SHARP

Worksheet 3 Memo: Number Patterns

Grade 11 Mathematics

1. Identify the type of pattern and give the next three terms:

a) $-\frac{3}{5}; -\frac{4}{5}; -1; -\frac{6}{5} \ldots$

linear; subtract $\frac{1}{5}$

$-\frac{7}{5}; -\frac{8}{5}; -\frac{9}{5}$

b) 12; 24; 48; 96… (R)

each term is multiplied by 2;

192; 384; 768

c) 1; 8; 27; 64 … (R)

each term position is cubed to give the next term; 125; 216; 343.

d) Quadratic (R)

1 6 14 25 39 56 76

5 8 11 14 17 20

3 3 3 3 3

e) 1; 4; 9; 16 ….

each term position is squared

25; 36; 49

f) 11; 16; 21; 26… (R)

linear, add 5;

31; 36; 41

g) $48; 16; \frac{16}{3}; \frac{16}{9} \ldots$ (R)

Divide each previous term by 3 $\rightarrow \frac{16}{27}; \frac{16}{81}; \frac{16}{243} \ldots$

h) Quadratic (R)

3 5 6 6 5 3 0

2 1 0 -1 -2 -3

-1 -1 -1 -1 -1

i) 0; 3; 8; 15…

each term position is squared and then 1 is subtracted: 24; 35; 48

j) 1; 10; 101; 1010… (R)

each term has a zero added after 1, or a 1 added after a 0; 10101; 101010; 1010101

2. a) 125; 119; 113; 107…

-6 -6 -6

$\therefore T_n = -6n + x$

$125 = -6(1) + x$

$x = 131$

$\therefore T_n = -6n + 131$

b) $-13\frac{1}{2}; -12\frac{3}{4}; -12; -11\frac{1}{4} \ldots$ (R)

$+\frac{3}{4}$ $+\frac{3}{4}$ $+\frac{3}{4}$

$\therefore T_n = \frac{3}{4}n + x$

$-13\frac{1}{2} = \frac{3}{4}(1) + x$

$x = -14\frac{1}{4}$

$\therefore T_n = \frac{3}{4}n - 14\frac{1}{4}$

c) 8 17 30 47

9 13 17

4 4

$a + b + c = 8$

$3a + b = 9$

$2a = 4$

$\therefore a = 2$

$\therefore b = 9 - 3(2) = 3$

$\therefore c = 8 - 2 - 3 = 3$

$\therefore T_n = 2n^2 + 3n + 3$

d) 5 8 11 14 (R)

3 3 3

$\therefore T_n = 3n + x$

$5 = 3(1) + x$

$x = 2$

$\therefore T_n = 3n + 2$

e) $12; 11\frac{1}{2}; 11; 10\frac{1}{2} \ldots$

$-\frac{1}{2}$ $-\frac{1}{2}$

$\therefore T_n = -\frac{1}{2}n + x$

$12 = -\frac{1}{2}(1) + x$

$12\frac{1}{2} = x$ (R)

$\therefore T_n = -\frac{1}{2}n + 12\frac{1}{2}$

f)

4 7 12 19

3 5 7

2 2

$a + b + c = 4$

$3a + b = 3$

$2a = 2$

$\therefore a = 1$

$\therefore b = 3 - 3(1) = 0$

$\therefore c = 4 - 1 - 0 = 3$

$\therefore T_n = n^2 + 3$

g) (R)

1 5 14 28

4 9 14

5 5

$a + b + c = 1$

$3a + b = 4$

$2a = 5$

$\therefore a = 2\frac{1}{2}$

$\therefore b = 4 - 3\left(2\frac{1}{2}\right) = -3\frac{1}{2}$

$\therefore c = 1 - 2\frac{1}{2} + 3\frac{1}{2} = 2$ (R)

$\therefore T_n = 2\frac{1}{2}n^2 - 3\frac{1}{2}n + 2$

h)

32 19 6 -7

-13 -13 -13

$\therefore T_n = -13n + x$

$32 = -13(1) + x$

$45 = x$

$\therefore T_n = -13n + 45$

i)

