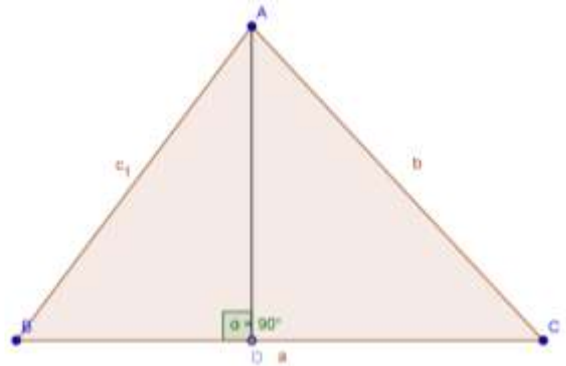
<u>**Worksheet 8 Memorandum: Trigonometry**</u>

<u>**Grade 11 Mathematics**</u>

1. a) Given a triangle ABC – draw in an altitude from vertex A:



The distance BD can be worked out using basic trigonometry ratios:

$\cos B = \frac{BD}{AB}$ $\therefore BD = c \cos B$

Now the distance CD can be given by subtracting BD from the total distance BC:

$a - c \cos B$

The altitude AD has a height which can be worked out using basic trigonometry ratios:

$\sin B = \frac{AD}{AB}$ $\therefore AD = c \sin B$

Now according to Pythagoras hypotenuse^2 (b^2) = the other two sides squared (for this we use ∆ADC):

$\therefore AC^2 = AD^2 + CD^2$

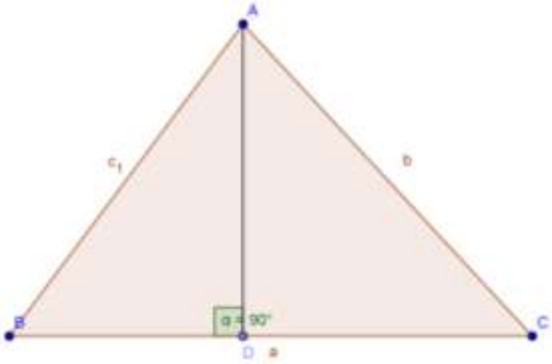
$\therefore b^2 = (c \sin B)^2 + (a - c \cos B)^2$

$\therefore b^2 = c^2 \sin^2 b + a^2 - 2a.c.\cos B + c^2 \cos^2 B$

$\therefore b^2 = a^2 + c^2(\sin^2 B + \cos^2 B) - 2ac \cos B$ $(\sin^2 B + \cos^2 B = 1)$

$\therefore b^2 = a^2 + c^2 - 2ac \cos B$

b) Given a triangle ABC – draw in an altitude from vertex A:



For this proof we are going to find to different ways of making AD from the two triangles.

In ∆ABD: and in ∆ADC:

$\sin B = \frac{AD}{AB}$ $\sin C = \frac{AD}{AC}$

$\therefore c \sin B = AD$ $\therefore b \sin C = AD$

Equating the two equations:

$c \sin B = b \sin C$ Now we manipulate the equation to get to the sin formula

$\therefore \frac{c \sin B}{b} = \sin C$

$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$

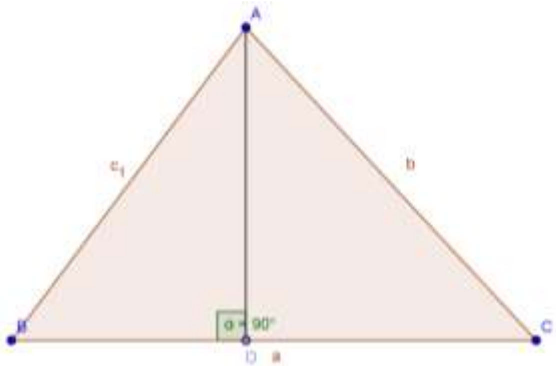
If we did the same thing with a perpendicular line from vertex B we could also prove that

$\frac{\sin A}{a} = \frac{\sin C}{c}$

And therefore: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

The inverse or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ can also be proved from the same method, but instead of dividing by the side, divide by the sin "A" .

c) Given ∆ABC – with D perpendicular to BC:



Area of a right angled triangle = $\frac{1}{2}\ base\ \times height$

In our triangle the base is BC.

