SHARP

Worksheet 5: Trigonometry (Compound Angles)

Grade 12 Mathematics CAPS

1. Complete the compound angle identity for each of these:

a) $\cos(x+y) = ?$ b) $\sin(\alpha + \beta) = ?$

c) $\cos 2c = ?$ (using only cos) d) $\cos(b-c) = ?$

e) $\sin 2x = ?$ f) $\sin(y-z) = ?$

g) $\cos 2d = ?$ (using only sin) h) $\cos 2e = ?$ (using both cos and sin)

2. Simplify the following without using a calculator:

a) $\sin(75°)$ b) $\sin(105°)$

c) $\sin(120°)$ d) $\sin(135°)$

e) $\cos(150°)$ f) $\cos(15°)$

g) $\cos(135°)$ h) $\cos(75°)$

3. Use the compound-angle identities to simplify each of these expressions to one term:

a) $\cos 23°\cos 22° - \sin 23°\ \sin 22°$ b) $1 - 2\sin 45°\cos 45°$

c) $\sin 18°\cos 19° + \sin 19°\cos 18°$ d) $2\cos^2 A - 1$

e) $\cos 81°\cos 11° + \sin 81°\cos 11°$ f) $2\sin 16°\cos 16°$

g) $\cos 25°\cos 25° - \sin 25°\ \sin 25°$ h) $\sin 28°\cos 8° - \sin 8°\cos 28°$

4. Prove the following identities:

a) $\cos(a+a) = 1 - 2\sin^2 a$ b) $\frac{\cos 2b+1}{\sin 2b} = \cot b$

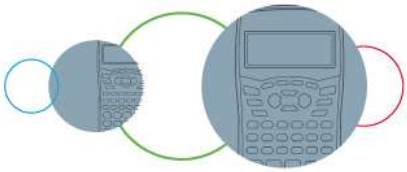
c) $\sin(d+45°) - \cos(d+45°) = \sqrt{2}\sin d$

d) $\frac{1-\sin 2w}{\sin 2w-\cos 2w-1} = \frac{1}{2}\tan w - \frac{1}{2}$ e) $\frac{\sin(b+30°)+\cos(60°+b)}{\sin 2b} = \frac{1}{2}cosec\ b$

f) $\sin 3a + \cos 3a = (\cos a - \sin a)(1 + 4\cos a \sin a)$

g) $\frac{\cos 2f-1}{\sin 2f} = -\tan f$ h) $\frac{1-2\sin d}{\cos d-\sin d} - \frac{2\cos d-1}{\cos d+\sin d} = \frac{2(\cos d+\sin d-1)}{\cos 2d}$

i) $\frac{1}{1-\sin A} + \frac{2\sin A}{1+\sin A} = 2 - \frac{1-3\sin A}{\cos^2 A}$ j) $1 - \frac{\sin 2B+1}{\cos 2B} = \frac{-2\sin B}{\cos B+\sin B}$



5. Give the general solution for each of the following equations:

a) $\sin a \sin b + \cos a \cos b = 0.5$ where $a = 2b$

b) $\sin 2c - \sin c = 0.74$

c) $\cos 2d + \cos d = 0$

d) $\cos 5d - \cos 4d = 0.33$

e) $\sin(\alpha + 30°) = \cos a$

6. If $5\sin(\theta + \mu) = 4$, $\tan(\theta + \mu) < 0$ and $3\cos 2\theta + 2 = 0$, $\tan 2\theta > 0$ determine, without a calculator and with the aid of a diagram the following:

a) $\sin\theta$

b) $\cos\theta$

c) $\tan(\theta + \mu)$

d) $\cos\mu$

7. Given that $A = \tan\varepsilon$ and $B = \sin\beta$, give each of the following in terms of A and B:

a) $\frac{\sin(\varepsilon + \beta)}{\sin 2\varepsilon}$

b) $\sin\varepsilon \times \tan\beta$

c) $\cos 2\beta$

d) $1 - \sin 2\beta$

8. Give the identity for each of these:

a) $\sin\frac{C}{2}$

b) $\cos\frac{D}{2}$

c) $\tan 2E$

d) $\sin\frac{3}{2}F$