therefore x = \frac{3}{2} \quad or \quad x = 15$

h) $18x^2 - 3x - 10 = 0$
$(6x - 5)(3x + 2) = 0$
$\therefore x = \frac{5}{6} \quad or \quad x = -\frac{2}{3}$

i) $6x^2 + 19x + 3$
$(6x + 1)(x + 3) = 0$
$\therefore x = -\frac{1}{6} \quad or \quad x = -3$

j) $4x^2 + 8x - 5 = 0$
$(2x - 1)(2x + 5) = 0$
$\therefore x = \frac{1}{2} \quad or \quad x = -\frac{5}{2}$

k) $x^2 - 6x - 91 = 0$
$(x - 13)(x + 7) = 0$
$\therefore x = 13 \quad or \quad x = -7$

l) $x^2 - 9x + 20 = 0$
$(x - 4)(x - 5) = 0$
$\therefore x = 4 \quad or \quad x = 5$

m) $3x^2 + 18x - 81 = 0$
$x^2 + 6x - 27 = 0$
$(x - 3)(x + 9) = 0$
$\therefore x = 3 \quad or \quad x = -9$

n) $\frac{1}{2}x^2 + \frac{7}{6}x - 1 = 0$
$3x^2 + 7x - 6 = 0$
$(3x - 2)(x + 3) = 0$
$\therefore x = \frac{2}{3} \quad or \quad x = -3$

2. *Solve for x by completing the square:*

a) $x^2 + 9x - \frac{1}{5} = 0$

$x^2 + 9x + \left(\frac{9}{2} \div 2\right)^2 = \frac{1}{5} + \left(\frac{9}{2} \div 2\right)^2$

$\left(x + \frac{9}{2}\right)^2 = 20\frac{9}{20}$

$\therefore x + \frac{9}{2} = 4{,}522\ldots$ OR $x + \frac{9}{2} = -4{,}522$

$\therefore x = 0{,}022$ $x = -9{,}022$

b) $2x^2 + x - 8 = 0$

$x^2 + \frac{1}{2}x - 4 = 0$

$x^2 + \frac{1}{2}x + \frac{1}{16} = 4 + \frac{1}{16}$

$\left(x + \frac{1}{4}\right)^2 = \frac{65}{16}$

$\therefore x + \frac{1}{4} = \frac{\sqrt{65}}{4}$ OR $x + \frac{1}{4} = -\frac{\sqrt{65}}{4}$

$\therefore x = \frac{-1+\sqrt{65}}{4}$ $x = \frac{-1-\sqrt{65}}{4}$

$\therefore x \approx 1{,}766$ $x \approx -2{,}266$

c) $-2x^2 + 2x + \frac{2}{3} = 0$

$x^2 + x - \frac{1}{3} = 0$

$\left(x + \frac{1}{2}\right)^2 = \frac{1}{3} + \frac{1}{4}$

$\left(x + \frac{1}{2}\right)^2 = \frac{7}{12}$

$\therefore x + \frac{1}{2} = \frac{\sqrt{21}}{6}$ OR $x + \frac{1}{2} = \frac{-\sqrt{21}}{6}$