To find the height: $\sin B = \frac{AD}{AB}$

$\therefore c \sin B = AD$

Now substitute these values into the area formula:

$\therefore Area = \frac{1}{2} \times a \times c \sin B$

$\therefore Area = \frac{1}{2} ac. \sin B$

2. a)

a = 5 | $b^2 = a^2 + c^2 - 2ac \cos B$

$\hat{B}$ = 35° | $\therefore b^2 = (5)^2 + (4)^2 - 2(5)(4)(\cos 35°)$

c = 4 | $\therefore b = \sqrt{8.2339}$

$\therefore b = 2.87$

b)

a = 2 | $b^2 = a^2 + c^2 - 2ac \cos B$

$\hat{B} = 59°$ | $\therefore b^2 = (2)^2 + (4)^2 - 2(2)(4) \cos 59\,°$

c = 4 | $\therefore b = \sqrt{11.7594}$

$\therefore b = 3.43$

c)

b = 19 | $a^2 = b^2 + c^2 - 2bc \cos A$

$\hat{A} = 104°$ | $\therefore a^2 = (19)^2 + (3)^2 - 2(19)(3) \cos 104°$

c = 3 | $\therefore a = \sqrt{397.5791}$

$\therefore a = 19.94$

d)

a = 4 | $b^2 = a^2 + c^2 - 2ac \cos B$

$\hat{B} = 14°$ | $\therefore b^2 = (4)^2 + (4)^2 - 2(4)(4)(\cos 14°)$

c = 4 | $\therefore b = \sqrt{0.9505}$

$\therefore b = 0.97$

e)

a = 4 | $b^2 = a^2 + c^2 - 2ac \cos B$

$\hat{B} = 76°$ | $\therefore b^2 = (4)^2 + (2)^2 - 2(4)(2)(\cos 76°)$

c = 2 | $\therefore b = \sqrt{16.1292}$

$\therefore b = 4.02$

3. a)

$\hat{B} = 180° - 65° - 49° = 66°$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \frac{a}{\sin 65°} = \frac{4}{\sin 49°}$ and $\therefore \frac{4}{\sin 49°} = \frac{b}{\sin 66°}$

$\therefore a = \frac{4 \sin 65°}{\sin 49°}$ | $\therefore b = \frac{4 \sin 66°}{\sin 49°}$

$\therefore a = 4.8$ | $\therefore b = 4.84$

b) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\therefore \frac{\sin 55°}{5} = \frac{\sin C}{6}$

$\therefore \sin C = \frac{6 \sin 55°}{5}$

$\therefore \sin C = 0.98298$

$\therefore \hat{C} = 79.41°$ $\therefore \hat{B} = 180° - 55° - 79.41° = 45.59°$

$\therefore \frac{5}{\sin 55°} = \frac{b}{\sin 45.59°}$

$\therefore b = \frac{5 \sin 45.59°}{\sin 55°}$

$\therefore b = 4.36$

c) $\hat{B} = 180° - 65° - 62° = 53°$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \frac{a}{\sin 65°} = \frac{5}{\sin 62°}$ and $\therefore \frac{b}{\sin 53°} = \frac{5}{\sin 62°}$

$\therefore a = \frac{5 \sin 65°}{\sin 62°}$ $\therefore b = \frac{5 \sin 53°}{\sin 62°}$

$\therefore a = 5.13$ $\therefore b = 4.52$

d) $\hat{A} = 180° - 101° - 39° = 40°$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \frac{5}{\sin 40°} = \frac{b}{\sin 101°}$ and $\therefore \frac{5}{\sin 40°} = \frac{c}{\sin 39°}$

$\therefore b = \frac{5 \sin 101°}{\sin 40°}$ $\therefore c = \frac{5 \sin 39°}{\sin 40°}$

$\therefore b = 7.64$ $\therefore c = 4.9$

e) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\therefore \frac{\sin 123°}{8} = \frac{\sin C}{3}$

$\therefore \sin C = \frac{3 \sin 123°}{8}$

$\therefore \sin C = 0.314501 \ldots$

$\therefore \hat{C} = 18.33°$ $\therefore \hat{B} = 180° - 123° - 18° = 39°$

$\therefore \frac{b}{\sin 39°} = \frac{8}{\sin 123°}$

$\therefore b = \frac{8 \sin 39°}{\sin 123°}$

$\therefore b = 6$

4.