2 6 13 23

4 7 10

3 3

$a + b + c = 2$

$3a + b = 4$

$2a = 3$

$\therefore a = \frac{3}{2}$

j) (R)

1 4 5 4

3 1 -1

-2 -2

$a + b + c = 1$

$3a + b = 3$

$2a = -2$

$\therefore a = -1$

$\therefore b = 4 - 3\left(\frac{3}{2}\right) = -\frac{1}{2}$

$\therefore c = 2 - \frac{3}{2} + \frac{1}{2} = 1$

$\therefore T_n = \frac{3}{2}n^2 - \frac{1}{2}n + 1$

$\therefore b = 3 - 3(-1) = 6$

$\therefore c = 1 - 6 + 1 = -4$

$\therefore T_n = -n^2 + 6n - 4$

3. a) $T_{11} = -6(11) + 131$

$\therefore T_{11} = 65$

b) $T_{11} = \frac{3}{4}(11) - 14\frac{1}{4}$

$\therefore T_{11} = -6$

c) $T_{11} = 2(11)^2 + 3(11) + 3$

$\therefore T_{11} = 278$

d) $T_{11} = 3(11) + 2$

$\therefore T_{11} = 35$

e) $T_{11} = -\frac{1}{2}(11) + 12\frac{1}{2}$

$\therefore T_{11} = 7$

f) $T_{11} = (11)^2 + 3$

$\therefore T_{11} = 124$

g) $T_{11} = 2\frac{1}{2}(11)^2 - 3\frac{1}{2}(11) + 2$

$\therefore T_{11} = 266$

h) $T_{11} = -13(11) + 45$

$\therefore T_{11} = -98$

i) $T_{11} = \frac{3}{2}(11)^2 - \frac{1}{2}(11) + 1$

$\therefore T_{11} = 177$

j) $T_{11} = -(11)^2 + 6(11) - 4$

$\therefore T_{11} = -59$

4. $1 \quad 6\frac{1}{3} \quad 11\frac{2}{3} \quad 17$

$5\frac{1}{3} \quad 5\frac{1}{3} \quad 5\frac{1}{3}$

a) $T_n = 5\frac{1}{3}n + x$

$\therefore 1 = 5\frac{1}{3}(1) + x$

$\therefore x = -4\frac{1}{3}$

$\therefore T_n = 5\frac{1}{3}n - 4\frac{1}{3}$

b) $T_{13} = 5\frac{1}{3}(13) - 4\frac{1}{3}$

$\therefore T_{13} = 65$

c) $33 = 5\frac{1}{3}n - 4\frac{1}{3}$

$\therefore 37\frac{1}{3} = 5\frac{1}{3}n$

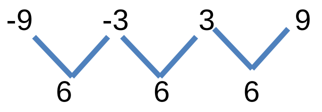
$\therefore n = 7$

d) $81 = 5\frac{1}{3}n - 4\frac{1}{3}$

$\therefore 85\frac{1}{3} = 5\frac{1}{3}n$

$\therefore n = 16$ $\quad\quad$ ∴ 81 is part of the sequence.

5.



b) $T_{20} = 6(20) - 15$

$\therefore T_{20} = 105$

a) $T_n = 6n + x$

$\therefore\ -9 = 6(1) + x$

$\therefore x = \ -15$

$\therefore T_n = 6n - 15$

c) $75 = 6n - 15$

$90 = 6n$

$\therefore 15 = n$

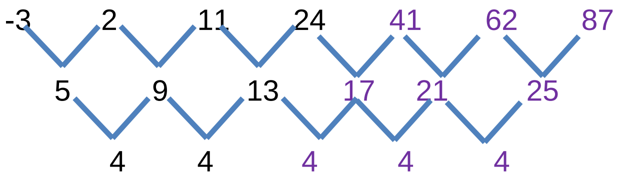
d) $T_5 = 6(5) - 15$

$T_5 = 15$

Sum $= -9 + (-3) + 3 + 9 + 15$

$= 15$

6. a)