$\therefore x = \frac{-3+\sqrt{21}}{6}$ $x = \frac{-3-\sqrt{21}}{6}$

$\therefore x = 0{,}264$ $x = -1{,}264$

d) $x^2 - 6x - 39 = 0$

$(x-3)^2 = 39 + 9$

$(x-3)^2 = 48$

$\therefore x - 3 = 4\sqrt{3}$ OR $x - 3 = -4\sqrt{3}$

$\therefore x = 3 + 4\sqrt{3}$ $x = 3 - 4\sqrt{3}$

$\therefore x = 9{,}928$ $x = -3{,}928$

e) $-3x^2 - 6x + 4 = 0$

$x^2 + 2x - \frac{4}{3} = 0$

$(x+1)^2 = \frac{4}{3} + 1$

$(x+1)^2 = \frac{7}{3}$

$\therefore x + 1 = \frac{\sqrt{21}}{3}$ OR $x + 1 = \frac{-\sqrt{21}}{3}$

$\therefore x = \frac{-3+\sqrt{21}}{3}$ $x = \frac{-3-\sqrt{21}}{3}$

$\therefore x = 0{,}528$ $x = -2{,}528$

f) $-\frac{3}{4}x^2 + \frac{375}{4} = 0$

$x^2 - 125 = 0$

$x^2 = 125$

$\therefore x = 5\sqrt{5}$ OR $x = -5\sqrt{5}$

$\therefore x = 11{,}18$ $x = -11{,}18$

g) $x^2 - 16x - 16 = 0$

$(x-8)^2 = 16 + 64$

$(x-4)^2 = 80$

$\therefore x - 4 = 4\sqrt{5}$ OR $x - 4 = -45$

$\therefore x = 4 + 4\sqrt{5}$ $x = 4 - 4\sqrt{5}$

$\therefore x = 16{,}944$ $x = -0{,}944$

h) $-4x^2 + 40x - 44 = 0$

$x^2 - 10x + 11 = 0$

$(x-5)^2 = -11 + 25$

$(x-5)^2 = 14$

$\therefore x - 5 = \sqrt{14}$ OR $x - 5 = -\sqrt{14}$

$\therefore x = 5 + \sqrt{14}$ $x = 5 - \sqrt{14}$

$\therefore x = 8{,}742$ $x = 1{,}258$

i) $2x^2 - 36x - 5 = 0$

$x^2 - 18x - \frac{5}{2} = 0$

$(x-9)^2 = \frac{5}{2} + 81$

$(x-9)^2 = 83\frac{1}{2}$

$\therefore x - 9 = \sqrt{83\frac{1}{2}}$ OR $x - 9 = -\sqrt{83\frac{1}{2}}$

$\therefore x = 9 + \sqrt{83\frac{1}{2}}$ $x = 9 - \sqrt{83\frac{1}{2}}$

$\therefore x = 18{,}138$ $x = -0{,}138$

j) $\frac{1}{3}x^2 - \frac{1}{4}x - \frac{3}{4} = 0$

$x^2 - \frac{3}{4}x - \frac{9}{4} = 0$

$\left(x - \frac{3}{8}\right)^2 = \frac{9}{4} + \frac{9}{64}$

$\left(x - \frac{3}{8}\right)^2 = 2\frac{25}{64}$

$\therefore x - \frac{3}{8} = \sqrt{\frac{153}{64}}$ OR $x - \frac{3}{8} = -\sqrt{\frac{153}{64}}$

$\therefore x = \frac{3}{8} + \sqrt{\frac{153}{64}}$ $x = \frac{3}{8} - \sqrt{\frac{153}{64}}$

$\therefore x = \frac{3+3\sqrt{17}}{8}$ $x = \frac{3-3\sqrt{17}}{8}$

$\therefore x = 1{,}921$ $x = -1{,}171$

3. *Solve the following inequalities:*

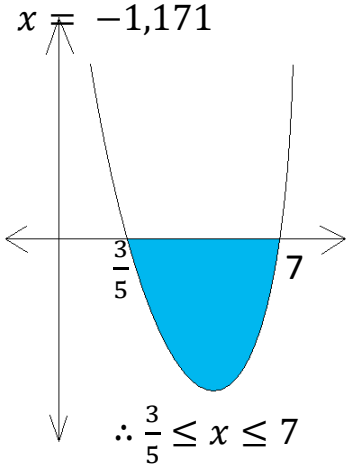
a) $5(x^2 - 1) \leq 2(19x - 13)$

$5x^2 - 5 \leq 38x - 26$

$5x^2 - 38x - 5 + 26 \leq 0$

$5x^2 - 38x + 21 \leq 0$

$(5x - 3)(x - 7)$



$\therefore \frac{3}{5} \leq x \leq 7$

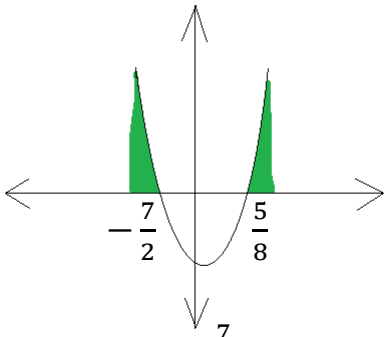
b) $16x(x+1) > 5(7-6x)$

$16x^2 + 16x > 35 - 30x$

$16x^2 + 16x + 30x - 35 > 0$

$16x^2 + 46x - 35 > 0$

$(8x-5)(2x+7) > 0$



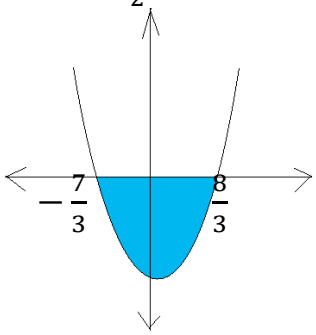
$\therefore x < -\frac{7}{2}$ OR $x > \frac{5}{8}$

