

SHARP

Werkkaart 5 Memorandum: Trigonometrie (Saamgestelde hoeke)

Graad 12 Wiskunde KABV

1. a) $\cos(x + y) = \cos x \cos y - \sin x \sin y$ b) $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$
c) $\cos 2c = 2 \cos^2 c - 1$ d) $\cos(b - c) = \cos b \cos c + \sin b \sin c$
e) $\sin 2x = 2 \sin x \cos x$ f) $\sin(y - z) = \sin y \cos z - \sin z \cos y$
g) $\cos 2d = 1 - 2 \sin^2 d$ h) $\cos 2e = \cos^2 e - \sin^2 e$
2. a) $\sin(75^\circ)$
 $= \sin(30^\circ + 45^\circ)$
 $= \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ$
 $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
 $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$
 $= \frac{\sqrt{6} + \sqrt{2}}{4}$
- b) $\sin(105^\circ)$
 $= \sin(60^\circ + 45^\circ)$
 $= \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ$
 $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
 $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$
 $= \frac{\sqrt{6} + \sqrt{2}}{4}$
- c) $\sin(120^\circ)$
 $= \sin(90^\circ + 30^\circ)$
 $= \sin 90^\circ \cos 30^\circ + \cos 90^\circ \sin 30^\circ$
 $= 1\left(\frac{\sqrt{3}}{2}\right) + 0\left(\frac{1}{2}\right)$
 $= \frac{\sqrt{3}}{2}$
- d) $\sin(135^\circ)$
 $= \sin(90^\circ + 45^\circ)$
 $= \sin 90^\circ \cos 45^\circ + \sin 45^\circ \cos 90^\circ$
 $= (1)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)(0)$
 $= \frac{\sqrt{2}}{2}$
- e) $\cos(150^\circ)$
 $= \cos(90^\circ + 60^\circ)$
 $= \cos 90^\circ \cos 60^\circ - \sin 60^\circ \sin 90^\circ$
 $= 0\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)(1)$
 $= -\frac{\sqrt{3}}{2}$
- f) $\cos(15^\circ)$
 $= \cos(45^\circ - 30^\circ)$
 $= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$
 $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
 $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$
 $= \frac{\sqrt{6} + \sqrt{2}}{4}$

$$\begin{aligned}
\text{g)} \quad & \cos(135^\circ) \\
& = \cos(90^\circ + 45^\circ) \\
& = \cos 90^\circ \cos 45^\circ - \sin 90^\circ \sin 45^\circ \\
& = 0 \left(\frac{\sqrt{2}}{2}\right) - 1 \left(\frac{\sqrt{2}}{2}\right) \\
& = -\frac{\sqrt{2}}{2}
\end{aligned}$$

$$\begin{aligned}
\text{h)} \quad & \cos(75^\circ) \\
& = \cos(45^\circ + 30^\circ) \\
& = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\
& = \left(\frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right) \left(\frac{1}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}
\end{aligned}$$

$$\begin{aligned}
3. \quad \text{a)} \quad & \cos 23^\circ \cos 22^\circ - \sin 23^\circ \sin 22^\circ \\
& = \cos(23^\circ + 22^\circ) \\
& = \cos 45^\circ
\end{aligned}$$

$$\begin{aligned}
\text{b)} \quad & 1 - 2 \sin 45^\circ \cos 45^\circ \\
& = 1 - 2 \sin 45^\circ \sin 45^\circ \\
& = 1 - 2 \sin^2 45^\circ \\
& = \cos 2(45^\circ) \\
& = \cos 90^\circ
\end{aligned}$$

$$\begin{aligned}
\text{c)} \quad & \sin 18^\circ \cos 19^\circ + \sin 19^\circ \cos 18^\circ \\
& = \sin(18^\circ + 19^\circ) \\
& = \sin 37^\circ
\end{aligned}$$

$$\begin{aligned}
\text{d)} \quad & 2 \cos^2 A - 1 \\
& = \cos 2A
\end{aligned}$$

$$\begin{aligned}
\text{e)} \quad & \cos 81^\circ \cos 11^\circ + \sin 81^\circ \sin 11^\circ \\
& = \cos(81^\circ - 11^\circ) \\
& = \cos 70^\circ
\end{aligned}$$

$$\begin{aligned}
\text{f)} \quad & 2 \sin 16^\circ \cos 16^\circ \\
& = \sin 2(16^\circ) \\
& = \sin 32^\circ
\end{aligned}$$

