Werkkaart 6 Memorandum: Kwartaal 1 Hersiening

Graad 12 Wiskunde

1\. a) 7 15 23 31…

8 8 8

Rekenkundig

b) 0 1 1 2 3 5…

1 0 1 1 2

-1 1 0 1

Geen *side note – dit is die Fibionacci Ry - die vorige term en huidige kwartaal bymekaar getel word om die volgende term te kry.

c) 48 36 27 $20\frac{1}{4}$…

$\times\frac{3}{4}$ $\times\frac{3}{4}$ $\times\frac{3}{4}$

Meetkundig

d) 88 95 107 126 154…

7 12 19 28

5 7 9

Geen

e) 38 58 68 68…

20 10 0

-10 -10

Kwadraties

f) 85 -51 $30\frac{3}{5}$ $-18\frac{9}{25}$…

$\times -\frac{3}{5}$ $\times -\frac{3}{5}$ $\times -\frac{3}{5}$

Meetkundig

g) 22 30 38 46…

8 8 8

Rekenkundig

h) 14 21 29 40 57…

7 8 11 17

1 3 6

Geen

i) 1 2 4 8…

$\times 2$ $\times 2$ $\times 2$

Meetkundig

j) 77 88 99 110 121…

11 11 11 11

Rekenkundig

2\. a) 7 15 23 31…

8 8 8

i) $T_n = 7 + 8(n-1)$

c) 48 36 27 $20\frac{1}{4}$

$\times\frac{3}{4}$ $\times\frac{3}{4}$ $\times\frac{3}{4}$

i) $T_n = 48\left(\frac{3}{4}\right)^{n-1}$

ii) $T_{22} = 7 + 8(22 - 1)$

$T_{22} = 175$

iii) $T_n = 7 + 8n - 8$

$T_n = 8n - 1$

$\therefore \sum_{i=1}^{n}(8i - 1)$

iv) $S_9 = \frac{9}{2}[2(7) + (9 - 1)(8)]$

$S_9 = 351$

ii) $T_{22} = 48\left(\frac{3}{4}\right)^{22-1}$

$T_{22} = 0.114163 \ldots \approx 0.11$

iii) $\sum_{i=1}^{n} 48\left(\frac{3}{4}\right)^{i-1}$

iv) $S_9 = \frac{48\left(\left(\frac{3}{4}\right)^9 - 1\right)}{\frac{3}{4} - 1}$

$S_9 = 177.58$

Konvergerend: $\therefore S_\infty = \frac{48}{1 - \frac{3}{4}}$

$S_\infty = 192$

f) 85- 51 $30\frac{3}{5}$ $-18\frac{9}{25}$

$\times -\frac{3}{5}$ $\times -\frac{3}{5}$ $\times -\frac{3}{5}$

i) $T_n = 85\left(-\frac{3}{5}\right)^{n-1}$

ii) $T_{22} = 85\left(-\frac{3}{5}\right)^{22-1}$

$T_{22} = -0.0019$

iii) $\sum_{i=1}^{n} 85\left(-\frac{3}{5}\right)^{i-1}$

iv) $S_9 = \frac{85\left(\left(-\frac{3}{5}\right)^9 - 1\right)}{-\frac{3}{5} - 1}$