b) $a + b + c = \ -3$

$3a + b = 5$

$2a = 4$

$\therefore a = 2$

$\therefore b = 5 - 3(2)$

$\therefore b = -1$

$\therefore c = \ -3 - 2 + 1$

$\therefore c = \ -4$

$\therefore T_n = 2n^2 - n - 4$

c) $T_{14} = 2(14)^2 - (14) - 4$

$\therefore T_{14} = 374$

d) $T_{12} = 2(12)^2 - (12) - 4$

$\therefore T_{12} = 272$

e) $116 = 2n^2 - n - 4$

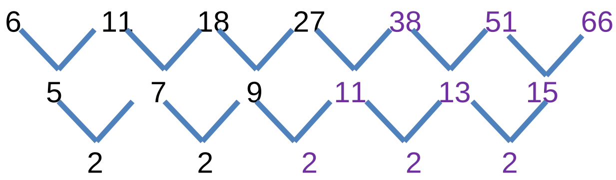
$0 = 2n^2 - n - 120$

$0 = (2n + 15)(n - 8)$

$\therefore n = -\frac{15}{2} \quad or\ n = 8$

N/A

7. a)



b) $a + b + c = 6$

$3a + b = 5$

$2a = 2$

$\therefore a = 1$

$\therefore b = 5 - 3(1)$

$\therefore b = 2$

$\therefore c = 6 - 2 - 1$

$\therefore c = 3$

$\therefore T_n = n^2 + 2n + 3$

c) $T_{10} = (10)^2 + 2(10) + 3$

$\therefore T_{10} = 123$

d) $258 = n^2 + 2n + 3$

$0 = n^2 + 2n - 255$

$0 = (n - 15)(n + 17)$

$\therefore n = 15 \quad or\ n = -17$

N/A

e) $98 = n^2 + 2n + 3$

$0 = n^2 + 2n - 95$

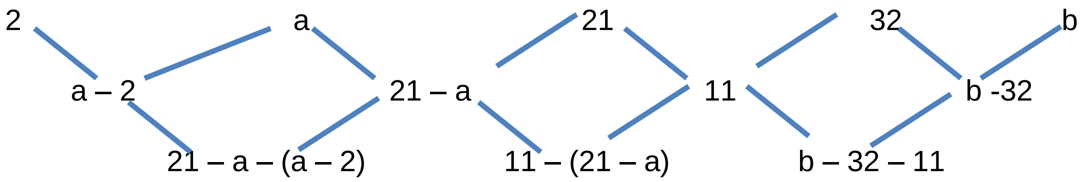
$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore n = \frac{-2 \pm \sqrt{2^2 - 4(1)(-95)}}{2(1)}$

$\therefore n = 8.80 \quad or \quad n = -10.80$

$\therefore$ 98 is not part of the sequence

8. a)



$\therefore 21 - a - a + 2 = 11 - 21 + a = b - 43$ (Ignore this one for now.)

$\therefore 23 - 2a = -10 + a$

$\therefore 33 = 3a$

$\therefore a = 11$

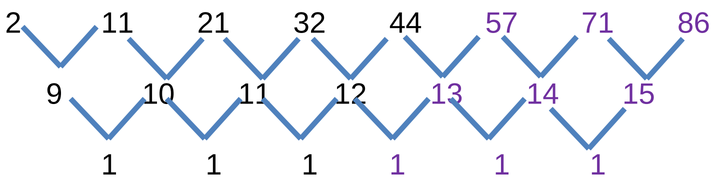
AND

$-10 + a = b - 43$

$\therefore -10 + 11 + 43 = b$

$\therefore b = 44$

b)



c) $a + b + c = 2$

$3a + b = 9$

$2a = 1$

$\therefore a = \frac{1}{2}$

$\therefore b = 9 - 3\left(\frac{1}{2}\right)$

$\therefore b = 7\frac{1}{2}$

$\therefore c = 2 - \frac{1}{2} - 7\frac{1}{2}$

$\therefore c = -6$

$$\therefore T_n = \frac{1}{2}n^2 + 7\frac{1}{2}n - 6$$

d) $T_{11} = \frac{1}{2}(11)^2 + 7\frac{1}{2}(11) - 6$

$\therefore T_{11} = 137$

f) $242 = \frac{1}{2}n^2 + 7\frac{1}{2}n - 6$

$0 = \frac{1}{2}n^2 + 7\frac{1}{2}n - 248$

$0 = n^2 + 15n - 496$

e) $176 = \frac{1}{2}n^2 + 7\frac{1}{2}n - 6$

$0 = \frac{1}{2}n^2 + 7\frac{1}{2}n - 182$

$0 = n^2 + 15n - 364$

$0 = (n - 13)(n + 28)$

$\therefore n = 13 \quad or \; n = -28$

N/A

$\therefore n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore n = \frac{-15 \pm \sqrt{15^2 - 4(1)(-496)}}{2(1)}$

$\therefore n = 16 \quad or \quad n = -31$

N/A

∴ 242 is a part of the sequence

It is the 16th term.