$$\begin{aligned}
\text{g)} \quad & \cos 25^\circ \cos 25^\circ - \sin 25^\circ \sin 25^\circ \\
& = \cos^2 25^\circ - \sin^2 25^\circ \\
& = \cos 2(25^\circ) \\
& = \cos 50^\circ
\end{aligned}$$

$$\begin{aligned}
\text{h)} \quad & \sin 28^\circ \cos 8^\circ - \sin 8^\circ \cos 28^\circ \\
& = \sin(28^\circ - 8^\circ) \\
& = \sin 20^\circ
\end{aligned}$$

$$\begin{aligned}
4. \quad \text{a)} \quad & \cos(a + a) = 1 - 2 \sin^2 a \\
\text{LK} \quad & = \cos a \cos a - \sin a \sin a \\
& = \cos^2 a - \sin^2 a \\
& = (1 - \sin^2 a) - \sin^2 a \\
& = 1 - 2 \sin^2 a \\
\therefore \text{LK} & = \text{RK}
\end{aligned}$$

$$\begin{aligned}
\text{b)} \quad & \frac{\cos 2b+1}{\sin 2b} = \cot b \\
\text{LK} \quad & = \frac{2 \cos^2 b - 1 + 1}{2 \sin b \cos b} \\
& = \frac{2 \cos^2 b}{2 \sin b \cos b} \\
& = \frac{\cos b}{\sin b} \\
& = \cot b \\
\therefore \text{LK} & = \text{RK}
\end{aligned}$$

$$c) \quad \sin(d + 45^\circ) - \cos(d + 45^\circ) = \sqrt{2} \sin d$$

$$\text{LK} = \sin d \cos 45^\circ + \sin 45^\circ \cos d - (\cos d \cos 45^\circ - \sin d \sin 45^\circ)$$

$$= \frac{\sqrt{2}}{2} \sin d + \frac{\sqrt{2}}{2} \cos d - \frac{\sqrt{2}}{2} \cos d + \frac{\sqrt{2}}{2} \sin d$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) \sin d$$

$$= \sqrt{2} \sin d$$

$$\therefore \text{LK} = \text{RK}$$

$$d) \quad \frac{1 - \sin 2w}{\sin 2w - \cos 2w - 1} = \frac{1}{2} \tan w - \frac{1}{2}$$

$$\text{LK} = \frac{1 - 2 \sin w \cos w}{2 \sin w \cos w - (2 \cos^2 w - 1) - 1}$$

$$= \frac{\cos^2 w + \sin^2 w - 2 \sin w \cos w}{2 \sin w \cos w - 2 \cos^2 w + 1 - 1}$$

$$= \frac{\sin^2 w - 2 \sin w \cos w + \cos^2 w}{2 \sin w \cos w - 2 \cos^2 w}$$

$$= \frac{(\sin w - \cos w)(\sin w + \cos w)}{2 \cos w (\sin w - \cos w)}$$

$$= \frac{\sin w + \cos w}{2 \cos w}$$

$$= \frac{\sin w}{2 \cos w} + \frac{\cos w}{2 \cos w}$$

$$= \frac{1}{2} \tan w + \frac{1}{2}$$

$$\therefore \text{LK} = \text{RK}$$

$$e) \quad \frac{\sin(b+30^\circ) + \cos(60^\circ+b)}{\sin 2b} = \frac{1}{2} \operatorname{cosec} b$$

$$\text{LK} = \frac{\sin b \cos 30^\circ + \cos b \sin 30^\circ + \cos 60^\circ \cos b - \sin 60^\circ \sin b}{2 \sin b \cos b}$$

$$= \frac{\frac{\sqrt{3}}{2} \sin b + \frac{1}{2} \cos b + \frac{1}{2} \cos b - \frac{\sqrt{3}}{2} \sin b}{2 \sin b \cos b}$$

$$= \frac{\cos b}{2 \sin b \cos b}$$

$$= \frac{1}{2 \sin b}$$

$$= \frac{1}{2} \operatorname{cosec} b$$

$$\therefore \text{LK} = \text{RK}$$

$$f) \quad \sin 3a + \cos 3a = (\cos a - \sin a)(1 + 4 \cos a \sin a)$$

$$\text{LK} = \sin(2a + a) + \cos(2a + a)$$

$$= \sin 2a \cos a + \cos 2a \sin a + \cos 2a \cos a - \sin 2a \sin a$$

$$= \cos a (2 \sin a \cos a) + \sin a (\cos^2 a - \sin^2 a) + \cos a (\cos^2 a - \sin^2 a) - \sin a (2 \sin a \cos a)$$

