Worksheet 8 Memorandum: Functions – Polynomials (Factor and Remainder Theorem)

Grade 12 Mathematics

1. a) $3x^3 - 28x^2 + 52x + 48$

Factor: $x - 4$

$\therefore (x - 4)(3x^2 + kx - 12)$

$-12x^2 + kx^2 = -28x^2$

$\therefore kx^2 = -16x^2$

$\therefore k = -16$

$\therefore (x - 4)(3x^2 - 16x - 12)$

$\therefore (x - 4)(3x + 2)(x - 6)$

b) $2x^3 - 17x^2 + 41x - 30$

Factor: $x - 2$

$\therefore (x - 2)(2x^2 + kx + 15)$

$-4x^2 + kx^2 = -17x^2$

$\therefore kx^2 = -13x^2$

$\therefore k = -13$

$\therefore (x - 2)(2x^2 - 13x + 15)$

$\therefore (x - 2)(2x - 3)(x - 5)$

c) $30x^3 + 53x^2 - 4x - 15$

Factor: $2x - 1$

$\therefore (2x - 1)(15x^2 + kx + 15)$

$-15x^2 + 2kx^2 = 53x^2$

$\therefore 2kx^2 = 68x^2$

$\therefore k = 34$

$\therefore (2x - 1)(15x^2 + 34x + 15)$

$\therefore (2x - 1)(3x + 5)(5x + 3)$

d) $x^3 + 10x^2 + 8x - 64$

Factor: $x - 2$

$\therefore (x - 2)(x^2 + kx + 32)$

$-2x^2 + kx^2 = 10x^2$

$\therefore kx^2 = 12x^2$

$\therefore k = 12$

$\therefore (x - 2)(x^2 + 12x + 32)$

$\therefore (x - 2)(x + 4)(x + 8)$

e) $x^3 + 15x^2 + 75x + 125$

Factor: $x + 5$

$\therefore (x + 5)(x^2 + kx + 25)$

$5x^2 + kx^2 = 15x^2$

$\therefore kx^2 = 10x^2$

$\therefore k = 10$

$\therefore (x + 5)(x^2 + 10x + 25)$

$\therefore (x + 5)(x + 5)(x + 5)$

f) $x^3 - 7x^2 - 14x + 48$

Factor: $x - 2$

$\therefore (x - 2)(x^2 + kx - 24)$

$-2x^2 + kx^2 = -7x^2$

$kx^2 = -5x^2$

$k = -5$

$\therefore (x - 2)(x^2 - 5x - 24)$

$\therefore (x - 2)(x + 3)(x - 8)$

g) $2x^3 - 39x^2 + 157x + 330$

Factor: $x - 10$

$\therefore (x - 10)(2x^2 + kx - 33)$

$-20x^2 + kx^2 = -39x^2$

$kx^2 = -19x^2$

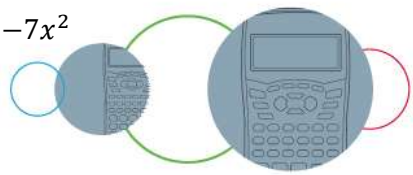
h) $15x^3 + 53x^2 - 58x - 120$

Factor: $x + 4$

$\therefore (x + 4)(15x^2 + kx - 30)$

$60x^2 + kx^2 = 53x^2$

$kx^2 = -7x^2$



$k = -19$

$\therefore (x-10)(2x^2-19x-33)$

$\therefore (x-10)(2x+3)(x-11)$

$k = -7$

$\therefore (x+4)(15x^2-7x-30)$

$\therefore (x+4)(5x+6)(3x-5)$

i) $x^3+14x^2+41x-56$

Factor: $x-1$

$\therefore (x-1)(x^2+kx+56)$

$-x^2+kx^2 = 14x^2$

$kx^2 = 15x^2$

$k = 15$

$\therefore (x-1)(x^2+15x+56)$

$\therefore (x-1)(x+7)(x+8)$