c) $-3x(3x-1) \geq -56$

$-9x^2 + 3x + 56 \geq 0$

$9x^2 - 3x - 56 \leq 0$

$(3x+7)(3x-8) \leq 0$



$\therefore -\frac{7}{3} \leq x \leq \frac{8}{3}$

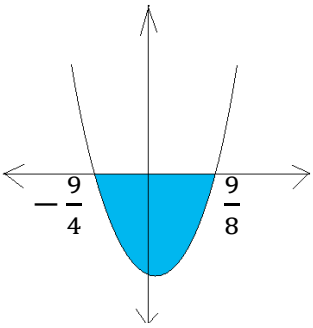
d) $4x(8x+7) \leq 81 - 8x$

$32x^2 + 28x \leq 81 - 8x$

$32x^2 + 28x + 8x - 81 \leq 0$

$32x^2 + 36x - 81 \leq 0$

$(4x+9)(8x-9) \leq 0$

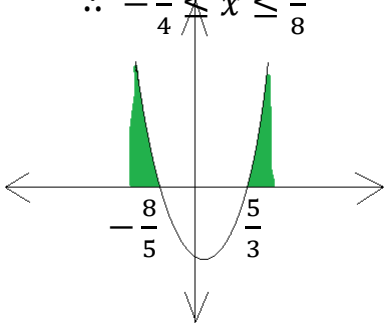


$\therefore -\frac{9}{4} \leq x \leq \frac{9}{8}$

e) $-5x^2 + \frac{x}{3} + 13\frac{1}{3} \leq 0$

$15x^2 - x - 40 \geq 0$

$(3x-5)(5x+8) \geq 0$



$\therefore x \leq -\frac{8}{5}$ *or* $x \geq \frac{5}{3}$

f) $8x(x+1) \geq 4(9-x)$

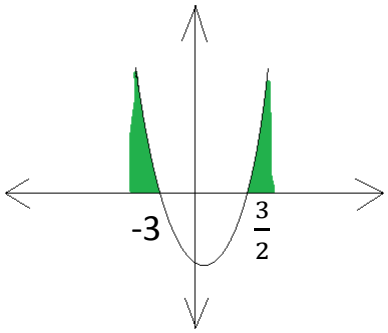
$8x^2 + 8x \geq 36 - 4x$

$8x^2 + 8x + 4x - 36 \geq 0$

$8x^2 + 12x - 36 \geq 0$

$2x^2 + 3x - 9 \geq 0$

$(2x-3)(x+3) \geq 0$



$\therefore x \leq -3$ OR $x \geq \frac{3}{2}$

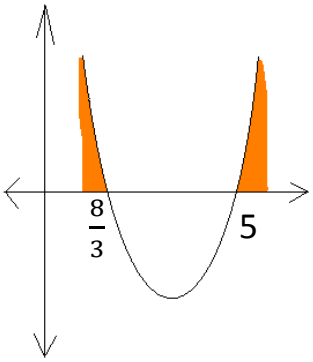
g) $3x(x-1) \geq 10(2x-4)$

$3x^2 - 3x \geq 20x - 40$

$3x^2 - 3x - 20x + 40 \geq 0$

$3x^2 - 23x + 40 \geq 0$

$(3x-8)(x-5) \geq 0$



$\therefore x \leq \frac{8}{3}$ OR $x \geq 5$

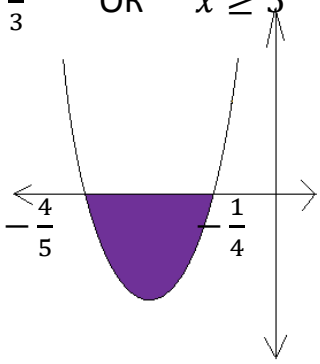
h) $5x(4x+5) < 4(x-1)$

$20x^2 + 25x < 4x - 4$

$20x^2 + 25x - 4x + 4 < 0$

$20x^2 + 21x + 4 < 0$

$(5x+4)(4x+1) < 0$

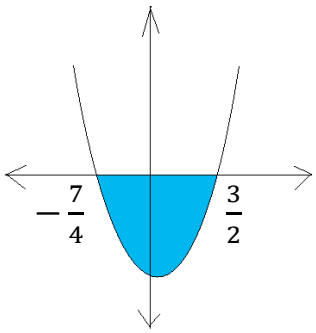


$\therefore -\frac{4}{5} < x < -\frac{1}{4}$

i) $4x\left(2x + \frac{1}{2}\right) \leq 21$

$8x^2 + 2x - 21 \leq 0$

$(2x-3)(4x+7) \leq 0$



$\therefore -\frac{7}{4} \leq x \leq \frac{3}{2}$

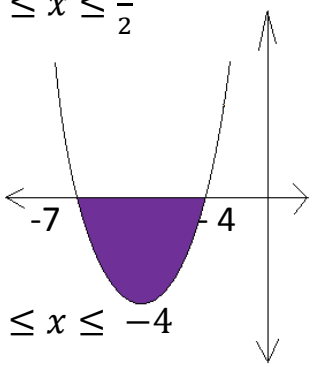
j) $-x(x+3) \geq 4(2x+7)$

$-x^2 - 3x \geq 8x + 28$

$-x^2 - 3x - 8x - 28 \geq 0$

$x^2 + 11x + 28 \geq 0$

$(x+4)(x+7) \geq 0$



$\therefore -7 \leq x \leq -4$

k) $\frac{4}{x} \leq \frac{7}{x-1}$

$\frac{4}{x} - \frac{7}{x-1} \leq 0$

$\frac{4(x-1)-7x}{x(x-1)} \leq 0$

$\frac{4x-4-7x}{x(x-1)} \leq 0$