$$= 2 \sin a \cos^2 a + \cos^2 a \sin a - \sin^3 a + \cos^3 a - \sin^2 a \cos a - 2 \sin^2 a \cos a$$

$$= 3 \sin a \cos^2 a + \cos^3 a - \sin^3 a - 3 \sin^2 a \cos a$$

$$= (3 \sin a \cos^2 a - 3 \sin^2 a \cos a) + (\cos^3 a - \sin^3 a)$$

$$= 3 \sin a \cos a (\cos a - \sin a) + (\cos a - \sin a)(\cos^2 a + \cos a \sin a + \sin^2 a)$$

$$= 3 \sin a \cos a (\cos a - \sin a) + (\cos a - \sin a)(1 + \cos a \sin a)$$

$$= (\cos a - \sin a)(3 \sin a \cos a + 1 + \sin a \cos a)$$

$$= (\cos a - \sin a)(1 + 4 \cos a \sin a)$$

$$\therefore \text{LK} = \text{RK}$$

$$g) \quad \frac{\cos 2f - 1}{\sin 2f} = -\tan f$$

$$\text{LK} = \frac{1 - 2\sin^2 f - 1}{2\sin f \cos f}$$

$$= \frac{-2\sin^2 f}{2\sin f \cos f}$$

$$= \frac{-\sin f}{\cos f}$$

$$= -\tan f$$

$$\therefore \text{LK} = \text{RK}$$

$$h) \quad \frac{1 - 2\sin d}{\cos d - \sin d} - \frac{2\cos d - 1}{\cos d + \sin d} = \frac{2(\cos d - 1)}{\cos 2d}$$

$$\text{RK} = \frac{2\cos d - 2}{\cos^2 d - \sin^2 d}$$

$$\text{LK} = \frac{(1 - 2\sin d)(\cos d + \sin d) - (2\cos d - 1)(\cos d - \sin d)}{(\cos d - \sin d)(\cos d + \sin d)}$$

$$= \frac{\cos d + \sin d - 2\sin d \cos d - 2\sin^2 d - (2\cos^2 d - 2\sin d \cos d - \cos d + \sin d)}{\cos^2 d - \sin^2 d}$$

$$= \frac{\cos d + \sin d - 2\sin d \cos d - 2\sin^2 d - 2\cos^2 d + 2\sin d \cos d + \cos d - \sin d}{\cos^2 d - \sin^2 d}$$

$$= \frac{2\cos d - 2\sin^2 d - 2\cos^2 d}{\cos^2 d - \sin^2 d}$$

$$= \frac{2\cos d - 2(\sin^2 d + \cos^2 d)}{\cos^2 d - \sin^2 d}$$

$$= \frac{2\cos d - 2}{\cos^2 d - \sin^2 d}$$

$$\therefore \text{LK} = \text{RK}$$

$$i) \quad \frac{1}{1 - \sin A} + \frac{2\sin A}{1 + \sin A} = 2 - \frac{1 - 3\sin A}{\cos^2 A}$$

$$\text{RK} = \frac{2\cos^2 A - 1 + 3\sin A}{\cos^2 A}$$

$$= \frac{\cos 2A + 3\sin A}{\cos^2 A}$$

$$\text{LK} = \frac{1 + \sin A + (1 - \sin A)(2\sin A)}{(1 - \sin A)(1 + \sin A)}$$

