SHARP

Worksheet 1 Memo: Exponents and Surds

Grade 11 Mathematics

1. i) When the same bases are multiplied the exponents are added → $a^n \times a^m = a^{n+m}$

   ii) When the same bases are divided the exponents are subtracted → $a^n \div a^m = a^{n-m}$

   iii) When a power is raised to an exponent, the exponents are multiplied → $(a^n)^m = a^{n \times m}$

   iv) When a power is under a root sign the exponent is divided by the root → $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

   v) Anything to the power of zero is one → $a^0 = 1$

   vi) When a power has a negative exponent, the power is inverted (placed under 1) and the exponent becomes positive. → $a^{-n} = \frac{1}{a^n}$

2. i) You may only add or subtract like terms e.g. $3\sqrt{a} + 4\sqrt{a} = 7\sqrt{a}$

   ii) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ or vice versa

   iii) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ or vice versa

3. a) $(ab^2c^3)^0$

   $= 1$

   b) $\left(\frac{1}{xy^3}\right)^2$

   $= \frac{1}{x^2y^6}$ (K)

   c) $(x^2yz^{-1})^3 \times (x^4yz^3)^{-1}$

   $= x^6y^3z^{-3} \times x^{-4}y^{-1}z^{-3}$

   $= x^2y^2z^{-6}$

   $= \frac{x^2y^2}{z^6}$

   d) $\left(\frac{1}{xy^2}\right)^{-2} \times \left(\frac{x^3}{y^2}\right)^{-1}$

   $= x^2y^4 \times \frac{y^2}{x^3}$

   $= x^{-1}y^6$

   $= \frac{y^6}{x}$ (R)

   e) $\frac{a^2b^{-2}c^3}{ab^{-1}c^2} \times \frac{(a^2b^{-2}c)^{-1}}{a^{-1}b^2c^3} \div \frac{ab}{c^4}$

   $= \frac{a^2bc^3}{ab^2c^2} \times \frac{a^{-2}b^2c^{-1}}{a^{-1}b^2c^3} \times \frac{c^4}{ab}$

   $= \frac{ac}{b} \times \frac{1}{ac^4} \times \frac{c^4}{ab}$

   $= a^{1-1-1}b^{-1-1}c^{1+4-4}$

   $= a^{-1}b^{-2}c$

   $= \frac{c}{ab^2}$

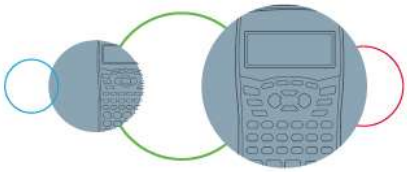
   f) $\frac{fg}{h^3} \times \left(\frac{h^4}{f^2g}\right)^{-2} \div \frac{a^0f^2g^{-2}}{h^{-3}}$

   $= \frac{fg}{h^3} \times \frac{f^4g^2}{h^8} \times \frac{h^{-3}}{f^2g^{-2}}$

   $= f^{1+4-2}g^{1+2-(-2)}h^{-3-3-8}$

   $= f^3g^5h^{-14}$

   $= \frac{f^3g^5}{h^{14}}$ (C)



g) $\left(\frac{1}{x}+\frac{x}{y}\right)^{-2}$

$=\left(\frac{y+x^2}{xy}\right)^{-2}$

$=\left(\frac{xy}{y+x^2}\right)^{2}$

$=\frac{x^2y^2}{y^2+2x^2y+x^4}$

h) $\frac{27^{x+1}.18^{x-1}}{36^{3-x}}$ (C)