9. a)

2		2		a		b		50
	0		a – 2		b – a		50 – b	
		a – 2 – 0		b – a – (a – 2)		50 – b – (b – a)		

$\therefore a - 2 = b - a - a + 2 = 50 - b - b + a$

$\therefore a - 2 = b - 2a + 2$ AND $a - 2 = 50 - 2b + a$

$\therefore \; -2 = 50 - 2b$

$\therefore \; -52 = -2b$

$\therefore b = 26$

AND

$\therefore a - 2 = 26 - 2a + 2$

$\therefore 3a = 30$

$\therefore a = 10$

b)

2		2		10		26		50		82		122		170
	0		8		16		24		32		40		48	
		8		8		8		8		8		8		

c) $a + b + c = 2$

$3a + b = 0$

$2a = 8$

$\therefore a = 4$

$\therefore b = 0 - 3(4)$

$\therefore b = -12$

$\therefore c = 2 - 4 + 12$

$\therefore c = 10$

$$\therefore T_n = 4n^2 - 12n + 10$$

d) $T_{10} = 4(10)^2 - 12(10) + 10$

$\therefore T_{10} = 290$

e) $962 = 4n^2 - 12n + 10$

$0 = 4n^2 - 12n - 952$

$0 = n^2 - 3n - 238$

$0 = (n - 17)(n + 14)$

$\therefore n = 17 \quad or\ n = -14$

N/A

f) $1001 = 4n^2 - 12n + 10$

$0 = 4n^2 - 12n - 991$

$\therefore n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore n = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(-991)}}{2(4)}$

$\therefore n = 17{,}31 \quad or \quad n = -14{,}31$

$\therefore$ 1001 is not part of the sequence.

10. a)

120 136 146 150 148 140 126

16 10 4 -2 -8 -14

-6 -6 -6 -6 -6

b) $a + b + c = 120$

$3a + b = 16$

$2a = -6$

$\therefore a = -3$

$\therefore b = 16 - 3(-3)$

$\therefore b = 25$

$\therefore c = 120 - 25 + 3$

$\therefore c = 98$

$$\therefore T_n = -3t^2 + 25t + 98$$

c) $0 = -3t^2 + 25t + 98$

$0 = 3t^2 - 25t - 98$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{25 \pm \sqrt{(-25)^2 - 4(3)(-98)}}{2(3)}$

$t = 11{,}24$ seconds

d) $y = -3t^2 + 25t + 98$

$y = -3\left(t^2 - \frac{25}{3}t - \frac{98}{3}\right)$

$y = -3\left(t^2 - \frac{25}{3}t + \left(\frac{25}{6}\right)^2 - \left(\frac{25}{6}\right)^2 - \frac{98}{3}\right)$

$y = -3\left[\left(t - \frac{25}{6}\right)^2 - 50\frac{1}{36}\right]$

$y = -3\left(t - \frac{25}{6}\right)^2 + 150{,}08m$

e) make $t = 0$

$T_0 = -3(0)^2 + 25(0) + 98$

$\therefore T_0 = 98$

$\therefore$ The building is 98m high

11. a)

7776m 7676m 7524m 7320m 7064m 6750m 6396m

-100 -152 -204 -256 -308 -360

-52 -52 -52 -52 -52

b) $a + b + c = 7776$

$3a + b = -100$

$2a = -52$

$\therefore a = -26$

$\therefore T_n = -26t^2 - 22t + 7824$

$\therefore b = -100 - 3(-26)$

$\therefore b = -22$

$\therefore c = 7776 + 22 + 26$

$\therefore c = 7824$

c) $0 = -26t^2 - 22t + 7824$

$0 = 13t^2 + 11t - 3912$

$\therefore t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore t = \frac{-11 \pm \sqrt{11^2 - 4(13)(-3912)}}{2(13)}$