$$= \frac{1 + \sin A + 2\sin A - 2\sin^2 A}{1 - \sin^2 A}$$

$$= \frac{1 - 2\sin^2 A + 3\sin A}{\cos^2 A}$$

$$= \frac{\cos 2A + 3\sin A}{\cos^2 A}$$

$$\therefore \text{LK} = \text{RK}$$

$$j) \quad 1 - \frac{\sin 2B + 1}{\cos 2B} = \frac{2\sin B}{\sin B - \cos B}$$

$$\text{LK} = \frac{\cos 2B - \sin 2B - 1}{\cos 2B}$$

$$= \frac{1 - 2\sin^2 B - 2\sin B \cos B - 1}{\cos^2 B - \sin^2 B}$$

$$= \frac{-2\sin^2 B - 2\sin B \cos B}{(\cos B - \sin B)(\cos B + \sin B)}$$

$$= \frac{-2\sin B(\sin B + \cos B)}{(\cos B - \sin B)(\cos B + \sin B)}$$

$$= \frac{-2\sin B}{\cos B - \sin B}$$

$$= \frac{2\sin B}{\sin B - \cos B}$$

$$\therefore \text{LK} = \text{RK}$$

$$5. \quad a) \quad \sin a \sin b + \cos a \cos b = 0.5 \text{ waar } a = 2b$$

$$\cos(a + b) = 0.5$$

$$\cos(2b + b) = 0.5$$

$$\cos 3b = 0.5$$

$$3b = 60^\circ + k360^\circ$$

$$b = 20^\circ + k120^\circ$$

$$\therefore a = 40^\circ + k240^\circ$$

$$\text{OF} \quad 3b = 360^\circ - 60^\circ + k.360^\circ$$

$$b = 100^\circ + k.120^\circ$$

$$a = 200^\circ + k240^\circ$$

b) $\sin 2c - \sin c = 0$
 $2 \sin c \cos c - \sin c = 0$
 $\sin c (2 \cos c - 1) = 0$
 $\therefore \sin c = 0 \quad \text{or} \quad 2 \cos c = 1$
 $\therefore c = 0 + k.360^\circ$
 OF $c = 180^\circ + k360^\circ$
 OF $\cos c = \frac{1}{2}$
 $\therefore c = 60^\circ + k.360^\circ$

c) $\cos 2d + \cos d = 0$
 $2 \cos^2 d - 1 + \cos d = 0$
 $2 \cos^2 d + \cos d - 1 = 0$
 $(2 \cos d - 1)(\cos d + 1) = 0$
 $\cos d = \frac{1}{2} \quad \cos d = -1$
 $\therefore d = 60^\circ + k.360^\circ$
 OF $d = 300^\circ + k360^\circ$
 OF $d = 180^\circ - 0^\circ + k.360^\circ$
 $d = 180^\circ + k.360^\circ$

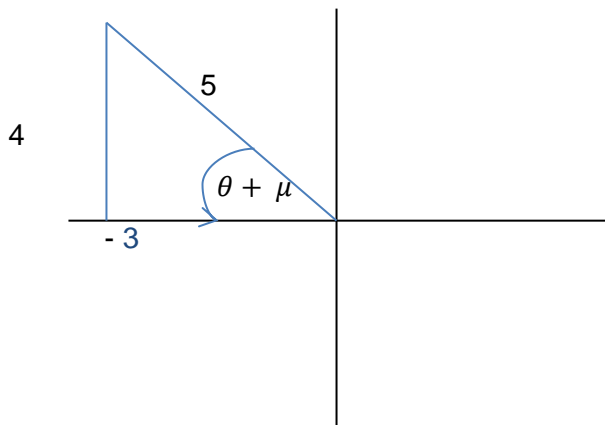
d) $\cos 3d \sin 4d - \sin 3d \cos 4d = 0.33$
 $\sin(4d - 3d) = 0.33$
 $\sin d = 0.33$

e) $\sin(\alpha + 30^\circ) = \cos \alpha$
 $\sin(\alpha + 30^\circ) = \sin(90^\circ - \alpha)$
 $\alpha + 30^\circ = 90^\circ - \alpha$

$d = 19.27^\circ + k.360^\circ$
 OF $d = 180^\circ - 19.27^\circ + k360^\circ$
 $d = 160.73^\circ + k360^\circ$

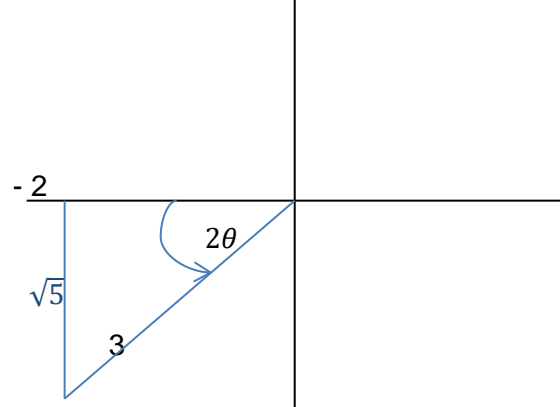
$2\alpha = 60^\circ + k.360$
 $\alpha = 30^\circ + k180^\circ$
 OF $2\alpha = 180^\circ - 60^\circ + k 360^\circ$
 $\alpha = 60^\circ + k180^\circ$

6. $5 \sin(\theta + \mu) = 4, \tan(\theta + \mu) < 0$
 $\sin(\theta + \mu) = \frac{4}{5} \quad \tan \text{ negatief}$



