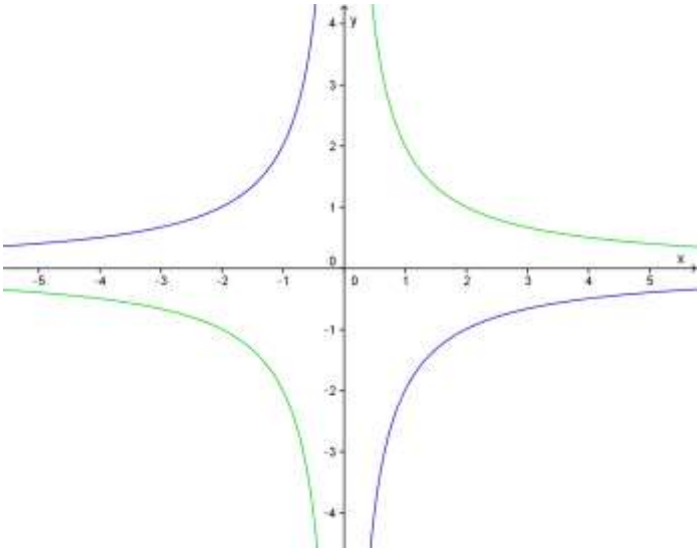
Worksheet 10 Memo – Functions: Hyperbolas, Parabolas and Exponential Graphs

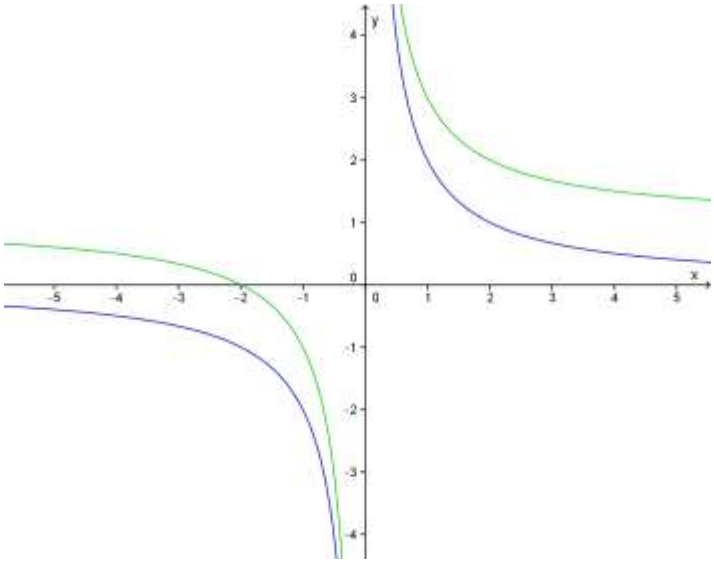
Grade 10 – Mathematics

1. a) $y = \frac{2}{x}$ and $y = \frac{-2}{x}$



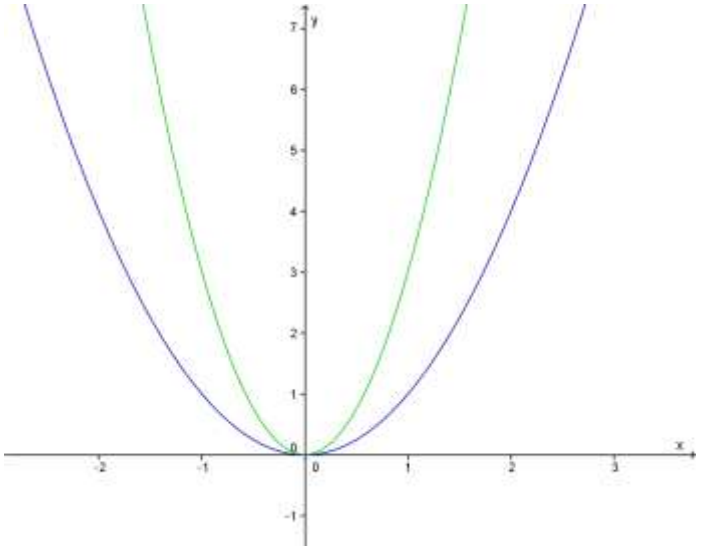
The graph was reflected about the y-axis or the x-axis.