$=\frac{3^{3(x+1)}.(3^2.2)^{x-1}}{(3^2.2^2)^{3-x}}$

$=\frac{3^{3x+3}.3^{2(x-1)}2^{x-1}}{3^{(6-2x)}2^{6-2x}}$

$=3^{3x+3+2x-2-(6-2x)}.2^{x-1-(6-2x)}$

$=3^{5x+1-6+2x}.2^{x-1-6+2x}$

$=3^{7x-5}.2^{3x-7}$

i) $\frac{8^{2x+1}+4^{3x-1}}{2^{3x}}$

$=\frac{(2^3)^{2x+1}+(2^2)^{3x-1}}{2^{3x}}$

$=\frac{2^{6x+3}+2^{6x-2}}{2^{3x}}$

$=\frac{2^{6x}(2^3+2^{-2})}{2^{3x}}$

$=2^{3x}\left(\frac{33}{4}\right)$

$=2^{3x-2}.33$

j) $\frac{54^{x+1}.36^{x-1}}{24^{2x-3}}$ (C)

$=\frac{(3^3.2)^{x+1}.(2^2.3^2)^{x-1}}{(2^3.3)^{2x-3}}$

$=\frac{3^{3x+3}.2^{x+1}.2^{2x-2}.3^{2x-2}}{2^{6x-9}.3^{2x-3}}$

$=3^{3x+3+2x-2-(2x-3)}.2^{x+1+2x-2-(6x-9)}$

$=3^{5x+1-2x+3}.2^{3x-1-6x+9}$

$=3^{3x+4}.2^{8-3x}$

4. a) $5^{3x}-5^{3x-1}=4$

$\therefore 5^{3x}(1-5^{-1})=4$

$\therefore 5^{3x}\left(\frac{4}{5}\right)=4$

$\therefore 5^{3x}=5$

$\therefore 3x=1$

$\therefore x=\frac{1}{3}$

b) $\frac{4^{x+2}+4^{x-1}}{5}=\frac{13}{16}$ (C)

$\therefore 4^{x+2}+4^{x-1}=\frac{65}{16}$

$\therefore 4^{x}(4^2+4^{-1})=\frac{65}{16}$

$\therefore 4^{x}\left(\frac{65}{4}\right)=\frac{65}{16}$

$\therefore 4^{x}=\frac{1}{4}=4^{-1}$

$\therefore x=-1$

c) $2^{x-1}+2^{2+x}=144$

$\therefore 2^{x}(2^{-1}+2^{2})=144$

$\therefore 2^{x}\left(4\frac{1}{2}\right)=144$

$\therefore 2^{x}=32$

$\therefore 2^{x}=2^{5}$

$\therefore x=5$

d) $3^{x+1}.5-4.3^{x+2}=-\frac{7}{3}$ (C)

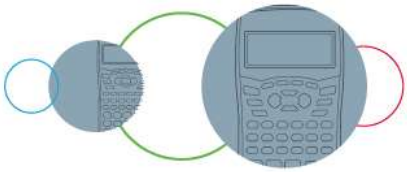
$\therefore 3^{x}(3.5-4.3^{2})=-\frac{7}{3}$

$\therefore 3^{x}(-21)=-\frac{7}{3}$

$\therefore 3^{x}=\frac{1}{9}$

$\therefore 3^{x}=3^{-2}$

$\therefore x=-2$



e) $2^x - 5.2^{x+1} = -144$

$\therefore 2^x(1 - 5.2) = -144$

$\therefore 2^x(-9) = -144$

$\therefore 2^x = 16$

$\therefore 2^x = 2^4$

$\therefore x = 4$

f) $5^{x+2} + 5^x = 26$ (R)

$\therefore 5^x(5^2 + 1) = 26$

$\therefore 5^x(26) = 26$

$\therefore 5^x = 1$

$\therefore 5^x = 5^0$

$\therefore x = 0$

g) $5.3^{x-2} - 5.3^{x+1} = -\frac{130}{729}$

$\therefore 5(3^{x-2} - 3^{x+1}) = -\frac{130}{729}$

$\therefore 3^x(3^{-2} - 3) = -\frac{26}{729}$

$\therefore 3^x\left(-\frac{26}{9}\right) = -\frac{26}{729}$

$\therefore 3^x = \frac{1}{81}$

$\therefore 3^x = 3^{-4}$

$\therefore x = -4$

h) $5^{2x+4} - 25^{x-1} = 78\,120$ (C)