j) $x^3+3x^2-88x+240$

Factor: $x-4$

$\therefore (x-4)(x^2+kx-60)$

$-4x^2+kx^2 = 3x^2$

$kx^2 = 7x^2$

$k = 7$

$\therefore (x-4)(x^2+7x-60)$

$\therefore (x-4)(x-5)(x+12)$

2. Solve for x in of the following equations:

a) $x^3-2x^2-x+2=0$

Factor: $x-1$

$\therefore (x-1)(x^2+kx-2)$

$-x^2+kx^2 = -2x^2$

$kx^2 = -x^2$

$k = -1$

$\therefore (x-1)(x^2-x-2)$

$\therefore (x-1)(x-2)(x+1)$

$\therefore x = 1 \ \ or \ \ x = 2 \ \ or \ \ x = -1$

b) $x(x^2-67) = 126$

$x^3 - 67x - 126 = 0$

Factor: $x+2$

$\therefore (x+2)(x^2+kx-63)$

$2x^2+kx^2 = 0$

$kx^2 = -2x^2$

$k = -2$

$\therefore (x+2)(x^2-2x-63)$

$\therefore (x+2)(x-9)(x+7)$

$\therefore x = -2 \ \ or \ \ x = 9 \ \ or \ \ x = -7$

c) $3(x^3+8) = 7x(x+10)$

$3x^3+24 = 7x^2+70x$

$3x^3-7x^2-70x+24 = 0$

Factor: $x+4$

$(x+4)(3x^2+kx+6)$

$12x^2+kx^2 = -7x^2$

$kx^2 = -19x^2$

$k = -19$

$\therefore (x+4)(3x^2-19x+6) = 0$

$\therefore (x+4)(3x-1)(x-6) = 0$

$\therefore x = -4 \ \ or \ \ x = \frac{1}{3} \ \ or \ x = 6$

d) $x(3x^2+8x-48) = 128$

$3x^3+8x^2-48x = 128$

$3x^3+8x^2-48x-128 = 0$

Factor: $x+4$

$(x+4)(3x^2+kx-32)$

$12x^2+kx^2 = 8x^2$

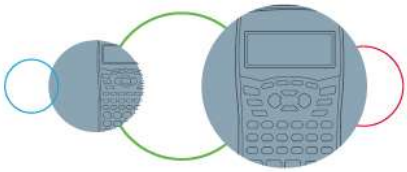
$kx^2 = -4x^2$

$k = -4$

$\therefore (x+4)(3x^2-4x-32) = 0$

$\therefore (x+4)(3x+8)(x-4) = 0$

$\therefore x = -4 \ \ or \ \ x = -\frac{8}{3} \ \ x = 4$



e) $10(x^3+10)=3x(11x+35)$

$10x^3+100=33x^2+105x$

$10x^3-33x^2-105x+100=0$

Factor: $x-5$

$\therefore (x-5)(10x^2+kx-20)$

$-50x^2+kx^2=-33x^2$

$kx^2=17x^2$

$k=17$

$\therefore (x-5)(10x^2+17x-20)=0$

$\therefore (x-5)(2x+5)(5x-4)=0$

$\therefore\ x=5\ \ or\ \ x=-2\frac{1}{2}\ \ or\ \ x=\frac{4}{5}$

f) $x^3-3x^2-126x+648=0$

Factor: $x-6$

$\therefore (x-6)(x^2+kx-108)=0$

$-6x^2+kx^2=-3x^2$

$kx^2=3x^2$

$k=3$

$\therefore (x-6)(x^2+3x-108)=0$

$\therefore (x-6)(x-9)(x+12)=0$

$\therefore x=6\ \ or\ \ x=9\ \ or\ \ x=-12$

g) $x(x^2-41)=-4(x^2+9)$

$x^3-41x=-4x^2-36$

$x^3+4x^2-41x+36=0$

Factor: $x-1$

$\therefore (x-1)(x^2+kx-36)=0$

$-x^2+kx^2=4x^2$

$kx^2=5x^2$

$k=5$

$\therefore (x-1)(x^2+5x-36)=0$

$\therefore (x-1)(x+9)(x-4)=0$

$\therefore\ x=1\ \ or\ \ x=-9\ \ or\ \ x=4$