b) $y = \frac{2}{x}$ and $y = \frac{2}{x} + 1$



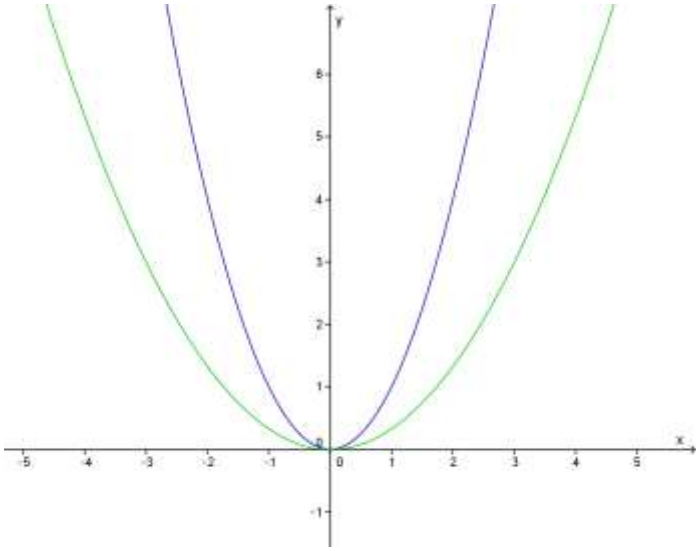
the graph was shifted down by 1 unit

c) $y = x^2$ and $y = 3x^2$



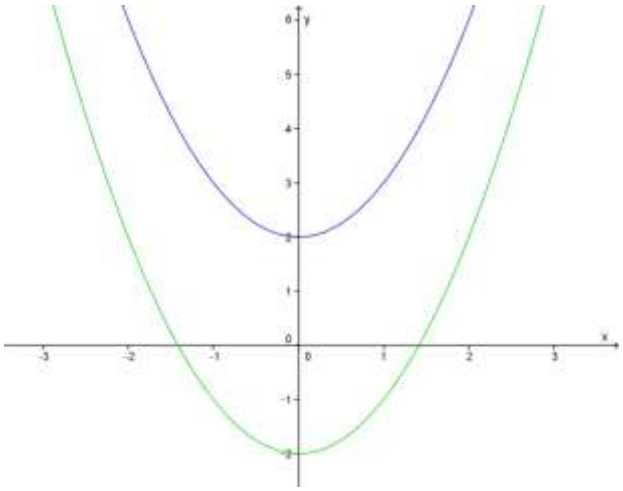
The graph becomes thinner

d) $y = x^2$ and $y = \frac{1}{3}x^2$



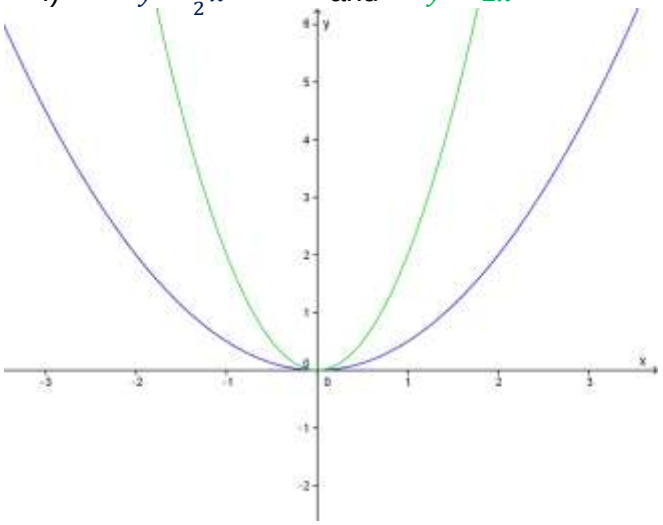
The graph becomes fatter.