2a) $Area = \frac{1}{2}bc\sin A$

$\therefore Area = \frac{1}{2}\ (5)(4)(\sin 35°)$

$\therefore Area = 5.74\ units^2$

2b) $Area = \frac{1}{2}ac\sin B$

$\therefore Area = \frac{1}{2}(2)(4)(\sin 59°)$

$\therefore Area = 3.43\ units^2$

2c) $Area = \frac{1}{2}bc\sin A$

$\therefore Area = \frac{1}{2}(19)(3)(\sin 104°)$

$\therefore Area = 27{,}65\ units^2$

2d) $Area = \frac{1}{2}ac\sin B$

$\therefore Area = \frac{1}{2}(4)(4)(\sin 14\,°)$

$\therefore Area = 1.94\ units^2$

2e) $Area = \frac{1}{2}ac\sin B$

$\therefore Area = \frac{1}{2}(4)(2)(\sin 76\,°)$

$\therefore Area = 3.88\ units^2$

3a) $Area = \frac{1}{2}bc\sin A$

$\therefore Area = \frac{1}{2}(4.84)(4)(\sin 65°)$

$\therefore Area = 8.77 units^2$

3b) $Area = \frac{1}{2}bc\sin A$

$\therefore Area = \frac{1}{2}(4.36)(6)(\sin 55°)$

$\therefore Area = 10.71\ units^2$

3c) $Area = \frac{1}{2}bc\sin A$

$\therefore Area = \frac{1}{2}(4.52)(5)(\sin 65°)$

$\therefore Area = 10.24\ units^2$

3d) $Area = \frac{1}{2}ac\sin B$

$\therefore Area = \frac{1}{2}(5)(4.9)(\sin 101\,°)$

$\therefore Area = 12.02\ units^2$

3e) $Area = \frac{1}{2}bc\sin A$

$\therefore Area = \frac{1}{2}(6)(3)(\sin 123\,°)$

$\therefore Area = 7.55\ units^2$

5.

a) $Area = \frac{1}{2}ab\sin C$

$\therefore 9 = \frac{1}{2}(6)(b)\sin 51°$

$\therefore b = \frac{9}{3\sin 51°}$

$\therefore b = 3.86$

$\therefore c^2 = a^2 + b^2 - 2ab\cos C$

$\therefore c^2 = (6)^2 + (3.86)^2 - 2(6)(3.86)\cos 51°$

$\therefore c = \sqrt{21.7495}$

$\therefore c = 4.66$

$\therefore \frac{\sin A}{a} = \frac{\sin C}{c}$

$\therefore \sin A = \frac{6\sin 51°}{4.66}$

$\therefore \sin A = 1.000$

$\therefore\ \hat{A} = 90°$

$\therefore\ \hat{B} = 180° - 90° - 51° = 39°$

b) $Area = \frac{1}{2} ac \sin B$

$\therefore 8 = \frac{1}{2}(3)(6) \sin B$

$\therefore \frac{8}{9} = \sin B$

$\therefore \ \hat{B} = 62.73°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$\therefore b^2 = (3)^2 + (6)^2 - 2(3)(6)(\cos 62.73°)$

$\therefore b = \sqrt{28.5054}$

$\therefore b = 5.34$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$

$\therefore \sin A = \frac{3 \sin 62.73}{5.34}$

$\therefore \sin A = 0.49935$

$\therefore \ \hat{A} = 29.96°$

$\therefore \ \hat{C} = 180° - 62.73° - 29.96° = 87.31°$

c) $Area = \frac{1}{2} ac \sin B$

$\therefore 7 = \frac{1}{2}(7)(3) \sin B$

$\therefore \frac{2}{3} = \sin B$

$\therefore \ \hat{B} = 41.81°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$\therefore b^2 = (7)^2 + (3)^2 - 2(7)(3)(\cos 41.81°)$

