

# SHARP

## Werkkaart 1 Memo: Eksponente en wortels

### Graad 11 Wiskunde

1. i) Wanneer dieselfde grondtalle vermenigvuldig word, word die eksponente bygevoeg  
 $\rightarrow a^n \times a^m = a^{n+m}$
- ii) Wanneer dieselfde grondtalle gedeel word, word die eksponente afgetrek  
 $\rightarrow a^n \div a^m = a^{n-m}$
- iii) Wanneer 'n mag tot 'n eksponent verhef word, word die eksponente vermenigvuldig  
 $\rightarrow (a^n)^m = a^{n \times m}$
- iv) Wanneer 'n mag onder 'n wortelteken is, word die eksponent gedeel deur die wortel  
 $\rightarrow \sqrt[m]{a^n} = a^{\frac{n}{m}}$
- v) Enigiets tot die mag van nul is een  
 $\rightarrow a^0 = 1$
- vi) Wanneer 'n mag 'n negatiewe eksponent het, word die mag omgekeer (onder 1 geplaas) en die eksponent word positief  
 $\rightarrow a^{-n} = \frac{1}{a^n}$
2. i) Jy mag slegs soortgelyke terme byvoeg of aftrek, bv.  $3\sqrt{a} + 4\sqrt{a} = 7\sqrt{a}$
- ii)  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$  of omgekeer
- iii)  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$  of omgekeer
3. a)  $(ab^2c^3)^0$   
 $= 1$
- b)  $\left(\frac{1}{xy^3}\right)^2$   
 $= \frac{1}{x^2y^6}$
- c)  $(x^2yz^{-1})^3 \times (x^4yz^3)^{-1}$   
 $= x^6y^3z^{-3} \times x^{-4}y^{-1}z^{-3}$   
 $= x^2y^2z^{-6}$   
 $= \frac{x^2y^2}{z^6}$
- d)  $\left(\frac{1}{xy^2}\right)^{-2} \times \left(\frac{x^3}{y^2}\right)^{-1}$   
 $= x^2y^4 \times \frac{y^2}{x^3}$   
 $= x^{-1}y^6$   
 $= \frac{y^6}{x}$

$$\begin{aligned}
 \text{e)} \quad & \frac{a^2b^{-2}c^3}{ab^{-1}c^2} \times \frac{(a^2b^{-2}c)^{-1}}{a^{-1}b^2c^3} \div \frac{ab}{c^4} \\
 & = \frac{a^2bc^3}{ab^2c^2} \times \frac{a^{-2}b^2c^{-1}}{a^{-1}b^2c^3} \times \frac{c^4}{ab} \\
 & = \frac{ac}{b} \times \frac{1}{ac^4} \times \frac{c^4}{ab} \\
 & = a^{1-1-1}b^{-1-1}c^{1+4-4} \\
 & = a^{-1}b^{-2}c \\
 & = \frac{c}{ab^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{g)} \quad & \left(\frac{1}{x} + \frac{x}{y}\right)^{-2} \\
 & = \left(\frac{y+x^2}{xy}\right)^{-2} \\
 & = \left(\frac{xy}{y+x^2}\right)^2 \\
 & = \frac{x^2y^2}{y^2+2x^2y+x^4}
 \end{aligned}$$

$$\begin{aligned}
 \text{i)} \quad & \frac{8^{2x+1}+4^{3x-1}}{2^{3x}} \\
 & = \frac{(2^3)^{2x+1}+(2^2)^{3x-1}}{2^{3x}} \\
 & = \frac{2^{6x+3}+2^{6x-2}}{2^{3x}} \\
 & = \frac{2^{6x}(2^3+2^{-2})}{2^{3x}} \\
 & = 2^{3x} \left(\frac{33}{4}\right) \\
 & = 2^{3x-2} \cdot 33
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \text{a)} \quad & 5^{3x} - 5^{3x-1} = 4 \\
 & \therefore 5^{3x}(1 - 5^{-1}) = 4 \\
 & \therefore 5^{3x} \left(\frac{4}{5}\right) = 4 \\
 & \therefore 5^{3x} = 5 \\
 & \therefore 3x = 1 \\
 & \therefore x = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{f)} \quad & \frac{fg}{h^3} \times \left(\frac{h^4}{f^2g}\right)^{-2} \div \frac{a^0f^2g^{-2}}{h^{-3}} \\
 & = \frac{fg}{h^3} \times \frac{f^4g^2}{h^8} \times \frac{h^{-3}}{f^2g^{-2}} \\
 & = f^{1+4-2}g^{1+2-(-2)}h^{-3-3-8} \\
 & = f^3g^5h^{-14} \\
 & = \frac{f^3g^5}{h^{14}}
 \end{aligned}$$

