SHARP

Worksheet 6 Memorandum: Term 1 Revision

Grade 12 Mathematics

1. a) 7 15 23 31…

   8 8 8

   Linear

   b) 0 1 1 2 3 5…

   1 0 1 1 2

   -1 1 0 1

   None *side note – this is the Fibionacci Sequence, the previous term and current term are added together to get the next term.

   c) 48 36 27 $20\frac{1}{4}$…

   $\times\frac{3}{4}$ $\times\frac{3}{4}$ $\times\frac{3}{4}$

   Geometric

   d) 88 95 107 126 154…

   7 12 19 28

   5 7 9

   None

   e) 38 58 68 68…

   20 10 0

   -10 -10

   Quadratic

   f) 85 -51 $30\frac{3}{5}$ $-18\frac{9}{25}$…

   $\times -\frac{3}{5}$ $\times -\frac{3}{5}$ $\times -\frac{3}{5}$

   Geometric

   g) 22 30 38 46…

   8 8 8

   Linear

   h) 14 21 29 40 57…

   7 8 11 17

   1 3 6

   None

   i) 1 2 4 8…

   $\times 2$ $\times 2$ $\times 2$

   Geometric

   j) 77 88 99 110 121…

   11 11 11 11

   Linear

2. a) 7 15 23 31…

   8 8 8

   i) $T_n = 7 + 8(n-1)$

   c) 48 36 27 $20\frac{1}{4}$

   $\times\frac{3}{4}$ $\times\frac{3}{4}$ $\times\frac{3}{4}$

   i) $T_n = 48\left(\frac{3}{4}\right)^{n-1}$

ii) $T_{22} = 7 + 8(22-1)$

$T_{22} = 175$

iii) $T_n = 7 + 8n - 8$

$T_n = 8n - 1$

$\therefore \sum_{i=1}^{n}(8i-1)$

iv) $S_9 = \frac{9}{2}[2(7) + (9-1)(8)]$

$S_9 = 351$

ii) $T_{22} = 48\left(\frac{3}{4}\right)^{22-1}$

$T_{22} = 0.114163\ldots \approx 0.11$

iii) $\sum_{i=1}^{n} 48\left(\frac{3}{4}\right)^{i-1}$

iv) $S_9 = \frac{48\left(\left(\frac{3}{4}\right)^9 - 1\right)}{\frac{3}{4}-1}$

$S_9 = 177.58$

Converging: $\therefore S_\infty = \frac{48}{1-\frac{3}{4}}$

$S_\infty = 192$

f) 85 -51 $30\frac{3}{5}$ $-18\frac{9}{25}$

$\times -\frac{3}{5}$ $\times -\frac{3}{5}$ $\times -\frac{3}{5}$

i) $T_n = 85\left(-\frac{3}{5}\right)^{n-1}$

ii) $T_{22} = 85\left(-\frac{3}{5}\right)^{22-1}$

$T_{22} = -0.0019$

iii) $\sum_{i=1}^{n} 85\left(-\frac{3}{5}\right)^{i-1}$

iv) $S_9 = \frac{85\left(\left(-\frac{3}{5}\right)^9 - 1\right)}{-\frac{3}{5}-1}$