$\therefore b = \sqrt{26.6949}$

$\therefore b = 5.17$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$

$\therefore \sin A = \frac{7 \sin 41.81°}{5.17}$

$\therefore \sin A = 0.9026 \ldots$

$\therefore \ \hat{A} = 64.51°$

$\therefore \ \hat{C} = 180° - 41.81° - 64.51° = 73.68°$

d) $Area = \frac{1}{2} ab \sin C$

$\therefore 20 = \frac{1}{2}(5)(b)(\sin 104°)$

$\therefore b = \frac{20}{\frac{1}{2} \times 5 \times \sin 104°}$

$\therefore b = 8.24$

$\therefore \ c^2 = a^2 + b^2 - 2ab \cos C$

$\therefore c^2 = (5)^2 + (8.24)^2 - 2(5)(8.24)(\cos 104°)$

$\therefore c = \sqrt{112.83196}$

$\therefore c = 10.62$

$\therefore \frac{\sin A}{a} = \frac{\sin C}{c}$

$\therefore \sin A = \frac{5 \sin 104°}{10.62}$

$\therefore \sin A = 0.4568 \ldots$

$\therefore \ \hat{A} = 27.18°$

$\therefore \ \hat{B} = 180° - 104° - 27.18° = 48.82°$

e) $Area = \frac{1}{2}ac\sin B$

$\therefore 14 = \frac{1}{2}(7)(10)\sin B$

$\therefore \sin B = 0.4$

$\therefore \hat{B} = 23.58°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$\therefore b^2 = (7)^2 + (10)^2 - 2(7)(10)(\cos 23.58°)$

$\therefore b = \sqrt{20.68966}$

$\therefore b = 5.55$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b}$

$\therefore \sin A = \frac{7\sin 23.58°}{5.55}$

$\therefore \sin A = 0.50454 \ldots$

$\therefore \hat{A} = 30.3°$

$\therefore \hat{C} = 180° - 23.58° - 30.3° = 126.12°$

6. a) Area ΔABD = Area ΔABC – ΔArea ADC

∴ Area ΔABD = 28 – 16 = 12units^2

$\therefore Area = \frac{1}{2}ad\sin B$

$\therefore 12 = \frac{1}{2}(BD)(7)\sin 67°$

$\therefore BD = \frac{12}{\frac{1}{2}(7)(\sin 67°)}$

$\therefore BD = 3.72$

$b^2 = a^2 + c^2 - 2ac\cos B$

$\therefore (AD)^2 = (3.72)^2 + (7)^2 - 2(3.72)(7)(\cos 67°)$

$\therefore AD = \sqrt{42.48912}$

$\therefore AD = 6.52$

b) $Area = \frac{1}{2}ac\sin B$

$\therefore 28 = \frac{1}{2}(BC)(7)\sin 67°$

$\therefore BC = \frac{28}{\frac{1}{2}(7)(\sin 67°)}$

$\therefore BC = 8.69$

$b^2 = a^2 + c^2 - 2ac\cos B$

$\therefore (AC)^2 = (8.69)^2 + (7)^2 - 2(8.69)(7)(\cos 67°)$

$\therefore AC = \sqrt{76.97975}$

$\therefore AC = 8.77$

c) DC = BC – BD

∴ DC = 8.69 – 3.72 = 4.97

d) $\therefore d^2 = c^2 + a^2 - 2ac\cos D$

$\therefore d^2 - c^2 - a^2 = -2ac\cos D$

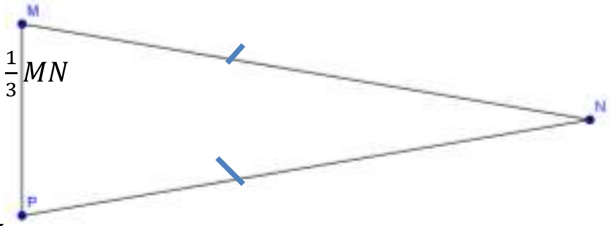
$\therefore \cos D = \frac{a^2+c^2-d^2}{2ac}$