$\therefore t = 16{,}93$ seconds

e) $4436 = -26t^2 - 22t + 7824$

$0 = -26t^2 - 22t + 3388$

$0 = 13t^2 + 11 - 1694$

$\therefore t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore t = \frac{-11 \pm \sqrt{11^2 - 4(13)(-1694)}}{2(13)}$

$\therefore t = 11 \quad or \quad t = -11\frac{11}{13}$ N/A

d) $T_{15} = -26(15)^2 - 22(15) + 7824$

$\therefore T_{15} = 1644m$ from the surface

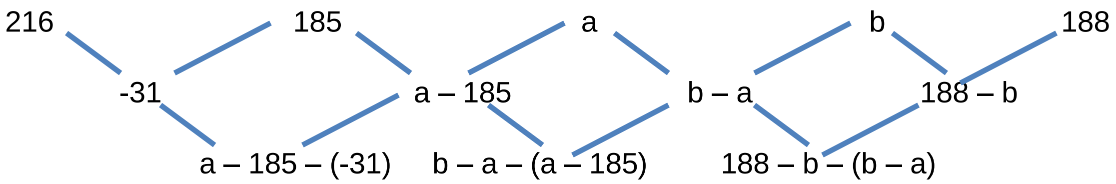
f) if $t = 17$ because it reaches the surface at 16,93 seconds

Then $T_{17} = -26(17)^2 - 22(17) + 7824$

$\therefore T_{17} = -64m$

In other words, 64m above the surface (i.e. in the air) which is impossible ∴ no the pattern cannot continue once the submarine has reached the surface.

12. a)



$\therefore a - 185 + 31 = b - a - a + 185 = 188 - b - b + a$

$\therefore a - 154 = b - 2a + 185$ AND $b - 2a + 185 = 188 - 2b + a$

$\therefore 3a - b = 339$ $3b - 3a = 3$

$\therefore b = 3a - 339 \; \ldots 1$ $b - a = 1$

$b = a + 1 \ldots 2$

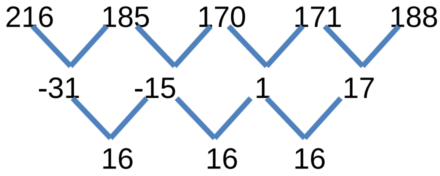
Subs 1 into 2:

$3a - 339 = 1 + a$

$\therefore 2a = 340$ $\therefore b = 1 + 170$

$\therefore a = 170$ $\therefore b = 171$

b)



$a + b + c = 216$

$3a + b = -31$

$2a = 16$

$\therefore a = 8$

$\therefore b = -31 - 3(8)$

$\therefore b = -55$

$\therefore c = 216 + 55 - 8$

$\therefore c = 263$

$$\therefore T_n = 8w^2 - 55w + 263$$

c) 3 months → 3 × 4 = 12 weeks $\therefore w = 12$

$\therefore T_{12} = 8(12)^2 - 55(12) + 263$

$\therefore T_{12} = R755$

d) $1000 = 8w^2 - 55w + 263$

$0 = 8w^2 - 55w - 737$

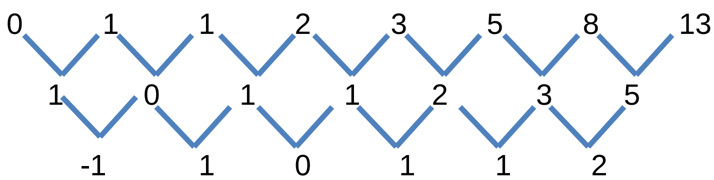
$\therefore w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore w = \frac{55 \pm \sqrt{55^2 - 4(8)(-737)}}{2(8)}$

$\therefore w = 13{,}63$ or $w = -6{,}76$

∴ after 14 weeks Sally will receive R1000 or more.

13.



∴ no the Fibonacci sequence is not a second common differences pattern as there is no common second difference.