$x^2 = r^2 - y^2$
 $x = \sqrt{(5)^2 - (4)^2}$
 $x = -3$

$3 \cos 2\theta + 2 = 0, \tan 2\theta > 0$
 $\cos 2\theta = -\frac{2}{3} \quad \tan \text{ positief}$



$y^2 = r^2 - x^2$
 $y = \sqrt{(3)^2 - (-2)^2}$
 $y = \sqrt{5}$

a) $\sin \theta$

$$\cos 2\theta = \frac{-2}{3}$$

$$1 - 2 \sin^2 \theta = -\frac{2}{3}$$

$$-2 \sin^2 \theta = -1\frac{2}{3}$$

$$\sin^2 \theta = \frac{5}{6}$$

$$\sin \theta = \sqrt{\frac{5}{6}}$$

b) $\cos \theta$

$$\cos 2\theta = -\frac{2}{3}$$

$$2 \cos^2 \theta - 1 = -\frac{2}{3}$$

$$2 \cos^2 \theta = \frac{1}{3}$$

$$\cos^2 \theta = \frac{1}{6}$$

$$\cos \theta = \sqrt{\frac{1}{6}}$$

c) $\tan(\theta + \mu)$

$$= \frac{4}{3}$$

d) $\cos \mu$

$$\sin(\theta + \mu) = \frac{4}{5}$$

en $\cos(\theta + \mu) = \frac{-3}{5}$

$$\sin \theta \cos \mu + \sin \mu \cos \theta = \frac{4}{5}$$

$$\cos \theta \cos \mu - \sin \theta \sin \mu = \frac{-3}{5} \quad \dots 2$$

$$\sin \mu \cos \theta = \frac{4}{5} - \sin \theta \cos \mu$$

$$\sin \mu = \frac{\left(\frac{4}{5} - \left(\sqrt{\frac{5}{6}}\right) \cos \mu\right)}{\sqrt{\frac{1}{6}}} \quad \dots 1$$

Subs 1 in 2:

$$\sqrt{\frac{1}{6}} (\cos \mu) - \sqrt{\frac{5}{6}} \left(\frac{\left(\frac{4}{5} - \left(\sqrt{\frac{5}{6}}\right) \cos \mu\right)}{\sqrt{\frac{1}{6}}} \right) = \frac{-3}{5}$$

$$\sqrt{\frac{1}{6}} (\cos \mu) - \frac{\sqrt{5}}{6} \left(\frac{4}{5} - \sqrt{\frac{5}{6}} \cos \mu \right) = \frac{-3}{5}$$

$$\sqrt{\frac{1}{6}} (\cos \mu) - \frac{2\sqrt{5}}{15} + \frac{5\sqrt{6}}{36} \cos \mu = \frac{-3}{5}$$

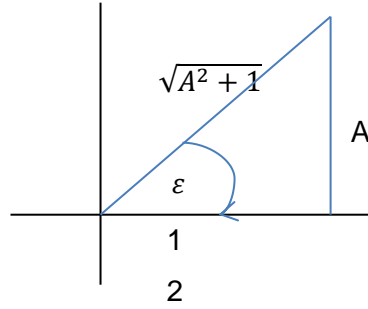
$$\frac{11\sqrt{6}}{36} \cos \mu = \frac{-3}{5} + \frac{2\sqrt{5}}{15}$$

$$\therefore \cos \mu = \frac{-9+2\sqrt{5}}{15} \div \frac{11\sqrt{6}}{36}$$

$$\therefore \cos \mu = \frac{4\sqrt{30}-18\sqrt{6}}{55}$$

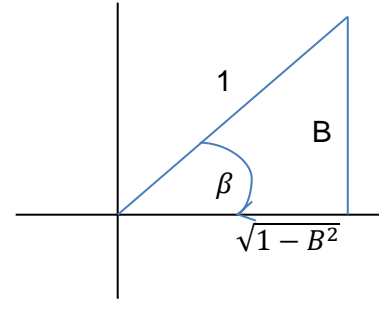
$$\therefore \cos \mu = -0.403$$

7. $A = \tan \varepsilon$ en $B = \sin \beta$



$$r^2 = x^2 + y^2$$

$$r = \sqrt{1 + A^2}$$



$$x^2 = r^2 - y^2$$

$$x = \sqrt{1 - B^2}$$

a) $\frac{\sin(\varepsilon + \beta)}{\sin 2\varepsilon}$

$$= \frac{\sin \varepsilon \cos \beta + \sin \beta \cos \varepsilon}{2 \sin \varepsilon \cos \varepsilon}$$