$\therefore \cos D = \frac{4.97^2+6.52^2-8.77^2}{2(4.97)(6.52)}$

$\therefore \cos D = -0.149695 \ldots$

$\therefore \widehat{D'} = 81{,}39°$ $\qquad \therefore A\hat{D}C = 180° - 81.39° = 98.61°$

7. Draw a diagram:



Let $MN = x$:

$\therefore n^2 = m^2 + p^2 - 2mp\cos N$

$\therefore \left(\frac{1}{3}x\right)^2 = (x)^2 + (x)^2 - 2(x)(x)\cos N$

$\therefore \frac{1}{9}x^2 = 2x^2 - 2x^2\cos N$

$\therefore \frac{1}{9}x^2 = 2x^2(1 - \cos N)$

$\therefore \frac{\frac{1}{9}x^2}{2x^2} = 1 - \cos N$

$\therefore \frac{1}{18} = 1 - \cos N$

$\therefore -\frac{17}{18} = -\cos N$

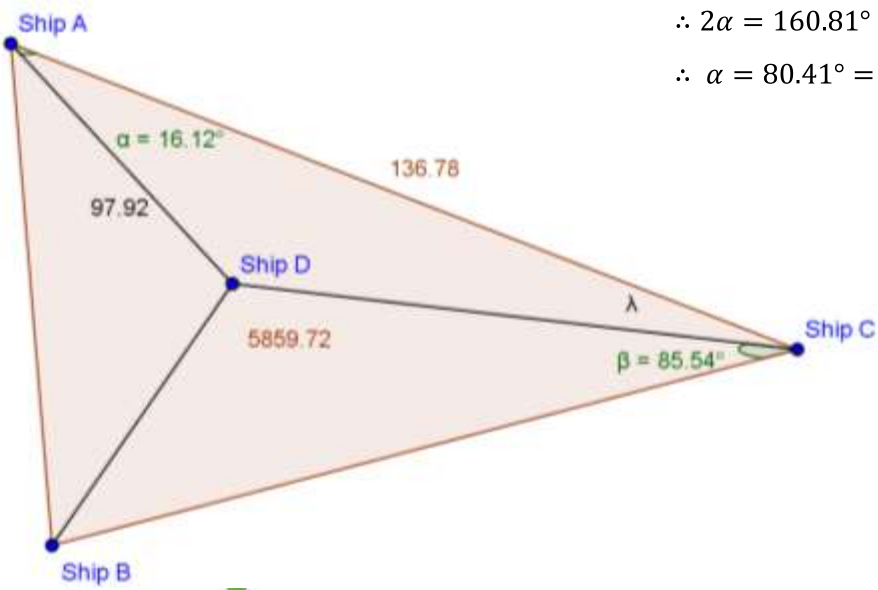
$\therefore \cos N = \frac{17}{18}$

$\therefore \hat{N} = 19.19°$ $\qquad \hat{M} = \hat{P} = \alpha$ $\qquad \therefore 180° = 19.19° + \alpha + \alpha$

$\therefore 2\alpha = 160.81°$

$\therefore \alpha = 80.41° = \hat{M} = \hat{P}$

8.



$a^2 = c^2 + d^2 - 2cd\cos A$

$\therefore a^2 = (97.92)^2 + (136.78)^2 - 2(97.92)(136.78)\,(\cos 16.12°)$

$\therefore a = \sqrt{2563.302787}$

$\therefore a = 50.63$ km between Ship D and Ship C

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\therefore \sin\lambda = \frac{97.92\sin 16.12°}{50.63}$

$\therefore \sin\lambda = 0.5369\ldots$

$\therefore\ \lambda = 32.48° = A\hat{C}D$ $\therefore A\hat{C}B = A\hat{C}D + D\hat{C}B = 32.48° + 85.54° = 118.02°$