$\therefore S_9 = 53.66$

Converging: $\therefore S_\infty = \frac{85}{1-\left(-\frac{3}{5}\right)}$

$S_\infty = 53\frac{1}{8}$

g) 22 30 38 46

8 8 8

i) $T_n = 22 + 8(n-1)$

ii) $T_{22} = 22 + 8(22-1)$

$T_{22} = 190$

iii) $T_n = 22 + 8n - 8$

$T_n = 8n + 14$

$\sum_{i=1}^{n}(8i+14)$

iv) $S_9 = \frac{9}{2}[2(22) + 8(9-1)]$

$S_9 = 486$

i) 1 2 4 8

x2 x2 x2

i) $T_n = 1(2)^{n-1}$

ii) $T_{22} = 2^{22-1}$

$T_{22} = 2\,097\,152$

iii) $\sum_{i=1}^{n} 2^{i-1}$

iv) $S_9 = \frac{1(2^9-1)}{2-1}$

$S_9 = 511$

j) 77 88 99 110 121

11 11 11 11

i) $T_n = 77 + 11(n-1)$

ii) $T_{22} = 77 + 11(22-1)$

$T_{22} = 308$

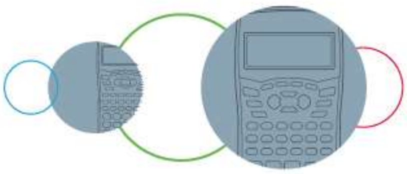
iii) $T_n = 77 + 11n - 11$

$T_n = 11n + 66$

$\sum_{i=1}^{n}(11i+66)$

iv) $S_9 = \frac{9}{2}[2(77) + 11(9-1)]$

$S_9 = 1089$



3. $S_9 = \frac{9}{2}[2a_A + d(9-1)] = T_5 = a_G r^4$ and $T_3 = a_G r^2 = T_8 = a_A + d(8-1) - 1$

a) $a_A : a_G = 25:16$ $\therefore 16a_A = 25a_G$ and $a_A = 9$

$\therefore a_G = \frac{16}{25}(9) = 5\frac{19}{25}$

So for equation 1 we now have:

$\frac{9}{2}[2(9) + d(8)] = 5\frac{19}{25}(r)^4$ to get d on its own:

$81 + 36d = 5\frac{19}{25}r^4$

$36d = 5\frac{19}{25}r^4 - 81$

$d = \frac{4}{25}r^4 - 2\frac{1}{4}$ …1

And for the second equation we have:

$5\frac{19}{25}\, r^2 = 8 + d(7)$ …2

Subs 1 into 2:

$\therefore 5\frac{19}{25}r^2 = 8 + 7(\frac{4}{25}r^4 - 2\frac{1}{4})$

$\therefore\ 0 = 8 + \frac{28}{25}r^4 - 15\frac{3}{4} - 5\frac{19}{25}r^2$

$\therefore 0 = \frac{28}{25}\, r^4 - 5\frac{19}{25}r^2 - 7\frac{3}{4}$ (multiply out by 100)

$\therefore 0 = 112r^4 - 576r^2 - 775$ (factorise as a trinomial)

$\therefore r^2 = \frac{576 \pm \sqrt{(-576)^2 - 4(112)(-775)}}{2(112)}$

$\therefore r^2 = -\frac{31}{28}$... not possible for r^2 to be negative therefore N/A

OR $r^2 = 6\frac{1}{4} = \frac{25}{4}$

$\therefore r = 2\frac{1}{2}\ or \frac{5}{2}$

Subs back into 1:

$d = \frac{4}{25}\left(\frac{5}{2}\right)^4 - 2\frac{1}{4}$

$d = 4$

$\therefore d = 4 \quad r = 2\frac{1}{2} \quad and \quad a_g = 5\frac{19}{25}$

b) The geometric series is diverging as r > 1

c) $T_n = 9 + 4(n-1)$

$\therefore 53 = 9 + 4(n-1)$

$\therefore 44 = 4(n-1)$

$\therefore 11 = n - 1$

$\therefore n = 12$

d) $T_n = 5\frac{19}{25}\left(\frac{5}{2}\right)^{n-1}$

$\therefore 8\,789\frac{1}{16} = 5\frac{19}{25}\left(\frac{5}{2}\right)^{n-1}$

$\therefore \frac{390\,625}{256} = \left(\frac{5}{2}\right)^{n-1}$

$\therefore\ \log_{\frac{5}{2}}\left(\frac{390\,625}{256}\right) = n - 1$

$\therefore n - 1 = 8$

$\therefore n = 9$

4. a) $y = Log_3 x$

i) Function (one to one)

ii) N/A

iii) $x = \log_3 y$

$\therefore y = 3^x$

b) $y^2 = 25 - x^2$

i) not a function (many to many)

ii) $0 \leq x \leq 5$

iii) $x^2 = 25 - y^2$

$\therefore x^2 + y^2 = 25$

$\therefore y^2 = 25 - x^2$

$\therefore y = \sqrt{25 - x^2}$

c) $y = 3x^2 - 7$

i) function (many to one)