$$= \frac{\frac{A}{\sqrt{A^2+1}} \left(\frac{\sqrt{1-B^2}}{1} \right) + (B) \left(\frac{1}{\sqrt{A^2+1}} \right)}{2 \left(\frac{A}{\sqrt{A^2+1}} \right) \left(\frac{1}{\sqrt{A^2+1}} \right)}$$

$$= \frac{\frac{A\sqrt{1-B^2}}{\sqrt{A^2+1}} + \frac{B}{\sqrt{A^2+1}}}{\frac{2A}{A^2+1}}$$

$$= \left(\frac{A\sqrt{1-B^2} + B}{\sqrt{A^2+1}} \right) \times \frac{A^2+1}{2A}$$

$$= \frac{\sqrt{A^2+1}(A\sqrt{1-B^2} + B)}{2A}$$

b) $\sin \varepsilon \times \tan \beta$

$$= \frac{A}{\sqrt{A^2+1}} \times \frac{B}{\sqrt{1-B^2}}$$

$$= \frac{AB}{\sqrt{(A^2+1)(1-B^2)}}$$

$$= \frac{AB}{\sqrt{A^2 - A^2B^2 - B^2 + 1}}$$

c) $\cos 2\beta$

$$= \cos^2 \beta - \sin^2 \beta$$

$$= \left(\frac{\sqrt{1-B^2}}{1} \right)^2 - \left(\frac{B}{1} \right)^2$$

$$= 1 - B^2 - B^2$$

$$= 1 - 2B^2$$

d) $1 - \sin 2\beta$

$$= 1 - 2 \sin \beta \cos \beta$$

$$= 1 - 2 \left(\frac{B}{1} \right) \left(\frac{\sqrt{1-B^2}}{1} \right)$$

$$= 1 - 2B\sqrt{1-B^2}$$

8. a) $\sin \frac{C}{2}$

$$\cos 2 \left(\frac{C}{2} \right) = 1 - 2 \sin^2 \left(\frac{C}{2} \right)$$

$$\cos C = 1 - 2 \sin^2 \left(\frac{C}{2} \right)$$

$$\cos C - 1 = -2 \sin^2 \left(\frac{C}{2} \right)$$

$$-\frac{1}{2}(\cos C - 1) = \sin^2 \left(\frac{C}{2} \right)$$

$$\therefore \sin \left(\frac{C}{2} \right) = \sqrt{-\frac{1}{2}(\cos C - 1)}$$

b) $\cos \frac{D}{2}$

$$\cos 2 \left(\frac{D}{2} \right) = 2 \cos^2 \left(\frac{D}{2} \right) - 1$$

$$\cos D = 2 \cos^2 \left(\frac{D}{2} \right) - 1$$

$$\cos D + 1 = 2 \cos^2 \left(\frac{D}{2} \right)$$

$$\frac{1}{2}(\cos D + 1) = \cos^2 \left(\frac{D}{2} \right)$$

$$\therefore \cos \left(\frac{D}{2} \right) = \sqrt{\frac{1}{2}(\cos D + 1)}$$

$$\text{c) } \tan 2E$$

$$= \frac{\sin 2E}{\cos 2E}$$

$$= \frac{2 \sin E \cos E}{\cos^2 E - \sin^2 E}$$

$$= \frac{2 \sin E \cos E}{\cos^2 E - \sin^2 E} \times \frac{\frac{1}{\cos^2 E}}{\frac{1}{\cos^2 E}}$$

$$= \frac{\frac{2 \sin E \cos E}{\cos^2 E}}{\frac{\cos^2 E - \sin^2 E}{\cos^2 E}}$$

$$= \frac{\frac{2 \sin E}{\cos E}}{1 - \frac{\sin^2 E}{\cos^2 E}}$$

$$= \frac{2 \tan E}{1 - \tan^2 E}$$

$$\text{d) } \sin \frac{3}{2}F$$

$$= \sin \left(F + \frac{1}{2}F \right)$$

$$= \sin F \cos \frac{1}{2}F + \sin \frac{1}{2}F \cos F$$

$$= \sin F \left(\sqrt{\frac{1}{2}(\cos F + 1)} \right) + \left(\sqrt{-\frac{1}{2}(\cos F - 1)} \right) \cos F$$