$$\begin{aligned}
 \text{h)} \quad & \frac{27^{x+1} \cdot 18^{x-1}}{36^{3-x}} \\
 & = \frac{3^{3(x+1)} \cdot (3^2 \cdot 2)^{x-1}}{(3^2 \cdot 2^2)^{3-x}} \\
 & = \frac{3^{3x+3} \cdot 3^2(x-1) \cdot 2^{x-1}}{3^{6-2x} \cdot 2^{6-2x}} \\
 & = 3^{3x+3+2x-2-(6-2x)} \cdot 2^{x-1-(6-2x)} \\
 & = 3^{5x+1-6+2x} \cdot 2^{x-1-6+2x} \\
 & = 3^{7x-5} \cdot 2^{3x-7}
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & \frac{54^{x+1} \cdot 36^{x-1}}{24^{2x-3}} \\
 & = \frac{(3^3 \cdot 2)^{x+1} \cdot (2^2 \cdot 3^2)^{x-1}}{(2^3 \cdot 3)^{2x-3}} \\
 & = \frac{3^{3x+3} \cdot 2^{x+1} \cdot 2^{2x-2} \cdot 3^{2x-2}}{2^{6x-9} \cdot 3^{2x-3}} \\
 & = 3^{3x+3+2x-2-(2x-3)} \cdot 2^{x+1+2x-2-(6x-9)} \\
 & = 3^{5x+1-2x+3} \cdot 2^{3x-1-6x+9} \\
 & = 3^{3x+4} \cdot 2^{8-3x}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \frac{4^{x+2}+4^{x-1}}{5} = \frac{13}{16} \\
 & \therefore 4^{x+2} + 4^{x-1} = \frac{65}{16} \\
 & \therefore 4^x(4^2 + 4^{-1}) = \frac{65}{16} \\
 & \therefore 4^x \left(\frac{65}{4}\right) = \frac{65}{16} \\
 & \therefore 4^x = \frac{1}{4} = 4^{-1} \\
 & \therefore x = -1
 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad 2^{x-1} + 2^{2+x} &= 144 \\ \therefore 2^x(2^{-1} + 2^2) &= 144 \\ \therefore 2^x\left(4\frac{1}{2}\right) &= 144 \\ \therefore 2^x &= 32 \\ \therefore 2^x &= 2^5 \\ \therefore x &= 5 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad 2^x - 5 \cdot 2^{x+1} &= -144 \\ \therefore 2^x(1 - 5 \cdot 2) &= -144 \\ \therefore 2^x(-9) &= -144 \\ \therefore 2^x &= 16 \\ \therefore 2^x &= 2^4 \\ \therefore x &= 4 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad 5 \cdot 3^{x-2} - 5 \cdot 3^{x+1} &= -\frac{130}{729} \\ \therefore 5(3^{x-2} - 3^{x+1}) &= -\frac{130}{729} \\ \therefore 3^x(3^{-2} - 3) &= -\frac{26}{729} \\ \therefore 3^x\left(-\frac{26}{9}\right) &= -\frac{26}{729} \\ \therefore 3^x &= \frac{1}{81} \\ \therefore 3^x &= 3^{-4} \\ \therefore x &= -4 \end{aligned}$$

$$\begin{aligned} \text{i)} \quad 3^{2-x} - 3^{-x-3} &= \frac{242}{9} \\ \therefore 3^{-x}(3^2 - 3^{-3}) &= \frac{242}{9} \\ \therefore 3^{-x}\left(\frac{242}{27}\right) &= \frac{242}{9} \\ \therefore 3^{-x} &= 3 \\ \therefore -x &= 1 \end{aligned}$$