e) $y = x^2 + 2$ and $y = x^2 - 2$



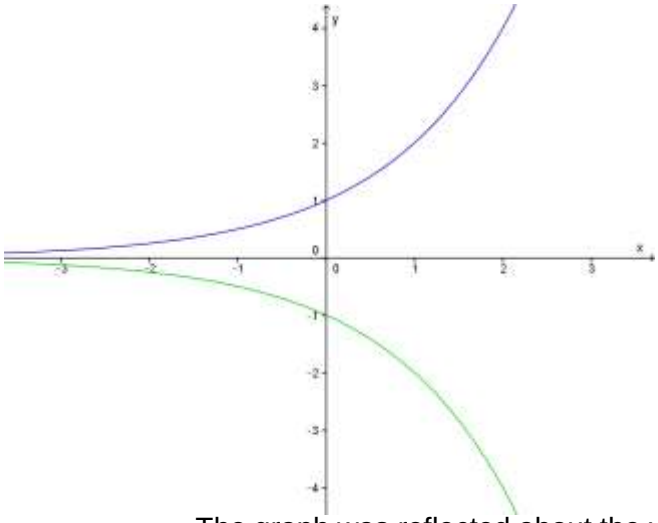
The graph was shifted 4 units down

f) $y = \frac{1}{2}x^2$ and $y = 2x^2$



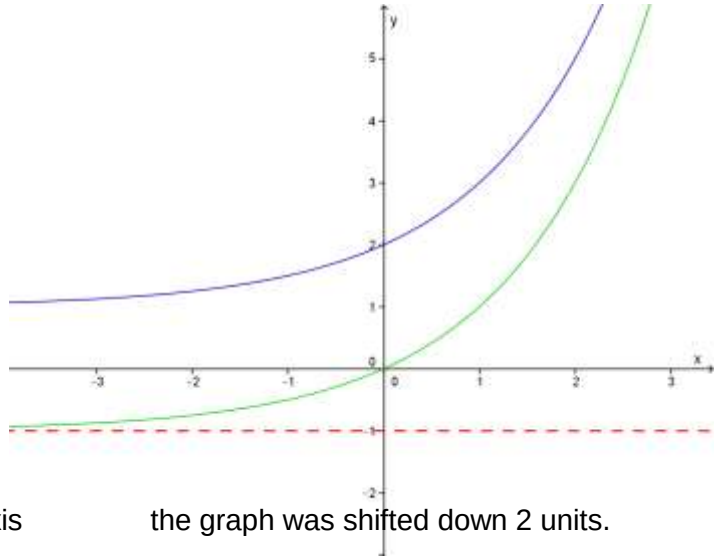
the graph became thinner

g) $y = 2^x$ and $y = -2^x$



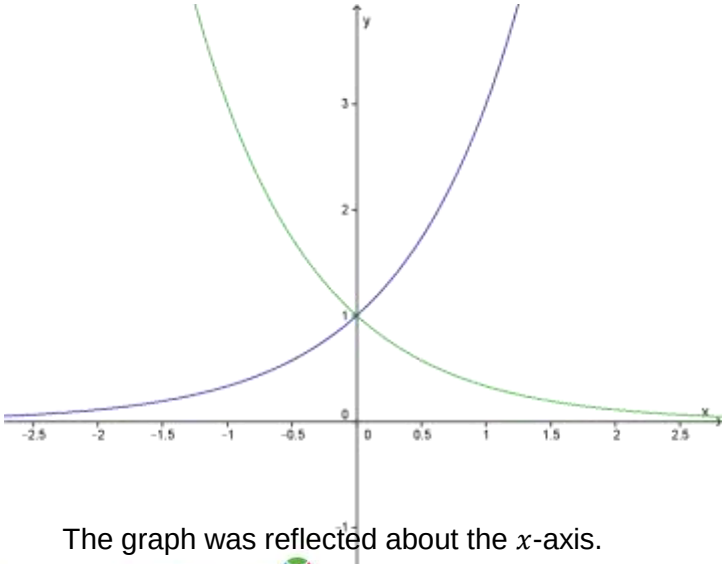
The graph was reflected about the y-axis

h) $y = 2^x + 1$ and $y = 2^x - 1$



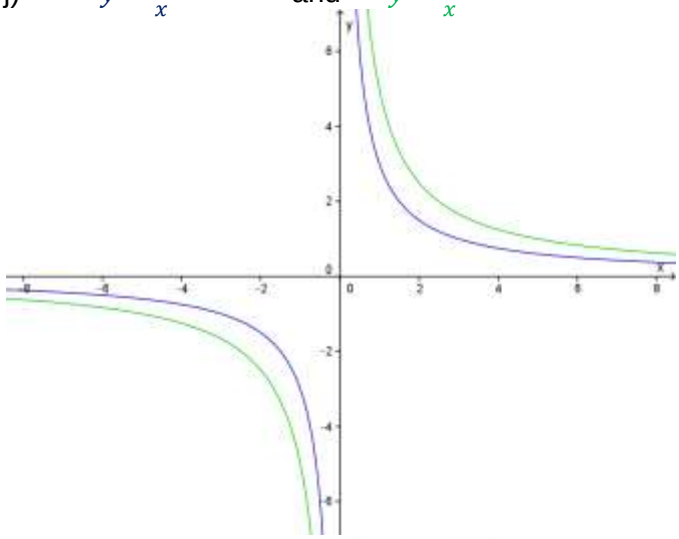
the graph was shifted down 2 units.