ii) N/A

iii) $x = 3y^2 - 7$

$x + 7 = 3y^2$

$\frac{1}{3}(x + 7) = y^2$

$\therefore y = \sqrt{\frac{1}{3}(x + 7)}$

d) $x = y^2 + 2$

i) not a function (one to many)

ii) $y = \sqrt{x - 2}$

$\therefore x \geq 2$

iii) $y = x^2 + 2$

e) $y = x^3$

i) function (many to one)

ii) N/A

iii) $x = y^3$

$\therefore y = \sqrt[3]{x}$

f) $y = 3^x$

i) function (one to one)

ii) N/A

iii) $x = 3^y$

$\therefore y = \log_3 x$

g) $y = \sqrt{x^3 - 9}$

i) not a function (many to many)

ii) $y > 0 \quad and \; x > \sqrt[3]{9}$

iii) $x = \sqrt{y^3 - 9}$

$x^2 = y^3 - 9$

$x^2 + 9 = y^3$

$\therefore y = \sqrt[3]{x^2 + 9}$

h) $y^3 = x^2$

i) not a function (many to many)

ii) $x > 0$

iii) $x^3 = y^2$

$\therefore y = \sqrt{x^3}$

5. P = 20 000 n = 3 years

a) A = ?

P = 20 000

$i = 8.5\% = 0.085$

n = 3 years

$A = P(1 + i)^n$

$A = 20\,000(1 + 0.085)^3$

$A = 25\,545.78$

b) A = ? $A = P(1-i)^n$

P = 20 000 $A = 20\,000(1-0.076)^3$

$i = 7.6\% = 0.076$ $A = 15\,777.78$

n = 3 years

c) P = 20 000 $P = \frac{x[1-(1+i)^{-n}]}{i}$

$i = 11.2\% \div 12 = \frac{7}{750}$ $20\,000 = \frac{x\left[1-\left(1+\frac{7}{750}\right)^{-36}\right]}{\frac{7}{750}}$

$x = ?$ $186\frac{2}{3} = x[0.28426242]$

n = 3 years x 12 = 36 $\therefore x = 656.67$

d) $F = 25\,545.78 - 15\,777.78 = 9768$

$i = 12.5\% \div 4 = \frac{1}{32}$ $F = \frac{x[(1+i)^n - 1]}{i}$

$x = ?$ $9\,768 = \frac{x\left[\left(1+\frac{1}{32}\right)^{12}-1\right]}{\frac{1}{32}}$

n = 3 years x 4 = 12 $305\frac{1}{4} = x[0.446663548]$

$\therefore x = 683.40$

e) for c) $i_{eff} = ?$ $i_{eff} = \left(1+\frac{i_m}{m}\right)^m - 1$

$i_m = 11.2\% = 0.122$ $i_{eff} = \left(1+\frac{0.112}{12}\right)^{12} - 1$

$m = 12$ $i_{eff} = 0.1179 = 11.79\%$

for d) $i_{eff} = ?$ $i_{eff} = \left(1+\frac{i_m}{m}\right)^m - 1$

$i_m = 12.5\% = 0.125$ $i_{eff} = \left(1+\frac{0.125}{4}\right)^4 - 1$

$m = 4$ $i_{eff} = 0.13098 = 13.1\%$

The effective interest rate in d) (the sinking fund) is better.

f) $x = 656.67 + 15\% = 755.17$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

P = 20 000 $20\,000 = \frac{755.17\left[1-\left(1+\frac{7}{750}\right)^{-12n}\right]}{\frac{7}{750}}$