$$\begin{aligned} \text{k)} \quad 5^x &= 200 \\ \text{beproeving en fout} \\ \therefore x &= 3.29 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad 3^{x+1} \cdot 5 - 4 \cdot 3^{x+2} &= -\frac{7}{3} \\ \therefore 3^x(3 \cdot 5 - 4 \cdot 3^2) &= -\frac{7}{3} \\ \therefore 3^x(-21) &= -\frac{7}{3} \\ \therefore 3^x &= \frac{1}{9} \\ \therefore 3^x &= 3^{-2} \\ \therefore x &= -2 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad 5^{x+2} + 5^x &= 26 \\ \therefore 5^x(5^2 + 1) &= 26 \\ \therefore 5^x(26) &= 26 \\ \therefore 5^x &= 1 \\ \therefore 5^x &= 5^0 \\ \therefore x &= 0 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad 5^{2x+4} - 25^{x-1} &= 78\,120 \\ \therefore 5^{2x+4} - 5^{2(x-1)} &= 78\,120 \\ \therefore 5^{2x+4} - 5^{2x-2} &= 78\,120 \\ \therefore 5^{2x}(5^4 - 5^{-2}) &= 78\,120 \\ \therefore 5^{2x}\left(\frac{15624}{25}\right) &= 78\,120 \\ \therefore 5^{2x} &= 125 \\ \therefore 5^{2x} &= 5^3 \\ \therefore 2x &= 3 \\ \therefore x &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{j)} \quad 4^{x+1} \cdot 3 + 5 \cdot 2^{2x-1} - 7 &= \frac{1}{4} \\ \therefore 2^{2(x+1)} \cdot 3 + 5 \cdot 2^{2x-1} &= 7\frac{1}{4} \\ \therefore 2^{2x+2} \cdot 3 + 5 \cdot 2^{2x-1} &= 7\frac{1}{4} \\ \therefore 2^{2x}(3 \cdot 2^2 + 5 \cdot 2^{-1}) &= 7\frac{1}{4} \\ \therefore 2^{2x}\left(14\frac{1}{2}\right) &= 7\frac{1}{4} \\ \therefore 2^{2x} &= \frac{1}{2} \\ \therefore 2^{2x} &= 2^{-1} \\ \therefore 2x &= -1 \\ \therefore x &= -\frac{1}{2} \end{aligned}$$

l)  $3^x \cdot 4 + 5 = 25$   
 $\therefore 3^x \cdot 4 = 20$   
 $\therefore 3^x = 5$       probeer en fout  $\rightarrow x = 1.46$

5. a)  $4^{\frac{3}{2}}$       b)  $64^{\frac{2}{3}}$       c)  $27^{\frac{2}{9}}$       d)  $49^{\frac{3}{4}}$   
 $= (2^2)^{\frac{3}{2}}$        $= (4^3)^{\frac{2}{3}}$        $= (3^3)^{\frac{2}{9}}$        $= (7^2)^{\frac{3}{4}}$   
 $= 2^3$        $= 4^2$        $= 3^{\frac{2}{3}}$        $= 7^{\frac{3}{2}}$   
 $= 8$        $= 16$        $= \sqrt[3]{9}$        $= \sqrt{343}$

e)  $81^{\frac{5}{4}}$       f)  $48^{-\frac{1}{2}}$       g)  $100^{-\frac{3}{2}}$       h)  $32^{-\frac{2}{5}}$   
 $= (3^4)^{\frac{5}{4}}$        $= \sqrt{\frac{1}{48}}$        $= (10^2)^{-\frac{3}{2}}$        $= (2^5)^{-\frac{2}{5}}$   
 $= 3^5$        $= \frac{1}{\sqrt{48}}$        $= 10^{-3}$        $= 2^{-2}$   
 $= 243$        $= \frac{1}{4\sqrt{3}}$        $= 0.001$        $= \frac{1}{4}$

i)  $36^{-\frac{5}{2}} = (6^2)^{-\frac{5}{2}}$       j)  $24^{\frac{1}{3}} = \sqrt[3]{24}$   
 $= 6^{-5}$        $= \sqrt[3]{8 \times 3}$   
 $= \frac{1}{6^5}$        $= 2^3 \sqrt{3}$

6. a) wortel      b) nie wortel      c) wortel      d) nie wortel  
e) nie wortel      f) wortel      g) nie wortel      h) nie wortel  
i) wortel      j) wortel