$\therefore S_9 = 53.66$

Konvergerend: $\therefore S_\infty = \frac{85}{1 - \left(-\frac{3}{5}\right)}$

$S_\infty = 53\frac{1}{8}$

g) 22 30 38 46

8 8 8

i) $T_n = 22 + 8(n - 1)$

ii) $T_{22} = 22 + 8(22 - 1)$

$T_{22} = 190$

iii) $T_n = 22 + 8n - 8$

$T_n = 8n + 14$

$\sum_{i=1}^{n}(8i + 14)$

iv) $S_9 = \frac{9}{2}[2(22) + 8(9 - 1)]$

$S_9 = 486$

i) 1 2 4 8

x2 x2 x2

i) $T_n = 1(2)^{n-1}$

ii) $T_{22} = 2^{22-1}$

$T_{22} = 2\ 097\ 152$

j) 77 88 99 110 121

11 11 11 11

i) $T_n = 77 + 11(n - 1)$

ii) $T_{22} = 77 + 11(22 - 1)$

$T_{22} = 308$

iii) $\sum_{i=1}^{n} 2^{i-1}$

iv) $S_9 = \frac{1(2^9-1)}{2-1}$

$S_9 = 511$

iii) $T_n = 77 + 11n - 11$

$T_n = 11n + 66$

$\sum_{i=1}^{n}(11i + 66)$

iv) $S_9 = \frac{9}{2}[2(77) + 11(9-1)]$

$S_9 = 1089$

3. $S_9 = \frac{9}{2}[2a_A + d(9-1)] = T_5 = a_G r^4$ en $T_3 = a_G r^2 = T_8 = a_A + d(8-1) - 1$

a) $a_A : a_G = 25:16$ $\quad \therefore 16a_A = 25a_G$ en $a_A = 9$

$\therefore a_G = \frac{16}{25}(9) = 5\frac{19}{25}$

So vir vergelyking 1 het ons nou:

$\frac{9}{2}[2(9) + d(8)] = 5\frac{19}{25}(r)^4$ om d op sy eie te kry:

$81 + 36d = 5\frac{19}{25}r^4$

$36d = 5\frac{19}{25}r^4 - 81$

$d = \frac{4}{25}r^4 - 2\frac{1}{4}$ ...1

En vir die tweede vergelyking het ons:

$5\frac{19}{25}\, r^2 = 8 + d(7)$ ...2

Subs 1 in 2:

$\therefore 5\frac{19}{25}r^2 = 8 + 7(\frac{4}{25}r^4 - 2\frac{1}{4})$

$\therefore \ 0 = 8 + \frac{28}{25}r^4 - 15\frac{3}{4} - 5\frac{19}{25}r^2$

$\therefore 0 = \frac{28}{25}\, r^4 - 5\frac{19}{25}r^2 - 7\frac{3}{4}$ (vermenigvuldig uit met 100)

$\therefore 0 = 112r^4 - 576r^2 - 775$ (faktoriseer as 'n drieterm)

$\therefore r^2 = \frac{576 \pm \sqrt{(-576)^2 - 4(112)(-775)}}{2(112)}$

$\therefore r^2 = -\frac{31}{28}$ ... nie moontlik om r^2 negatief te wees nie, daarom NVT

OF $r^2 = 6\frac{1}{4} = \frac{25}{4}$

$\therefore r = 2\frac{1}{2}\ or\ \frac{5}{2}$

Subs terug in 1:

$d = \frac{4}{25}\left(\frac{5}{2}\right)^4 - 2\frac{1}{4}$

$d = 4$

$\therefore d = 4 \quad r = 2\frac{1}{2} \quad en \ \ a_g = 5\frac{19}{25}$

b) Die meetkundige reeks divergeer as r > 1

c) $T_n = 9 + 4(n-1)$

$\therefore 53 = 9 + 4(n-1)$

$\therefore 44 = 4(n-1)$

$\therefore 11 = n - 1$

$\therefore n = 12$

d) $T_n = 5\frac{19}{25}\left(\frac{5}{2}\right)^{n-1}$

$\therefore 8\ 789\frac{1}{16} = 5\frac{19}{25}\left(\frac{5}{2}\right)^{n-1}$

$\therefore \frac{390\ 625}{256} = \left(\frac{5}{2}\right)^{n-1}$

$\therefore \log_{\frac{5}{2}}\left(\frac{390\ 625}{256}\right) = n - 1$

$\therefore n - 1 = 8$

$\therefore n = 9$

4. a) $y = Log_3 x$

i) Funksie (een tot een)

ii) NVT

iii) $x = \log_3 y$

$\therefore y = 3^x$

b) $y^2 = 25 - x^2$

i) nie 'n funksie nie (meer tot meer)

ii) $0 \leq x \leq 5$

iii) $x^2 = 25 - y^2$

$\therefore x^2 + y^2 = 25$

$\therefore y^2 = 25 - x^2$

$\therefore y = \sqrt{25 - x^2}$

c) $y = 3x^2 - 7$

i) funksie (veel tot een)