$Area = \frac{1}{2}ab\sin C$

$\therefore 5859.72 = \frac{1}{2}(BC)(136.78)(\sin 118.02°)$

$\therefore BC = \frac{5859.72}{\frac{1}{2}(136.78)(\sin 118.02°)}$

$\therefore BC = 97.11$km between Ship B and Ship C

$\therefore c^2 = b^2 + d^2 - 2bd\cos C$

$\therefore BD^2 = (50.63)^2 + (97.11)^2 - 2(50.63)(97.11)(\cos 118.02°)$

$\therefore BD = \sqrt{16\,613.26166}$

$\therefore BD = 128.89$ km between Ship B and D

$\therefore$ the ship closest to Ship D is Ship C.

9. $a^2 = b^2 + c^2 - 2bc\cos A$

$\therefore (BC)^2 = (x)^2 + (7)^2 - 2(x)(7)(\cos 95.26°)$

$\therefore BC^2 = x^2 + 49 - 14x(-0.091675\ldots)$

$\therefore BC^2 = x^2 + 49 + 1.283455852x$ … 1

And

$c^2 = d^2 + b^2 - 2bd\cos C$

$\therefore \left(\frac{5}{4}x\right)^2 = (BC)^2 + (BC)^2 - 2(BC)(BC)\cos 34.54°$ Given that BC = CD $\therefore$ d = b

$\therefore \frac{25}{16}x^2 = BC^2 + BC^2 - 2BC^2(0.823730561)$

$\therefore \frac{25}{16}x^2 = 2BC^2 - 1.647461123BC^2$

$\therefore \frac{25}{16}x^2 = 0.352538876BC^2$ … 2

Substitute 1 into 2:

$\therefore \frac{25}{16}x^2 = 0.352538876(x^2 + 49 + 1.283455852x)$

$\therefore 0 = -1.209961123x^2 + 0.452468083x + 17.27440492$

Substitute into the quadratic equation:

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\therefore x = \frac{-0.452468083\pm\sqrt{(0.452468083)^2-4(-1.209961123)(17.27440492)}}{2(-1.209961123)}$

$\therefore x = 3.97 \approx 4$ $\quad or \quad x = -3.6 \rightarrow$ N/A (length cannot be negative).

$\therefore BC^2 = x^2 + 49 + 1.283455852x$ Substitute in $x = 4$

$\therefore BC^2 = (4)^2 + 49 + 1.283455852(4)$

$\therefore BC = \sqrt{70.13382341}$

$\therefore BC = 8.37$

10. a) $\cos A = \frac{AB}{AC}$

$\therefore AB = 9.63 \cos 49.46°$

$\therefore AB = 6.26$

b) $a^2 = b^2 + d^2 - 2bd \cos A$

$\therefore (BD)^2 = (4.34)^2 + (6.26)^2 - 2(4.34)(6.26)(\cos 49.46°)$

$\therefore BD = \sqrt{22.70543447}$

$\therefore BD = 4.77$

c) $a^2 = b^2 + c^2 - 2bc \cos A$

$\therefore (BC)^2 = (9.63)^2 + (6.26)^2 - 2(9.63)(6.26) \cos 49.46°$

$\therefore BC = \sqrt{53.55812164}$

$\therefore BC = 7.32$

d) $\hat{C} = 180° - 90° - 49.46° = 40.54°$

DC = 9.63 – 4.34 = 5.29

$\frac{c}{\sin C} = \frac{e}{\sin E}$

$\therefore \frac{DE}{\sin 40.54°} = \frac{5.29}{\sin 114.51°}$

$\therefore DE = \frac{5.29 \sin 40.54°}{\sin 114.51°}$

$\therefore DE = 3.78$

e) $\widehat{D} = 180° - 114.51° - 40.54° = 24.95°$

$\frac{d}{\sin D} = \frac{e}{\sin E}$

$\therefore \frac{EC}{\sin 24.95°} = \frac{5.29}{\sin 114.51°}$

$\therefore EC = \frac{5.29 \sin 24.95°}{\sin 114.51°}$

$\therefore EC = 2.45$ ∴ BE = BC – EC = 7.32 – 2.45 = 4.87

f) $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\therefore \frac{\sin B}{4.34} = \frac{\sin 49.46°}{4.77}$