$\therefore 5^{2x+4} - 5^{2(x-1)} = 78120$

$\therefore 5^{2x+4} - 5^{2x-2} = 78\,120$

$\therefore 5^{2x}(5^4 - 5^{-2}) = 78\,120$

$\therefore 5^{2x}\left(\frac{15624}{25}\right) = 78\,120$

$\therefore 5^{2x} = 125$

$\therefore 5^{2x} = 5^3$

$\therefore 2x = 3$

$\therefore x = \frac{3}{2}$

i) $3^{2-x} - 3^{-x-3} = \frac{242}{9}$

$\therefore 3^{-x}(3^2 - 3^{-3}) = \frac{242}{9}$

$\therefore 3^{-x}\left(\frac{242}{27}\right) = \frac{242}{9}$

$\therefore 3^{-x} = 3$

$\therefore -x = 1$

j) $4^{x+1}.3 + 5.2^{2x-1} - 7 = \frac{1}{4}$

$\therefore 2^{2(x+1)}.3 + 5.2^{2x-1} = 7\frac{1}{4}$

$\therefore 2^{2x+2}.3 + 5.2^{2x-1} = 7\frac{1}{4}$

$\therefore 2^{2x}(3.2^2 + 5.2^{-1}) = 7\frac{1}{4}$

$\therefore 2^{2x}\left(14\frac{1}{2}\right) = 7\frac{1}{4}$

$\therefore 2^{2x} = \frac{1}{2}$

$\therefore 2^{2x} = 2^{-1}$

$\therefore 2x = -1$

$\therefore x = -\frac{1}{2}$

k) $5^x = 200$

trial and error

$\therefore x = 3.29$

l) $3^x.4 + 5 = 25$

$\therefore 3^x.4 = 20$

$\therefore 3^x = 5$ trial and error $\rightarrow x = 1.46$

5. a) $4^{\frac{3}{2}}$

$= (2^2)^{\frac{3}{2}}$

$= 2^3$

$= 8$

b) $64^{\frac{2}{3}}$

$= (4^3)^{\frac{2}{3}}$

$= 4^2$

$= 16$

c) $27^{\frac{2}{9}}$

$= (3^3)^{\frac{2}{9}}$

$= 3^{\frac{2}{3}}$

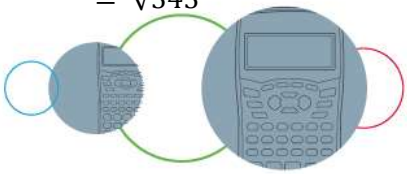
$= \sqrt[3]{9}$

d) $49^{\frac{3}{4}}$

$= (7^2)^{\frac{3}{4}}$

$= 7^{\frac{3}{2}}$

$= \sqrt{343}$



e) $81^{\frac{5}{4}}$

$= (3^4)^{\frac{5}{4}}$

$= 3^5$

$= 243$

f) $48^{-\frac{1}{2}}$

$= \sqrt{\frac{1}{48}}$

$= \frac{1}{\sqrt{48}}$

$= \frac{1}{4\sqrt{3}}$

g) $100^{-\frac{3}{2}}$

$= (10^2)^{-\frac{3}{2}}$

$= 10^{-3}$

$= 0.001$

h) $32^{-\frac{2}{5}}$

$= (2^5)^{-\frac{2}{5}}$

$= 2^{-2}$

$= \frac{1}{4}$

i) $36^{-\frac{5}{2}} = (6^2)^{-\frac{5}{2}}$

$= 6^{-5}$

$= \frac{1}{6^5}$

j) $24^{\frac{1}{3}} = \sqrt[3]{24}$

$= \sqrt[3]{8 \times 3}$

$= 2\sqrt[3]{3}$

6. a) surd b) not a surd c) surd d) not a surd
e) not a surd f) surd g) not a surd h) not a surd
i) surd j) surd