ii) NVT

iii) $x = 3y^2 - 7$

$x + 7 = 3y^2$

$\frac{1}{3}(x+7) = y^2$

$\therefore y = \sqrt{\frac{1}{3}(x+7)}$

d) $x = y^2 + 2$

i) nie 'n funksie nie (een tot meer)

ii) $y = \sqrt{x - 2}$

$\therefore x \geq 2$

iii) $y = x^2 + 2$

e) $y = x^3$

i) funksie (meer tot een)

ii) NVT

iii) $x = y^3$

$\therefore y = \sqrt[3]{x}$

f) $y = 3^x$

i) funksie (een tot een)

ii) NVT

iii) $x = 3^y$

$\therefore y = \log_3 x$

g) $y = \sqrt{x^3 - 9}$

i) nie 'n funksie nie (meer tot meer)

ii) $y > 0 \quad and\ x > \sqrt[3]{9}$

iii) $x = \sqrt{y^3 - 9}$

$x^2 = y^3 - 9$

$x^2 + 9 = y^3$

$\therefore y = \sqrt[3]{x^2 + 9}$

h) $y^3 = x^2$

i) nie 'n funksie nie (meer tot meer)

ii) $x > 0$

iii) $x^3 = y^2$

$\therefore y = \sqrt{x^3}$

5. a) A = ? $A = P(1+i)^n$

P = 20 000 $A = 20\,000(1+0.085)^3$

$i = 8.5\% = 0.085$ $A = 25\,545.78$

n = 3 jaar

b) A = ? $A = P(1-i)^n$

P = 20 000 $A = 20\,000(1-0.076)^3$

$i = 7.6\% = 0.076$ $A = 15\,777.78$

n = 3 jaar

c) P = 20 000 $P = \frac{x[1-(1+i)^{-n}]}{i}$

$i = 11.2\% \div 12 = \frac{7}{750}$ $20\,000 = \frac{x\left[1-\left(1+\frac{7}{750}\right)^{-36}\right]}{\frac{7}{750}}$

$x = ?$ $186\frac{2}{3} = x[0.28426242]$

n = 3 jaar x 12 = 36 $\therefore x = 656.67$

d) $F = 25\,545.78 - 15\,777.78 = 9768$

$i = 12.5\% \div 4 = \frac{1}{32}$ $F = \frac{x[(1+i)^n - 1]}{i}$

$x = ?$ $9\,768 = \frac{x\left[\left(1+\frac{1}{32}\right)^{12}-1\right]}{\frac{1}{32}}$

n = 3 jaar x 4 = 12 $305\frac{1}{4} = x[0.446663548]$

$\therefore x = 683.40$

e) vir c) $i_{eff} = ?$ $i_{eff} = \left(1+\frac{i_m}{m}\right)^m - 1$

$i_m = 11.2\% = 0.122$ $i_{eff} = \left(1+\frac{0.112}{12}\right)^{12} - 1$

$m = 12$ $i_{eff} = 0.1179 = 11.79\%$

e) vir d) $i_{eff} = ?$ $i_{eff} = \left(1+\frac{i_m}{m}\right)^m - 1$

$i_m = 12.5\% = 0.125$ $i_{eff} = \left(1+\frac{0.125}{4}\right)^4 - 1$

$m = 4$ $i_{eff} = 0.13098 = 13.1\%$

Die effektiewe rentekoers in d) (die delginsfonds) is beter.

f) $x = 656.67 + 15\% = 755.17 \quad P = \frac{x[1-(1+i)^{-n}]}{i}$

P = 20 000 $\qquad 20\ 000 = \frac{755.17\left[1-\left(1+\frac{7}{750}\right)^{-12n}\right]}{\frac{7}{750}}$