i) $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$

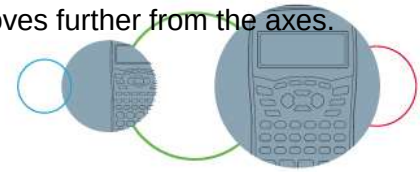


The graph was reflected about the x-axis.

j) $y = \frac{3}{x}$ and $y = \frac{5}{x}$



The graph moves further from the axes.



2. a) $x = 0$ and $y = 0$ b) $x = 0$ and $y = 1$

c) there are no asymptotes d) $x = 0$ and $y = 0$

e) $y = 0$ f) $x = 0$ and $y = -3$

g) $y = 1$ h) $y = 0$

i) $x = 0$ and $y = 2$ j) $x = 0$ and $y = -1$

3. a) $y = x$ and $y = -x$ b) $y = x$ and $y = -x$

c) $y = x + 1$ and $y = -x + 1$ d) $x = 0$

e) $x = 0$ f) $x = 0$

g) no axis of symmetry h) $y = x$ and $y = -x$

i) $y = x - 1$ and $y = -x - 1$ j) $y = x + 2$ and $y = -x + 2$

4. a) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 0$

b) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 0$

c) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 1$

d) Domain $\rightarrow x \in R$
Range $\rightarrow y \geq 0$

e) Domain $\rightarrow x \in R$
Range $\rightarrow y \geq 0$

f) Domain $\rightarrow x \in R$
Range $\rightarrow y \geq 2$

g) Domain $\rightarrow x \in R$
Range $\rightarrow y > 0$

h) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 0$

i) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq -1$

j) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 2$

5. a) (0; 0) b) (0; 0)

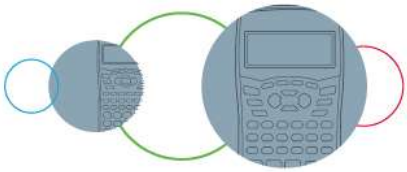
c) (0; 2) d) (0; -1)

6. a) $y = \frac{k}{x} + q$ $\left(-9; -\frac{1}{3}\right)$ and (3; 1)

$-\frac{1}{3} = \frac{k}{-9} + q$ and $1 = \frac{k}{3} + q$

$3 = k - 9q$ $3 = k + 3q$ …2

$3 + 9q = k$ …1



Subs 1 into 2

$\therefore 3 = 3 + 9q + 3q$

$\therefore 0 = 12q$

$\therefore q = 0$

$\therefore y = \frac{3}{x}$

Subs back into 1

$\therefore 3 + 9(0) = k$

$\therefore k = 3$

b) $y = ax^2 + q$ (0; 3) and (3; 12)

$\therefore y = ax^2 + 3$ Subs in (3; 12)

$\therefore 12 = a(3)^2 + 3$

$\therefore 9 = 9a$

$\therefore 1 = a$ $\therefore y = x^2 + 3$

c) $y = a^x + q$ $\left(-3; 1\frac{1}{8}\right)$ and y =1

$\therefore y = a^x + 1$ Subs in $\left(-3; 1\frac{1}{8}\right)$

$\therefore 1\frac{1}{8} = a^{-3} + 1$

$\therefore \frac{1}{8} = a^{-3}$

$\therefore 8 = a^3$

$\therefore \sqrt[3]{8} = a$

$\therefore a = 2$ $\therefore y = 2^x + 1$

d) $\perp$ to $y = \frac{1}{2}x + 3$ at (2; 4)

$\therefore m_1 \times \frac{1}{2} = -1$

$\therefore m_1 = -2$ $\therefore y = -2x + c$ Subs in (2; 4)

$\therefore 4 = -2(2) + c$

$\therefore 8 = c$ $\therefore y = -2x + 8$

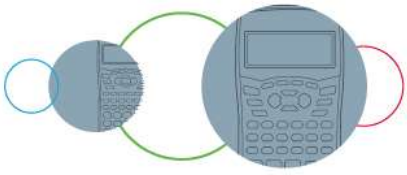
e) $y = \frac{k}{x} + q$ (-2; -4) and y = -2