7. a)  $\frac{\sqrt{80} + \sqrt{45}}{\sqrt{125}}$       b)  $\frac{\sqrt{8} - \sqrt{2}}{\sqrt{50}}$   
 $= \frac{\sqrt{16 \times 5} + \sqrt{9 \times 5}}{\sqrt{25 \times 5}}$        $= \frac{\sqrt{4 \times 2} - \sqrt{2}}{\sqrt{25 \times 2}}$   
 $= \frac{4\sqrt{5} + 3\sqrt{5}}{5\sqrt{5}}$        $= \frac{2\sqrt{2} - \sqrt{2}}{5\sqrt{2}}$   
 $= \frac{7\sqrt{5}}{5\sqrt{5}}$        $= \frac{\sqrt{2}}{5\sqrt{2}}$   
 $= \frac{7}{5} = 1\frac{2}{5}$        $= \frac{1}{5}$

c)  $\frac{\sqrt{48} + \sqrt{147}}{\sqrt{242}}$       d)  $\frac{\sqrt{2}(\sqrt{80} - \sqrt{45})}{\sqrt{1000}}$   
 $= \frac{\sqrt{16 \times 3} + \sqrt{49 \times 3}}{\sqrt{121 \times 2}}$        $= \frac{\sqrt{160} - \sqrt{90}}{\sqrt{1000}}$   
 $= \frac{4\sqrt{4} + 7\sqrt{3}}{11\sqrt{2}}$        $= \frac{\sqrt{16 \times 10} - \sqrt{9 \times 10}}{\sqrt{100 \times 10}}$

$$= \frac{11\sqrt{3}}{11\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \sqrt{\frac{3}{2}}$$

$$= \frac{4\sqrt{10}-3\sqrt{10}}{10\sqrt{10}}$$

$$= \frac{\sqrt{10}}{10\sqrt{10}}$$

$$= \frac{1}{10}$$

e)  $(5 + \sqrt{2})(5 - \sqrt{2})$   
 $= 25 - 5\sqrt{2} + 5\sqrt{2} - 2$   
 $= 25 - 2$   
 $= 23$

f)  $(3\sqrt{2} - 1)(3\sqrt{2} - 2)$   
 $= 9(2) - 6\sqrt{2} - 3\sqrt{2} + 2$   
 $= 18 - 9\sqrt{2} + 2$   
 $= 20 - 9\sqrt{2}$

g)  $(3\sqrt{2} + 2)^2$   
 $= 9(2) + 12\sqrt{2} + 4$   
 $= 18 + 12\sqrt{2} + 4$   
 $= 22 + 12\sqrt{2}$

h)  $(\sqrt{7} - \sqrt{2})(\sqrt{7} + \sqrt{2})$   
 $= 7 - \sqrt{14} + \sqrt{14} - 2$   
 $= 7 - 2$   
 $= 5$

i)  $(4 - 2\sqrt{5})(5 - 3\sqrt{5})$   
 $= 20 - 12\sqrt{5} - 10\sqrt{5} + 6(5)$   
 $= 20 - 22\sqrt{5} + 30$   
 $= 50 - 22\sqrt{5}$

j)  $\left(\frac{2}{\sqrt{3}} + 1\right)\left(\frac{\sqrt{3}}{2} - 1\right)$   
 $= 1 - \frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2} - 1$   
 $= -\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2}$   
 $= \frac{-4+3}{2\sqrt{3}}$   
 $= -\frac{1}{2\sqrt{3}}$

8. a)  $\sqrt{x-3} = 2$   
 $\therefore (\sqrt{x-3})^2 = (2)^2$   
 $\therefore x-3 = 4$   
 $\therefore x = 7$   
 Kontroleer:  $\sqrt{7-3} = 2 \checkmark$

b)  $\sqrt[3]{4+x} = 3$   
 $\therefore (\sqrt[3]{4+x})^3 = (3)^3$   
 $\therefore 4+x = 27$   
 $\therefore x = 23$   
 Kontroleer:  $\sqrt[3]{4+23} = 3 \checkmark$

c)  $\sqrt{-x-1} = x+1$   
 $\therefore (\sqrt{-x-1})^2 = (x+1)^2$   
 $\therefore -x-1 = x^2 + 2x + 1$   
 $\therefore 0 = x^2 + 3x + 2$   
 $\therefore 0 = (x+2)(x+1)$   
 $\therefore x = -2 \text{ OF } x = -1$

d)  $x = \sqrt{x+2}$   
 $\therefore (x)^2 = (\sqrt{x+2})^2$   
 $\therefore x^2 = x+2$   
 $\therefore x^2 - x - 2 = 0$   
 $\therefore (x-2)(x+1) = 0$   
 $\therefore x = 2 \text{ OF } x = -1$