$\therefore \sin B = \frac{4.34 \sin 49.46°}{4.77}$

$\therefore \sin B = 0.691445142$

$\therefore A\hat{B}D = 43.74°$ $\therefore D\hat{B}E = 90° - 43.74° = 46.26°$

$Area = \frac{1}{2} de \sin B$

$\therefore Area = \frac{1}{2}(4.87)(4.77)(\sin 46.26°)$

$\therefore Area = 8.39\ units^2$

11. a) $\hat{B} = 180° - 118.88° - 35.33° = 25.79°$

$\frac{b}{\sin B} = \frac{e}{\sin E}$

$\therefore \frac{EC}{\sin 25.79°} = \frac{15.4}{\sin 118.88°}$

$\therefore EC = \frac{15.4 \sin 25.79°}{\sin 118.88°}$

$\therefore EC = 7.65m$

b) $c^2 = a^2 + b^2 - 2ab \cos C$

$\therefore AB^2 = (15.4)^2 + (18.54)^2 - 2(15.4)(18.54) \cos 35.33°$

$\therefore AB = \sqrt{115.0237581}$

$\therefore AB = 10.72m$

c) $b^2 = a^2 + c^2 - 2ac \cos B$

$\therefore b^2 - a^2 - c^2 = -2ac \cos B$

$\therefore \cos B = \frac{a^2+c^2-b^2}{2ac}$

$\therefore \cos B = \frac{(15.4)^2+(10.72)^2-(18.54)^2}{2(15.4)(10.72)}$

$\therefore \cos B = 0.02527985$

$\therefore \hat{B} = 88.55°$

d) AE = AC – EC = 18.54 – 7.65 = 10.89m

i) $e^2 = a^2 + d^2 - 2ad\cos E$

$\therefore \cos E = \frac{a^2+d^2-e^2}{2ad}$

$\therefore \cos E = \frac{(13.7)^2+(10.89)^2-(17.32)^2}{2(13.7)(10.89)}$

$\therefore \cos E = 0.021112585$

$\therefore A\hat{E}D = 88.79°$

ii) $D\hat{E}C = 180° - 88.79° = 91.21°$

$Area = \frac{1}{2}cd\sin E$

$\therefore Area = \frac{1}{2}(13.7)(7.65)(\sin 91.21°)$

$\therefore Area = 52.39\ m^2$

iii) $e^2 = c^2 + d^2 - 2cd\cos E$

$\therefore (DC)^2 = (13.7)^2 + (7.65)^2 - 2(13.7)(7.65)\cos 91.21°$

$\therefore DC = \sqrt{250.6388164}$

$\therefore DC = 15.83$

12. a) $b^2 = a^2 + d^2 - 2ad\cos B$

$\therefore (AD)^2 = \left(\frac{3}{4}x\right)^2 + \left(\frac{3}{4}x\right)^2 - 2\left(\frac{3}{4}x\right)\left(\frac{3}{4}x\right)\cos 90°$

$\therefore AD^2 = \frac{9}{16}x^2 + \frac{9}{16}x^2 - \frac{9}{8}x^2(0)$

$\therefore AD^2 = \frac{9}{8}x^2$

$\therefore AD = \frac{3x\sqrt{2}}{4}$

b) $c^2 = b^2 + d^2 - 2bd\cos C$

$\therefore \cos C = \frac{b^2+d^2-c^2}{2bd}$

$\therefore \cos C = \frac{(x)^2+(x)^2-\left(\frac{3}{4}x\right)^2}{2(x)(x)}$

$\therefore \cos C = \frac{2x^2-\frac{9}{16}x^2}{2x^2}$

$\therefore \cos C = \frac{\frac{23}{16}x^2}{2x^2}$

$\therefore \cos C = \frac{23}{32}$

$\therefore B\hat{C}D = 44.05°$