7. a) $\frac{\sqrt{80}+\sqrt{45}}{\sqrt{125}}$

$= \frac{\sqrt{16\times5}+\sqrt{9\times5}}{\sqrt{25\times5}}$

$= \frac{4\sqrt{5}+3\sqrt{5}}{5\sqrt{5}}$

$= \frac{7\sqrt{5}}{5\sqrt{5}}$

$= \frac{7}{5} = 1\frac{2}{5}$

b) $\frac{\sqrt{8}-\sqrt{2}}{\sqrt{50}}$

$= \frac{\sqrt{4\times2}-\sqrt{2}}{\sqrt{25\times2}}$

$= \frac{2\sqrt{2}-\sqrt{2}}{5\sqrt{2}}$

$= \frac{\sqrt{2}}{5\sqrt{2}}$

$= \frac{1}{5}$

(R)

c) $\frac{\sqrt{48}+\sqrt{147}}{\sqrt{242}}$

$= \frac{\sqrt{16\times3}+\sqrt{49\times3}}{\sqrt{121\times2}}$

$= \frac{4\sqrt{4}+7\sqrt{3}}{11\sqrt{2}}$

$= \frac{11\sqrt{3}}{11\sqrt{2}}$

$= \frac{\sqrt{3}}{\sqrt{2}}$

$= \sqrt{\frac{3}{2}}$

d) $\frac{\sqrt{2}(\sqrt{80}-\sqrt{45})}{\sqrt{1000}}$

$= \frac{\sqrt{160}-\sqrt{90}}{\sqrt{1000}}$

$= \frac{\sqrt{16\times10}-\sqrt{9\times10}}{\sqrt{100\times10}}$

$= \frac{4\sqrt{10}-3\sqrt{10}}{10\sqrt{10}}$

$= \frac{\sqrt{10}}{10\sqrt{10}}$

$= \frac{1}{10}$

(R)

e) $(5+\sqrt{2})(5-\sqrt{2})$

$= 25 - 5\sqrt{2} + 5\sqrt{2} - 2$

$= 25 - 2$

$= 23$

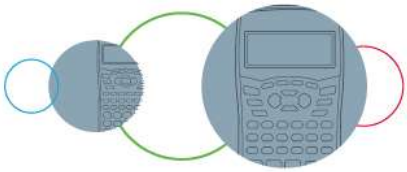
f) $(3\sqrt{2}-1)(3\sqrt{2}-2)$

$= 9(2) - 6\sqrt{2} - 3\sqrt{2} + 2$

$= 18 - 9\sqrt{2} + 2$

$= 20 - 9\sqrt{2}$

(R/C)



g) $(3\sqrt{2}+2)^2$

$= 9(2) + 12\sqrt{2} + 4$

$= 18 + 12\sqrt{2} + 4$

$= 22 + 12\sqrt{2}$

h) $(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2})$ (C)

$= 7 - \sqrt{14} + \sqrt{14} - 2$

$= 7 - 2$

$= 5$

i) $(4-2\sqrt{5})(5-3\sqrt{5})$

$= 20 - 12\sqrt{5} - 10\sqrt{5} + 6(5)$

$= 20 - 22\sqrt{5} + 30$

$= 50 - 22\sqrt{5}$

j) $\left(\frac{2}{\sqrt{3}}+1\right)\left(\frac{\sqrt{3}}{2}-1\right)$ (P)

$= 1 - \frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2} - 1$

$= -\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2}$

$= \frac{-4+3}{2\sqrt{3}}$

$= -\frac{1}{2\sqrt{3}}$

8. a) $\sqrt{x-3} = 2$

$\therefore (\sqrt{x-3})^2 = (2)^2$

$\therefore x - 3 = 4$

$\therefore x = 7$

Check: $\sqrt{7-3} = 2$ ✓

b) $\sqrt[3]{4+x} = 3$ (P)

$\therefore (\sqrt[3]{4+x})^3 = (3)^3$

$\therefore 4 + x = 27$

$\therefore x = 23$

Check: $\sqrt[3]{4+23} = 3$ ✓

c) $\sqrt{-x-1} = x + 1$

$\therefore (\sqrt{-x-1})^2 = (x+1)^2$

$\therefore -x - 1 = x^2 + 2x + 1$

$\therefore 0 = x^2 + 3x + 2$

$\therefore 0 = (x+2)(x+1)$

$\therefore x = -2$ OR $x = -1$

Check: $\sqrt{2-1} \neq -2 + 1$ ×

Check: $\sqrt{1-1} = -1 + 1$ ✓

d) $x = \sqrt{x+2}$ (P)