$\therefore y = \frac{k}{x} - 2$ Subs in (-2; -4)

$\therefore -4 = \frac{k}{-2} - 2$

$\therefore -2 = \frac{k}{-2}$

$\therefore k = 4$ $\therefore y = \frac{4}{x} - 2$



f) $y = ax^2 + q$ (-2; 7) and (5; 49)

$\therefore 7 = a(-2)^2 + q$ and $49 = a(5)^2 + q$

$\therefore 7 = 4a + q$ $49 = 25a + q$ … 2

$\therefore 7 - 4a = q$ …1 Subs 1 into 2

$\therefore 49 = 25a + 7 - 4a$

$\therefore 42 = 21a$

$\therefore 2 = a$

Subs back into 1:

$\therefore 7 - 4(2) = q$

$\therefore q = 7 - 8$

$\therefore q = -1$ $\therefore y = 2x^2 - 1$

g) $y = \frac{k}{x} + q$ (-5; 3) and y = 2

$\therefore y = \frac{k}{x} + 2$ Subs in (-5; 3)

$\therefore 3 = \frac{k}{-5} + 2$

$\therefore 1 = \frac{k}{-5}$

$\therefore k = -5$ $\therefore y = -\frac{5}{x} + 2$

h) $y = -a^x$ $\left(-1; -\frac{1}{3}\right)$

$\therefore -\frac{1}{3} = -a^{-1}$

$\therefore \frac{1}{3} = a^{-1}$

$\therefore 3 = a$ $\therefore y = -(3)^x$

i) $y = ax^2 + q$ (-5; -21) and (0; 4)

$\therefore y = ax^2 + 4$ Subs in (-5; -21)

$\therefore -21 = a(-5)^2 + 4$

$\therefore -25 = 25a$

$\therefore a = -1$ $\therefore y = -x^2 + 4$

j) $y = mx + c$ $\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$ (-4; -17)

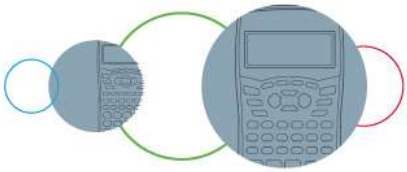
$\therefore m = \frac{-17-(-2)}{-4-1}$ (1; -2)

$\therefore m = \frac{-15}{-5} = 3$

$\therefore y = 3x + c$ Subs in (1; -2)

$\therefore -2 = 3(1) + c$

$\therefore c = -5$ $\therefore y = 3x - 5$



7. a) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 0$

b) Domain $\rightarrow x \in R$
Range $\rightarrow y \geq 3$

c) Domain $\rightarrow x \in R$
Range $\rightarrow y > 1$

d) Domain $\rightarrow x \in R$
Range $\rightarrow y \in R$

e) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq -2$

f) Domain $\rightarrow x \in R$
Range $\rightarrow y \geq -1$

g) Domain $\rightarrow x \in R; x \neq 0$
Range $\rightarrow y \in R; y \neq 2$

h) Domain $\rightarrow x \in R$
Range $\rightarrow y < 0$

i) Domain $\rightarrow x \in R$
Range $\rightarrow y \leq 4$

j) Domain $\rightarrow x \in R$
Range $\rightarrow y \in R$

8. a) $g(x) = mx + c$ A (-1; 0)
B (0; 2) → y –intercept ∴ c = 2

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m = \frac{2-0}{0-(-1)}$

$\therefore m = \frac{2}{1}$

$\therefore m = 2$ $\therefore g(x) = 2x + 2$

And $f(x) = ax^2 + b$ A (-1; 0)
E (0; 5) → y-intercept ∴ b = 5

$\therefore 0 = a(-1)^2 + 5$

$\therefore -5 = a$ $\therefore f(x) = -5x^2 + 5$

b) $\therefore g(x) = f(x)$

$\therefore 2x + 2 = -5x^2 + 5$

$\therefore 0 = -5x^2 - 2x + 5 - 2$

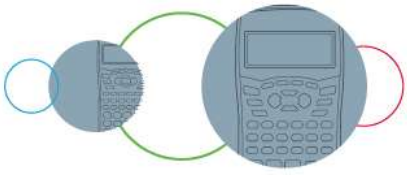
$\therefore 0 = -5x^2 - 2x + 3$

$\therefore 0 = 5x^2 + 2x - 3$

$\therefore 0 = (5x - 3)(x + 1)$

$\therefore x = \frac{3}{5}$ OR $x = -1 \rightarrow$ A $\therefore y = 2\left(\frac{3}{5}\right) + 2$