h) $2x^3=3x^2+98x+48$

$2x^3-3x^2-98x-48=0$

Factor: $x+6$

$\therefore (x+6)(2x^2+kx-8)=0$

$12x^2+kx^2=-3x^2$

$k\,x^2=-15x^2$

$k=-15$

$\therefore (x+6)(2x^2-15x-8)=0$

$\therefore (x+6)(2x+1)(x-8)=0$

$\therefore x=-6\ \ or\ \ x=-\frac{1}{2}\ \ or\ \ x=8$

i) $5x^3-31x^2=4(17x+8)$

$5x^3-31x^2=68x+32$

$5x^3-31x^2-68x-32=0$

Factor: $x+1$

$\therefore (x+1)(5x^2+kx-32)$

$5x^2+kx^2=-31x^2$

$kx^2=-36x^2$

$\therefore (x+1)(5x^2-36x-32)$

$\therefore (x+1)(5x+4)(x-8)=0$

$\therefore x=-1\ \ or\ \ x=-\frac{4}{5}\ \ or\ \ x=8$

j) $3x^3+7x^2-22x-8=0$

Factor: $x-2$

$\therefore (x-2)(3x^2+kx+4)$

$-6x^2+kx^2=7x^2$

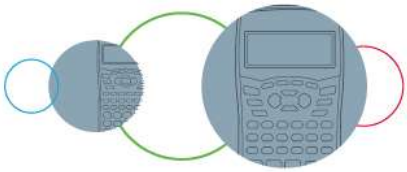
$kx^2=13x^2$

$k=9$

$\therefore (x-2)(3x^2+13x+4)=0$

$\therefore (x-2)(3x+1)(x+4)=0$

$\therefore x=2\ \ or\ \ x=-\frac{1}{3}\ \ or\ \ x=-4$



3. a) Factor: $x-1$ Expression: $f(x)=x^3+2x^2-x-2$

$x=1$ $\therefore f(1)=(1)^3+2(1)^2-(1)-2$

$\therefore f(1)=0$ $\therefore x-1$ is a factor

b) Factor: $x-4$ Expression: $f(x)=x^3-9x^2+26x-24$

$x=4$ $\therefore f(4)=(4)^3-9(4)^2+26(4)-24$

$\therefore f(4)=64-144+104-24$

$\therefore f(4)=0$ $\therefore x-4$ is a factor

c) Factor: $x+6$ Expression: $f(x)=x^3+13x^2+54x+72$

$x=-6$ $\therefore f(-6)=(-6)^3+13(-6)^2+54(-6)+72$

$\therefore f(-6)=-216+468-324+72$

$\therefore f(-6)=0$ $\therefore x+6$ is a factor

d) Factor: $2x+1$ Expression: $f(x)=2x^3+11x^2-23x-14$

$x=-\frac{1}{2}$ $\therefore f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+11\left(-\frac{1}{2}\right)^2-23\left(-\frac{1}{2}\right)-14$

$\therefore f\left(-\frac{1}{2}\right)=-\frac{1}{4}+2\frac{3}{4}+11\frac{1}{2}-14$

$\therefore f\left(-\frac{1}{2}\right)=0$ $\therefore 2x+1$ is a factor

e) Factor: $5x-1$ Expression: $f(x)=5x^3-26x^2-65x+14$

$x=\frac{1}{5}$ $\therefore f\left(\frac{1}{5}\right)=5\left(\frac{1}{5}\right)^3-26\left(\frac{1}{5}\right)^2-65\left(\frac{1}{5}\right)+14$

$\therefore f\left(\frac{1}{5}\right)=\frac{1}{25}-\frac{26}{25}-13+14$

$\therefore f\left(\frac{1}{5}\right)=0$ $\therefore 5x-1$ is a factor

4. a) When $f(x)=px^2+25x+4$ is divided by $x-2$ the remainder is 78.