$i = 11.2\% \div 12 = \frac{7}{750}$ $186\frac{2}{3} = 755.17\left[1-\left(1+\frac{7}{750}\right)^{-12n}\right]$

n = n x 12 = 12n $0.24718496 = 1 - \left(\frac{757}{750}\right)^{-12n}$

$$-0.752815039 = -\left(\frac{757}{750}\right)^{-12n}$$

$$\therefore\ -12n = \log_{\frac{757}{750}} 0.752815039$$

$$\therefore\ -12n = -30.56343163$$

$$\therefore n = 2.54\ years\ or\ 2\ years\ and\ 6.56\ months$$

6. a) $\frac{\cos 2a}{1+\sin 2a} = \frac{\cos a - \sin a}{\cos a + \sin a}$

LHS $= \frac{\cos^2 a - \sin^2 a}{\cos^2 a + \sin^2 a + 2\sin a \cos a}$

$= \frac{(\cos a - \sin a)(\cos a + sin a)}{(\cos a + \sin a)(\cos a + \sin a)}$

$= \frac{\cos a - \sin a}{\cos a + \sin a}$

∴ LHS = RHS

b) $\sin 3b = \sin b\,(4\cos^2 b - 1)$

LHS $= \sin(2b + b)$

$= \sin 2b \cos b + \sin b \cos 2b$

$= 2\sin b \cos b \cos b + \sin b\,(2\cos^2 b - 1)$

$= 2\sin b \cos^2 b + 2\cos^2 b \sin b - \sin b$

$= 4\cos^2 b \sin b - \sin b$

$= \sin b(4\cos^2 b - 1)$

∴ LHS = RHS

c) $\cos 3a - \sin 3a = (\cos a + \sin a)(1 - 4\cos a \sin a)$

LHS $= \cos(2a + a) - \sin(2a + a)$

$= \cos 2a \cos a - \sin 2a \sin a - (\sin 2a \cos a + \sin a \cos 2a)$

$= (\cos^2 a - \sin^2 a)(\cos a) - 2\sin a\ \cos a \sin a - 2\sin a\ \cos a \cos a - (\sin a)(\cos^2 a - \sin^2 a)$

$= \cos^3 a - \sin^2 a \cos a - 2\sin^2 a \cos a - 2\sin a \cos^2 a - \cos^2 a \sin a + \sin^3 a$

$= \cos^3 a - 3\sin^2 a \cos a - 3\cos^2 a \sin a + \sin^3 a$

$= (\cos^3 a + \sin^3 a) - 3\sin a \cos a\,(\sin a + \cos a)$

$= (\cos a + \sin a)(\cos^2 a - \sin a \cos a + \sin^2 a) - 3\sin a \cos a\,(\cos a + \sin a)$

$= (\cos a + \sin a)(1 - \sin a \cos a) - 3\sin a \cos a(\cos a + \sin a)$

$= (\cos a + \sin a)(1 - \sin a \cos a - 3\sin a \cos a)$

$= (\cos a + \sin a)(1 - 4\sin a \cos a)$

∴ LHS = RHS

d) $\sin 2f - \tan f = \tan f \,.\, \cos 2f$

RHS $= \tan f\,(\cos^2 f - \sin^2 f)$

$= \frac{\sin f}{\cos f}(\cos^2 f - \sin^2 f)$

$= \sin f \cos f - \frac{\sin^3 f}{\cos f}$

$= \frac{\sin f cos^2 f - \sin^3 f}{\cos f}$

$= \frac{\sin f(\cos 2f)}{\cos f}$

LHS $= 2\sin f \cos f - \frac{\sin f}{\cos f}$

$= \frac{2\sin f \cos^2 f - \sin f}{\cos f}$

$= \frac{\sin f(2\cos^2 - 1)}{\cos f}$

$= \frac{\sin f(\cos 2f)}{\cos f}$

$\therefore$ LHS = RHS

e) $\frac{\sin 2a + \sin 3a}{\sin a} = 4\cos^2 a + 2\cos a - 1$

LHS $= \frac{2\sin a\cos a + \sin(2a+a)}{\sin a}$

$= \frac{2\sin a\cos a + \sin 2a\cos a + \cos 2a\sin a}{\sin a}$

$= \frac{2\sin a\cos a + 2\sin a\cos a\cos a + (2\cos^2 a - 1)(\sin a)}{\sin a}$

$= \frac{2\sin a\cos a + 2\sin a\cos^2 a + 2\cos^2 a\sin a - \sin a}{\sin a}$

$= \frac{\sin a(2\cos a + 2\cos^2 a + 2\cos^2 a - 1)}{\sin a}$

$= 4\cos^2 a + 2\cos a - 1$

LHS = RHS