$i = 11.2\% \div 12 = \frac{7}{750}$ $\qquad 186\frac{2}{3} = 755.17\left[1-\left(1+\frac{7}{750}\right)^{-12n}\right]$

n = nx 12 = 12n $\qquad 0.24718496 = 1 - \left(\frac{757}{750}\right)^{-12n}$

$-0.752815039 = -\left(\frac{757}{750}\right)^{-12n}$

$\therefore\ -12n = \log_{\frac{757}{750}} 0.752815039$

$\therefore\ -12n = -30.56343163$

$\therefore\ n = 2.54\ jaar\ of\ 2\ jaar\ en\ 6.56\ maande$

6. a) $\frac{\cos 2a}{1+\sin 2a} = \frac{\cos a - \sin a}{\cos a + \sin a}$

LK $= \frac{\cos^2 a - \sin^2 a}{\cos^2 a + \sin^2 a + 2\sin a \cos a}$

$= \frac{(\cos a - \sin a)(\cos a + sin a)}{(\cos a + \sin a)(\cos a + \sin a)}$

$= \frac{\cos a - \sin a}{\cos a + \sin a}$

∴ LK = RK

b) $\sin 3b = \sin b\,(4\cos^2 b - 1)$

LK $= \sin(2b + b)$

$= \sin 2b \cos b + \sin b \cos 2b$

$= 2\sin b \cos b \cos b + \sin b\,(2\cos^2 b - 1)$

$= 2\sin b \cos^2 b + 2\cos^2 b \sin b - \sin b$

$= 4\cos^2 b \sin b - \sin b$

$= \sin b(4\cos^2 b - 1)$

∴ LK = RK

c) $\cos 3a - \sin 3a = (\cos a + \sin a)(1 - 4\cos a \sin a)$

LK $= \cos(2a + a) - \sin(2a + a)$

$= \cos 2a \cos a - \sin 2a \sin a - (\sin 2a \cos a + \sin a \cos 2a)$

$= (\cos^2 a - \sin^2 a)(\cos a) - 2\sin a\ \cos a \sin a - 2\sin a\ \cos a \cos a - (\sin a)(\cos^2 a - \sin^2 a)$

$= \cos^3 a - \sin^2 a \cos a - 2\sin^2 a \cos a - 2\sin a \cos^2 a - \cos^2 a \sin a + \sin^3 a$

$= \cos^3 a - 3\sin^2 a \cos a - 3\cos^2 a \sin a + \sin^3 a$

$= (\cos^3 a + \sin^3 a) - 3\sin a \cos a\,(\sin a + \cos a)$

$= (\cos a + \sin a)(\cos^2 a - \sin a \cos a + \sin^2 a) - 3\sin a \cos a\,(\cos a + \sin a)$

$= (\cos a + \sin a)(1 - \sin a \cos a) - 3 \sin a \cos a(\cos a + \sin a)$

$= (\cos a + \sin a)(1 - \sin a \cos a - 3 \sin a \cos a)$

$= (\cos a + \sin a)(1 - 4 \sin a \cos a)$

∴ LK = RK

d) $\sin 2f - \tan f = \tan f \,.\cos 2f$

RK $= \tan f\,(\cos^2 f - \sin^2 f)$

$= \frac{\sin f}{\cos f}(\cos^2 f - \sin^2 f)$

$= \sin f \cos f - \frac{\sin^3 f}{\cos f}$

$= \frac{\sin f\, cos^2 f - \sin^3 f}{\cos f}$

$= \frac{\sin f(\cos 2f)}{\cos f}$

LK $= 2 \sin f \cos f - \frac{\sin f}{\cos f}$

$= \frac{2 \sin f \cos^2 f - \sin f}{\cos f}$

$= \frac{\sin f(2\cos^2 - 1)}{\cos f}$

$= \frac{\sin f(\cos 2f)}{\cos f}$

∴ LK = RK

e) $\frac{\sin 2a + \sin 3a}{\sin a} = 4\cos^2 a + 2\cos a - 1$

LK $= \frac{2 \sin a \cos a + \sin(2a + a)}{\sin a}$

$= \frac{2 \sin a \cos a + \sin 2a \cos a + \cos 2a \sin a}{\sin a}$

$= \frac{2 \sin a \cos a + 2 \sin a \cos a \cos a + (2\cos^2 a - 1)(\sin a)}{\sin a}$

$= \frac{2 \sin a \cos a + 2 \sin a \cos^2 a + 2\cos^2 a \sin a - \sin a}{\sin a}$

$= \frac{\sin a(2\cos a + 2\cos^2 a + 2\cos^2 a - 1)}{\sin a}$

$= 4\cos^2 a + 2\cos a - 1$

LK = RK