$\frac{-3x-4}{x(x-1)} \leq 0$

l) $\frac{x-2}{x+2} > \frac{7}{x-2}$

$\frac{x-2}{x+2} - \frac{8}{x-2} > 0$

$\frac{x^2-4x+4-7(x+2)}{(x+2)(x-2)} > 0$

$\frac{x^2-4x+4-7x-14}{(x+2)(x-2)} > 0$

$\frac{x^2-11x-12}{(x+2)(x-2)} > 0$

$\frac{3x+4}{x(x-1)} \geq 0$ $\frac{(x-12)(x+1)}{(x+2)(x-2)} > 0$

		$-\frac{4}{3}$	0	1
$3x+4$	-	+	+	+
x	-	-	+	+
$x-1$	-	-	-	+
	-	+	-	+

		-2	-1	2	12
$x-12$	-	-	-	-	+
$x+1$	-	-	+	+	+
$x+2$	-	+	+	+	+
$x-2$	-	-	-	+	+
	+	-	+	-	+

$\therefore -\frac{4}{3} \leq x < 0 \quad or \quad x > 1$

Remember $x \neq 0 \ and \ x \neq 1$

$\therefore x < -2 \ or -1 < x < 2 \ or \ x > 12$

m) $-3 \geq \frac{1}{x-2}$

$0 \geq \frac{3(x-2)+1}{x-2}$

$0 \geq \frac{3x-6+1}{x-2}$

$0 \geq \frac{3x-5}{x-2}$

		$\frac{5}{3}$	2
$3x-5$	-	+	+
$x-2$	-	-	+
	+	-	+

$\therefore \frac{5}{3} \leq x < 2 \quad (Restrictions \rightarrow x \neq 2)$

n) $\frac{x^2}{x+3} \leq 0$

x^2 is always positive

because $(-)^2 = + \ and \ (+)^2 = +$

$\therefore x + 3 < 0 \ (cannot = 0)$

$\therefore x < -3$

4. *Solve the following simultaneous questions*:

a) $y = 3x - 9 \ldots 1$ and $x^2 - 4x + y^2 + 6y = 77 \ldots 2$

Subs 1 into 2

$x^2 - 4x + (3x-9)^2 + 6(3x-9) = 77$

$x^2 - 4x + 9x^2 - 54x + 81 + 18x - 54 = 77$

$10x^2 - 40x - 50 = 0$

$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$\therefore x = -1$ OR $x = 5$

Substitute back into 1
$\therefore y = 3(-1) - 9$ OR $y = 3(5) - 9$
$\therefore y = -12$ $y = 6$

$\therefore (-1; -12)$ and $(5; 6)$

b) $y = 4x + 9$ …1 and $x^2 + 3x + y^2 - 6y = \frac{227}{4}$ …2
Substitute 1 into 2

$\therefore x^2 + 3x + (4x + 9)^2 - 6(4x + 9) = \frac{227}{4}$
$x^2 + 3x + 16x^2 + 72x + 81 - 24x - 54 = \frac{227}{4}$
$17x^2 + 51x - 29\frac{3}{4} = 0$
$x^2 + 3x - 1\frac{3}{4} = 0$
$4x^2 + 12x - 7 = 0$
$(2x + 7)(2x - 1) = 0$
$\therefore x = -3\frac{1}{2}$ OR $x = \frac{1}{2}$