Kontroleer:  $\sqrt{2-1} \neq -2+1x$

Kontroleer:  $\sqrt{1-1} = -1+1x$  ✓

Kontroleer:  $2 = \sqrt{2+2}$  ✓

Kontroleer:  $-1 = \sqrt{-1+2}x$

e)  $\sqrt{3x^2} - \sqrt{12} = 0$

$\therefore (\sqrt{3x^2})^2 = (\sqrt{12})^2$

$\therefore 3x^2 = 12$

$\therefore x^2 = 4$

$\therefore x = \pm 2$

Kontroleer:  $\sqrt{3(2)^2} = \sqrt{12}$  ✓

Kontroleer:  $\sqrt{3(-2)^2} = \sqrt{12}$  ✓

f)  $\sqrt{18} - x\sqrt{2} = \sqrt{32}$

$\therefore -x\sqrt{2} = \sqrt{32} - \sqrt{18}$

$\therefore -x\sqrt{2} = 4\sqrt{2} - 3\sqrt{2}$

$\therefore -x\sqrt{2} = \sqrt{2}$

$\therefore -x = 1$

$\therefore x = -1$

g)  $\sqrt{x+2} = 9$

$\therefore (\sqrt{x+2})^2 = (9)^2$

$\therefore x+2 = 81$

$\therefore x = 79$

Kontroleer:  $\sqrt{79+2} = 9$  ✓

h)  $(x-3)^{\frac{1}{2}} = 4$

$\therefore [(x-3)^{\frac{1}{2}}]^2 = (4)^2$

$\therefore x-3 = 16$

$\therefore x = 19$

Kontroleer:  $(19-3)^{\frac{1}{2}} = 4$  ✓

i)  $\sqrt{6-2x} + 3 = x$

$\therefore (\sqrt{6-2x})^2 = (x-3)^2$

$\therefore 6-2x = x^2 - 6x + 9$

$\therefore 0 = x^2 - 4x + 3$

$\therefore 0 = (x-3)(x-1)$

$\therefore x = 3$  OF  $x = 1$

Kontroleer:  $\sqrt{6-2(3)} + 3 = 3$  ✓

Kontroleer:  $\sqrt{6-2(1)} + 3 = 1x$

j)  $x^2 - 5 = \sqrt{x^2 + 1}$

$\therefore (x^2 - 5)^2 = (\sqrt{x^2 + 1})^2$

$\therefore x^4 - 10x^2 + 25 = x^2 + 1$

$\therefore x^4 - 11x^2 + 24 = 0$

$\therefore (x^2 - 8)(x^2 - 3) = 0$

$\therefore x^2 = 8$  OF  $x^2 = 3$

Kontroleer:  $8 - 5 = \sqrt{8 + 1}$  ✓

$\therefore x = \pm\sqrt{8}$

Kontroleer:  $3 - 5 = \sqrt{3 + 1}x$

9. a)  $\frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

b)  $\frac{2}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

$$\begin{aligned}
 \text{c) } & \frac{1+\sqrt{5}}{\sqrt{5}} \\
 &= \frac{1+\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\
 &= \frac{\sqrt{5}+5}{5} \\
 &= \frac{\sqrt{5}}{5} + 1
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{\sqrt{3}+2}{\sqrt{3}} \\
 &= \frac{\sqrt{3}+2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{3+2\sqrt{3}}{3} \\
 &= 1 + \frac{2\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } & \frac{3}{1-\sqrt{2}} \quad \text{Uitdaging ☺} \\
 &= \frac{3}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \\
 &= \frac{3+3\sqrt{2}}{1-2} \\
 &= -3 - 3\sqrt{2}
 \end{aligned}$$