$\therefore y = \frac{6}{5} + 2 = 3\frac{1}{5}$ $\therefore$ C $\left(\frac{3}{5}; 3\frac{1}{5}\right)$



c) EB = 5 -2
= 3 units

d) $x = 0$

e) $\perp$ to $g(x)$ $\quad m_g = 2$

$\therefore m_h \times 2 = -1$

$\therefore m_h = -\frac{1}{2}$ $\quad \therefore y = -\frac{1}{2}x + c$ Subs in D (1; 0)

$\therefore 0 = -\frac{1}{2}(1) + c$

$\therefore c = \frac{1}{2}$ $\quad \therefore h(x) = -\frac{1}{2}x + \frac{1}{2}$

f) $g(x) = h(x)$

$\therefore 2x + 2 = -\frac{1}{2}x + \frac{1}{2}$

$\therefore \frac{5}{2}x = -\frac{3}{2}$

$\therefore x = -\frac{3}{5}$ and $y = 2\left(-\frac{3}{5}\right) + 2$

$y = -\frac{6}{5} + 2$

$y = \frac{4}{5}$ $\quad \therefore g(x)$ and $h(x)$ intersect at $\left(-\frac{3}{5}; \frac{4}{5}\right)$

And $h(x) = f(x)$

$\therefore -\frac{1}{2}x + \frac{1}{2} = -5x^2 + 5$

$\therefore 0 = -5x^2 + \frac{1}{2}x + 4\frac{1}{2}$

$\therefore 0 = 10x^2 - x - 9$

$\therefore 0 = (10x + 9)(x - 1)$

$\therefore x = -\frac{9}{10}$ $\quad OR \quad x = 1 \rightarrow$ D (1; 0). and $y = -\frac{1}{2}\left(-\frac{9}{10}\right) + \frac{1}{2}$

$\therefore y = \frac{19}{20}$ $\quad \left(-\frac{9}{10}; \frac{19}{20}\right)$

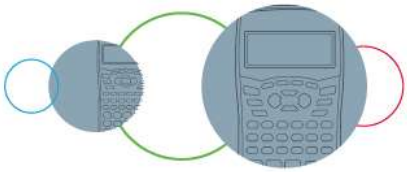
9. a) $m(x) = \frac{k}{x} + q$ $\quad$ And $p(x) = mx + c$ $\quad$ (0; 3)

$\therefore m(x) = \frac{k}{x} + 1$ Subs in (-3; 0) $\therefore p(x) = mx + 3$ Subs in (-3; 0)

$\therefore 0 = \frac{k}{-3} + 1$ $\quad \therefore 0 = m(-3) + 3$

$\therefore -1 = \frac{k}{-3}$ $\quad \therefore -3 = -3m$

$\therefore k = 3$ $\quad \therefore m(x) = \frac{3}{x} + 1$ $\quad \therefore m = 1$ $\quad \therefore p(x) = x + 3$



b) $p(x) = m(x)$ $\therefore x + 3 = \frac{3}{x} + 1$

$\therefore x^2 + 3x = 3 + x$

$\therefore x^2 + 2x - 3 = 0$

$\therefore (x + 3)(x - 1) = 0$

$\therefore x = -3$ (B) and $x = 1$ $\therefore y = 1 + 3 = 4$

$\therefore$ A (1; 4)

c) $x = 0$

d) $y = x + 1$ and $y = -x + 1$

e) $m = 1$

$q(x) = x + c$ Subs in (-1; 0)

$\therefore 0 = -1 + c$

$\therefore x = 1$ $\therefore y = x + 1$

f) The line in question e is one of the axes of symmetry for the function of $m(x)$

g) $q(x) = m(x)$

$\therefore x + 1 = \frac{3}{x} + 1$

$\therefore x = \frac{3}{x}$

$\therefore x^2 = 3$

$\therefore x = \pm\sqrt{3}$

$\therefore y = 1 + \sqrt{3}$ Or $y = 1 - \sqrt{3}$

$\therefore q(x)$ intersects $m(x)$ at the points $(\sqrt{3}; 1 + \sqrt{3})$ and $(-\sqrt{3}; 1 - \sqrt{3})$.