Substitute back into 1
$\therefore y = 4\left(-3\frac{1}{2}\right) + 9$ or $y = 4\left(\frac{1}{2}\right) + 9$
$\therefore y = -5$ $y = 11$

$\left(-3\frac{1}{2}; -5\right)$ and $\left(\frac{1}{2}; 11\right)$

c) $2y = 2x + 14$ and $x^2 + 8x + y^2 - 4y = 41$ …2
$\therefore y = x + 7$…1
Substitute 1 into 2

$x^2 + 8x + (x + 7)^2 - 4(x + 7) = 41$
$x^2 + 8x + x^2 + 14x + 49 - 4x - 28 - 41 = 0$
$2x^2 + 18x - 20 = 0$

$x^2 + 9x - 10 = 0$
$(x + 10)(x - 1) = 0$
$\therefore x = -10$ OR $x = 1$

Substitute back into 1
$\therefore y = -10 + 7$ OR $y = 1 + 7$
$\therefore y = -3$ $y = 8$

$(-10; -3)$ and $(1; 8)$

d) $16y = 2x - 55$ and $4x^2 - 20x + 4y^2 - 40y = 395$
$y = \frac{1}{8}x - \frac{55}{16}$...1 $x^2 - 5x + y^2 - 10y = \frac{395}{4}$ ...2
Substitute 1 into 2

$x^2 - 5x + \left(\frac{1}{8}x - \frac{55}{16}\right)^2 - 10\left(\frac{1}{8}x - \frac{55}{16}\right) = \frac{395}{4}$
$x^2 - 5x + \frac{1}{64}x^2 - \frac{55}{64}x + \frac{3025}{256} - \frac{5}{4}x + \frac{275}{8} - \frac{395}{4} = 0$
$64x^2 - 320x + x^2 - 55x + \frac{3025}{4} - 80x + 2200 - 6320 = 0$
$65x^2 - 455x - 3363\frac{3}{4} = 0$
$x^2 - 7x - 51\frac{3}{4} = 0$
$4x^2 - 28x - 207 = 0$
$(2x + 9)(2x - 23) = 0$
$\therefore \ x = -4\frac{1}{2} \quad or \quad x = 11\frac{1}{2}$

Substitute back into 1
$\therefore y = \frac{1}{8}\left(-4\frac{1}{2}\right) - \frac{55}{16}$ OR $y = \frac{1}{8}\left(11\frac{1}{2}\right) - \frac{55}{16}$
$\therefore y = -4$ $y = -2$

$\left(-4\frac{1}{2}; -4\right)$ and $\left(11\frac{1}{2}; -2\right)$

e) $4y = -5x + 38$ and $x^2 - 12x + y^2 - 4y = 1$ …2

$y = -\frac{5}{4}x + 9\frac{1}{2}$ …1

Substitute 1 into 2

$x^2 - 12x + \left(-\frac{5}{4}x + 9\frac{1}{2}\right)^2 - 4\left(-\frac{5}{4}x + 9\frac{1}{2}\right) = 1$

$x^2 - 12x + \frac{25}{16}x^2 - \frac{95}{4}x + 90\frac{1}{4} + 5x - 38 = 1$

$16x^2 - 192x + 25x^2 - 380x + 1444 + 80x - 608 - 16 = 0$

$41x^2 = 492x + 820 = 0$

$x^2 - 12x + 20 = 0$

$(x - 2)(x - 10) = 0$

$\therefore x = 2$ OR $x = 10$

Substitute back into 1

$\therefore y = -\frac{5}{4}(2) + 9\frac{1}{2}$ OR $y = -\frac{5}{4}(10) + 9\frac{1}{2}$

$\therefore y = 7$ $y = -3$

$(2; 7)$ and $(10; -3)$

f) $y + x + 3 = 0$ and $x^2 - 2x + y^2 + 10y - 155 = 0$ …2

$y = -x - 3$…1

Substitute 1 into 2

$x^2 - 2x + (-x - 3)^2 + 10(-x - 3) - 155 = 0$

$x^2 - 2x + x^2 + 6x + 9 - 10x - 30 - 155 = 0$

$2x^2 - 6x - 176 = 0$

$x^2 - 3x - 88 = 0$

$(x + 8)(x - 11) = 0$

$\therefore x = -8$ OR $x = 11$

Substitute back into 1

$\therefore y = -(-8) - 3$ or $y = -(11) - 3$

$\therefore y = 5$ $y = -14$

$(-8; 5)$ and $(11; -14)$

g) $10y + 8x = 49$ and $4x^2 - 32x + 4y^2 + 4y = 147$ …2

$y = -\frac{8}{10}x + \frac{49}{10}$…1

Substitute 1 into 2

$4x^2 - 32x + 4\left(-\frac{4}{5}x + \frac{49}{10}\right)^2 + 4\left(-\frac{4}{5}x + \frac{49}{10}\right) = 147$