$\therefore (x)^2 = (\sqrt{x+2})^2$

$\therefore x^2 = x + 2$

$\therefore x^2 - x - 2 = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ OR $x = -1$

Check: $2 = \sqrt{2+2}$ ✓

Check: $-1 = \sqrt{-1+2}$ ×

e) $\sqrt{3x^2} - \sqrt{12} = 0$

$\therefore (\sqrt{3x^2})^2 = (\sqrt{12})^2$

$\therefore 3x^2 = 12$

$\therefore x^2 = 4$

$\therefore x = \pm 2$

Check: $\sqrt{3(2)^2} = \sqrt{12}$ ✓

Check: $\sqrt{3(-2)^2} = \sqrt{12}$ ✓

f) $\sqrt{18} - x\sqrt{2} = \sqrt{32}$ (C)

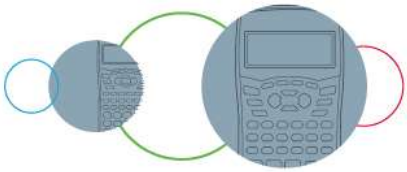
$\therefore -x\sqrt{2} = \sqrt{32} - \sqrt{18}$

$\therefore -x\sqrt{2} = 4\sqrt{2} - 3\sqrt{2}$

$\therefore -x\sqrt{2} = \sqrt{2}$

$\therefore -x = 1$

$\therefore x = -1$



g) $\sqrt{x+2} = 9$

$\therefore \left(\sqrt{x+2}\right)^2 = (9)^2$

$\therefore x + 2 = 81$

$\therefore x = 79$

Check: $\sqrt{79+2} = 9$ ✓

h) $(x-3)^{\frac{1}{2}} = 4$ (C)

$\therefore \left[(x-3)^{\frac{1}{2}}\right]^2 = (4)^2$

$\therefore x - 3 = 16$

$\therefore x = 19$

Check: $(19-3)^{\frac{1}{2}} = 4$ ✓

i) $\sqrt{6-2x} + 3 = x$

$\therefore \left(\sqrt{6-2x}\right)^2 = (x-3)^2$

$\therefore 6 - 2x = x^2 - 6x + 9$

$\therefore 0 = x^2 - 4x + 3$

$\therefore 0 = (x-3)(x-1)$

$\therefore x = 3 \;\; OR \;\; x = 1$

Check: $\sqrt{6-2(3)} + 3 = 3$ ✓

Check: $\sqrt{6-2(1)} + 3 = 1$ ×

j) $x^2 - 5 = \sqrt{x^2+1}$ (P)

$\therefore (x^2-5)^2 = \left(\sqrt{x^2+1}\right)^2$

$\therefore x^4 - 10x^2 + 25 = x^2 + 1$

$\therefore x^4 - 11x^2 + 24 = 0$

$\therefore (x^2-8)(x^2-3) = 0$

$\therefore x^2 = 8 \;\; OR \;\; x^2 = 3$

Check: $8 - 5 = \sqrt{8+1}$ ✓ $\quad \therefore x = \pm\sqrt{8}$

Check: $3 - 5 = \sqrt{3+1}$ ×

9. a) $\frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

b) $\frac{2}{\sqrt{2}}$ (P)

$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

c) $\frac{1+\sqrt{5}}{\sqrt{5}}$

$= \frac{1+\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}+5}{5}$

$= \frac{\sqrt{5}}{5} + 1$

d) $\frac{\sqrt{3}+2}{\sqrt{3}}$ (P)

$= \frac{\sqrt{3}+2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3+2\sqrt{3}}{3}$

$= 1 + \frac{2\sqrt{3}}{3}$

e) $\frac{3}{1-\sqrt{2}}$ Challenge ☺ (P)

$= \frac{3}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$= \frac{3+3\sqrt{2}}{1-2}$

$= -3 - 3\sqrt{2}$