$4x^2 - 32x + 4\left(\frac{16}{25}x^2 - \frac{196}{25}x + \frac{2401}{100}\right) - \frac{16}{5}x + \frac{98}{5} - 147 = 0$

$4x^2 - 32x + \frac{64}{25}x^2 + \frac{784}{25}x + \frac{2401}{25} - \frac{16}{5}x + \frac{98}{5} - 147 = 0$

$100x^2 - 800x + 64x^2 - 784x + 2401 - 80x + 490 - 3675 = 0$

$164x^2 - 1664x - 784 = 0$

$41x^2 - 416x - 196 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{416 \pm \sqrt{(-416)^2 - 4(41)(-196)}}{2(41)}$

$\therefore x = 10{,}6$ OR $x = 0{,}45$

Substitute back into 1

$\therefore y = -\frac{4}{5}(10{,}6) + \frac{49}{10}$ or $y = -\frac{4}{5}(0{,}45) + \frac{49}{10}$

$\therefore y = -3{,}58$ $y = 4{,}54$

$(10{,}6;\ -3{,}58)$ and $(0{,}45;\ 4{,}54)$

h) $5y = 4x + 14\frac{1}{2}$ and $2x^2 - 14x + 2y^2 + 10y - 127 = 0$ …2

$y = \frac{4}{5}x + \frac{29}{10}$ …1

Substitute 1 into 2

$2x^2 - 14x + 2\left(\frac{4}{5}x + \frac{29}{10}\right)^2 + 10\left(\frac{4}{5}x + \frac{19}{10}\right) - 127 = 0$

$2x^2 - 14x + 2\left(\frac{16}{25}x^2 + \frac{116}{25}x + \frac{841}{100}\right) + 8x + 29 - 127 = 0$

$2x^2 - 14x + \frac{32}{25}x^2 + \frac{232}{25}x + \frac{841}{50} + 8x + 29 - 127 = 0$

$100x^2 - 700x + 64x^2 + 464x + 841 + 400x + 1450 - 6350 = 0$

$164x^2 + 164x - 4059 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-164 \pm \sqrt{(164)^2 - 4(164)(-4059)}}{2(164)}$ $\therefore x = 4\frac{1}{2}$ OR $x = -5\frac{1}{2}$

Substitute back into 1

$\therefore y = \frac{4}{5}\left(4\frac{1}{2}\right) + \frac{29}{10}$ OR $y = \frac{4}{5}\left(-5\frac{1}{2}\right) + \frac{29}{10}$

$\therefore y = 6\frac{1}{2}$ $y = -1\frac{1}{2}$

$\left(4\frac{1}{2}; 6\frac{1}{2}\right)$ and $\left(-5\frac{1}{2}; -1\frac{1}{2}\right)$

i) $3y = 4x - 7$ and $x^2 - 5x + y^2 - 2y = \frac{71}{4}$ …2

$y = \frac{4}{3}x - \frac{7}{3}$…1

Substitute 1 into 2

$x^2 - 5x + \left(\frac{4}{3}x - \frac{7}{3}\right)^2 - 2\left(\frac{4}{3}x - \frac{7}{3}\right) = \frac{71}{4}$

$x^2 - 5x + \frac{16}{9}x^2 - \frac{56}{9}x + \frac{49}{9} - \frac{8}{3}x + \frac{14}{3} - \frac{71}{4} = 0$

$9x^2 - 45x + 16x^2 - 56x + 49 - 24x + 42 - \frac{639}{4} = 0$

$25x^2 - 125x - \frac{275}{4} = 0$

$x^2 - 5x - \frac{11}{4} = 0$

$4x^2 - 20x - 11 = 0$

$(2x + 1)(2x - 11) = 0$

$\therefore x = -\frac{1}{2}$ OR $x = 5\frac{1}{2}$

Substitute back into 1

$\therefore y = \frac{4}{3}\left(-\frac{1}{2}\right) - \frac{7}{3}$ OR $y = \frac{4}{3}\left(5\frac{1}{2}\right) - \frac{7}{3}$

$\therefore y = -3$ $y = 5$

$\left(-\frac{1}{2}; -3\right)$ and $\left(5\frac{1}{2}; 5\right)$

j) $y = 13x - 3$ …1 and $x^2 - 14x + y^2 - 6y = 27$ …2

Substitute 1 into 2

$x^2 - 14x + (13x - 3)^2 - 6(13x - 3) - 27 = 0$

$x^2 - 14x + 169x^2 - 78x + 9 - 78x + 18 - 27 = 0$

$170x^2 - 170x = 0$

$170x(x - 1) = 0$ $\therefore x = 0$ OR $x = 1$

Substitute back into 1

$\therefore y = 13(0) - 3$ OR $y = 13(1) - 3$

$\therefore y = -3$ $y = 10$

$(0; -3)$ and $(1; 10)$

5. *Solve for x:*

a) $\frac{3}{x-2} = \frac{x}{5}$

$15 = x^2 - 2x$

$0 = x^2 - 2x - 15$

$0 = (x-5)(x+3)$

$\therefore x = 5$ *or* $x = -3$

b) $\frac{x}{2x+4} = \frac{1}{x+2}$ Restrictions:

$\frac{x}{2(x+2)} = \frac{1}{x+2}$ $x \neq -2$

$x = 2$

Restrictions: $x \neq 2$

c) $\frac{6}{x-1} + \frac{6}{x-6} = 0$ Restrictions: $x \neq 1;\ x \neq 6$

$6(x-6) + 6(x-1) = 0$

$6x - 36 + 6x - 6 = 0$

$12x - 42 = 0$

$12x = 42$

$\therefore x = 3\frac{1}{2}$

d) $\frac{-4}{x-5} = \frac{3}{x+1} - \frac{5}{2x}$ Restrictions: $x \neq 5;\ x \neq -1;\ x \neq 0$

$-4(x+1)(2x) = 3(x-5)(2x) - 5(x-5)(x+1)$

$-8x(x+1) = 6x\ (x-5) - 5(x^2 - 4x - 5)$

$-8x^2 - 8x = 6x^2 - 30x - 5x^2 + 20x + 25$

$-9x^2 - 8x + 30x - 20x - 25 = 0$

$-9x^2 + 2x - 25 = 0$

$9x^2 - 2x + 25 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4(9)(25)}}{2(9)}$

$\therefore x$ is non-real / imaginary

e) $\frac{1}{x+2} = \frac{5}{3x}$ Restrictions: $x \neq -2; \quad x \neq 0$

$3x = 5x + 10$
$-2x = 10$
$\therefore x = -5$

f) $\frac{2}{x-6} + \frac{6}{x-1} = -\frac{2}{3x}$ Restrictions: $x \neq 6; \quad x \neq 1; \quad x \neq 0$

$2(3x)(x-1) + 6(3x)(x-6) = -2(x-6)(x-1)$
$6x^2 - 6x + 18x^2 - 108x = -2(x^2 - 7x + 6)$
$24x^2 - 114x = -2x^2 + 14x - 12$
$26x^2 - 128x + 12 = 0$
$13x^2 - 64x + 6 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\therefore x = \frac{64 \pm \sqrt{(-64)^2 - 4(13)(6)}}{2(13)}$
$\therefore x = 4{,}827$ OR $x = 0{,}096$

g) $\frac{6}{x-2} = -\frac{1}{2x-1}$ Restrictions: $x \neq 2; \quad x \neq \frac{1}{2}$

$12x - 6 = -x + 2$
$13x = 8$
$\therefore x = \frac{8}{13}$

h) $\frac{1}{x-5} + \frac{7}{x-6} = \frac{4}{-5x}$ Restrictions: $x \neq 5; \quad x \neq 6; \quad x \neq 0$

$1(x-6)(-5x) + 7(x-5)(-5x) = 4(x-5)(x-6)$
$-5x^2 + 30x - 35x^2 + 175x = 4(x^2 - 11x + 30)$
$-40x^2 + 205x = 4x^2 - 44x + 120$
$-44x^2 + 249x - 120 = 0$
$44x^2 - 249x + 120 = 0$
$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\therefore x = \frac{249 \pm \sqrt{(-249)^2 - 4(44)(120)}}{2(44)}$

$\therefore x = 5{,}127$ OR $x = 0{,}532$

i) $\frac{1}{x-10} - \frac{1}{3x+1} = 3$ Restrictions: $x \neq 10;\ x \neq -\frac{1}{3}$

$3x + 1 - x + 10 = 3(x - 10)(3x + 1)$
$2x + 11 = 3(3x^2 - 29x - 10)$
$2x + 11 = 9x^2 - 87x - 30$
$0 = 9x^2 - 89x - 41$
$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\therefore x = \frac{89 \pm \sqrt{(-89)^2 - 4(9)(41)}}{2(9)}$
$\therefore x = 10{,}33$ OR $x = -0{,}441$

j) $\frac{4}{x-2} + \frac{4}{x-3} = -1$ Restrictions: $x \neq 2;\ x \neq 3$

$4x - 12 + 4x - 8 = -1(x - 2)(x - 3)$
$8x - 20 = -1(x^2 - 5x + 6)$
$0 = -x^2 + 5x - 6 - 8x + 20$
$0 = -x^2 - 3x + 14$
$0 = x^2 + 3x - 14$
$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\therefore x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-14)}}{2(1)}$
$\therefore x = 2{,}531$ or $x = -5{,}531$

6. *Solve for x:*

a) $4^x = 8$
$\log_4 8 = x$
$\therefore x = 1\frac{1}{2}$

b) $4^{x+2} - 4^x = 960$
$4^x(4^2 - 1) = 960$
$4^x(15) = 960$
$4^x = 64$
$\log_4 64 = x$
$\therefore x = 3$

c) $7.5^x - 5^{2+x} = -3\frac{3}{5}$

$5^x(7 - 5^2) = -3\frac{3}{5}$

$5^x(-18) = -3\frac{3}{5}$

$5^x = \frac{1}{5}$

$\therefore x = -1$

d) $x^4 = 16$

$\therefore x = \sqrt[4]{16}$

$\therefore x = 2$

e) $x = 52^0$

$\therefore x = 1$

f) $3.(-5)^x = -375$

$(-5)^x = -125$

$(-5)^x = (-5)^3$

$\therefore x = 3$

g) $3^x.2^{x+1} = 72$

$3^x.2^x.2 = 72$

$3^x.2^x = 36$

$6^x = 36$

$\therefore x = 2$

h) $3^{x+1} + 2.3^{x+1} = 3$

$3^x(3 + 2.3) = 3$

$3^x(9) = 3$

$3^x = \frac{1}{3}$

$\therefore x = -1$

i) $3.2^{x-1} - 2^x = 1$

$2^x\left(3.\frac{1}{2} - 1\right) = 1$

$2^x\left(\frac{1}{2}\right) = 1$

$2^x = 2$

$\therefore x = 1$

j) $5^x - 5^{x-1} = 500$

$5^x(1 - 5^{-1}) = 500$

$5^x\left(\frac{4}{5}\right) = 500$

$5^x = 625$

$\log_5 625 = x$

$\therefore x = 4$

Story Sums:

7. Width of inner frame $= x - 1\frac{1}{2} \times 2$

Height of inner frame $= 2x - 3 - 1\frac{1}{2} \times 2$

$\therefore Area = l \times b$

$\therefore 99 = (x - 3)(2x - 6)$

$\therefore 99 = 2x^2 - 6x - 6x + 18$

$\therefore 0 = 2x^2 - 12x - 81$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{12 \pm \sqrt{(-12)^2 - 4(2)(-81)}}{2(2)}$

$\therefore x = 10{,}036$ OR $x = -4{,}036$

N/A (distance cannot be negative).

$\therefore x = 8{,}92$cm

8. $(3x + 4)(2x - 1) = 10$

$6x^2 - 3x + 8x - 4 - 10 = 0$

$6x^2 + 5x - 14 = 0$

$(6x - 7)(x + 2) = 0$

$\therefore x = \frac{7}{6} \quad or \quad x = -2$

N/A → answer can't be negative

$\therefore$ Factor $x = 1{,}666$ …

9. $x \rightarrow$ chocolate chip

$3x - 10 \rightarrow$ white chocolate chip

$500 \div 2$

$x + 3x - 10 = 250$

$4x - 10 = 250$

$4x = 260$

$\therefore x = 65$ chocolate chips

$\therefore 3(65) - 10 = 185$ white chocolate chips

10. $x = 0$ OR $x = 10$

$\therefore -x(x - 10) = 0$

$\therefore y = -x^2 + 10x$

$\therefore y = -x^2 + 10x - (5)^2 + (5)^2$

$\therefore y = -(x - 5)^2 + 25$

$\therefore$ Maximum strength of 25 at distance of 5m.