When $x=2$ then $f(2)=78$

$\therefore 78=p(2)^2+25(2)+4$

$\therefore 78=4p+54$

$\therefore 24=4p$

$\therefore p=6$

b) When $f(x)=x^2-7x-p$ is divided by $x+1$ the remainder is -52.

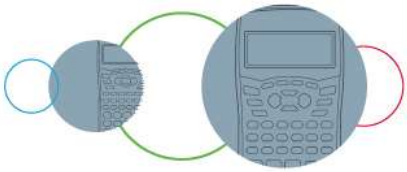
When $x=-1$ then $f(-1)=-52$

$\therefore -52=(-1)^2-7(-1)-p$

$\therefore -52=1+7-p$

$\therefore -60=-p$

$\therefore p=60$



c) When $f(x) = 5x^3 + px^2 - 47x - 10$ is divided by $x + 4$ the remainder is 114.

When $x = -4$ then $f(-4) = 114$

$\therefore 114 = 5(-4)^3 + p(-4)^2 - 47(-4) - 10$

$\therefore 114 = -320 + 16p + 188 - 10$

$\therefore 114 = 16p - 142$

$\therefore 256 = 16p$

$\therefore p = 16$

d) When $f(x) = x^2 + px - 21$ is divided by $x - 5$ the remainder is -16.

When $x = 5$ then $f(5) = -16$

$\therefore -16 = (5)^2 + p(5) - 21$

$\therefore -16 = 25 + 5p - 21$

$\therefore -16 = 5p + 4$

$\therefore -20 = 5p$

$\therefore p = -4$

e) When $f(x) = x^3 + 11x^2 + px - 216$ is divided by $x - 7$ it gives a remainder of 624.

When $x = 7$ then $f(7) = 624$

$\therefore 624 = (7)^3 + 11(7)^2 + p(7) - 216$

$\therefore 624 = 343 + 539 + 7p - 216$

$\therefore 624 = 7p + 666$

$\therefore -42 = 7p$

$\therefore p = -6$

5. Determine the remainder if each of these equations are divided by the factor given

a) $f(x) = x^3 - 5x^2 + 2x + 8$ divided by $x + 10$

When $x = -10$ then $f(-10) = (-10)^3 - 5(-10)^2 + 2(-10) + 8$

$\therefore f(-10) = -1000 - 500 - 20 + 8$

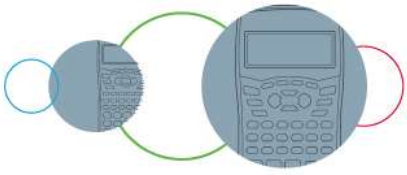
$\therefore f(-10) = -1\,512$ is the remainder

b) $f(x) = x^2 + 2x - 15$ divided by $x - 12$

When $x = 12$ then $f(12) = (12)^2 + 2(12) - 15$

$\therefore f(12) = 144 + 24 - 15$

$\therefore f(12) = 153$ is the remainder



c) $f(x) = x^2 - 14x + 40$ divided by $x + 5$

When $x = -5$ then $f(-5) = (-5)^2 - 14(-5) + 40$

$\therefore f(-5) = 25 + 70 + 40$

$\therefore f(-5) = 135$ is the remainder

d) $f(x) = 4x^3 - 12x^2 - 67x - 30$ divided by $x + 4$

When $x = -4$ then $f(-4) = 4(-4)^3 - 12(-4)^2 - 67(-4) - 30$

$\therefore f(-4) = -256 - 192 + 268 - 30$

$\therefore f(-4) = -210$ is the remainder

e) $f(x) = x^3 + 2x^2 - 111x + 108$ divided by $5x - 1$

When $x = \frac{1}{5}$ then $f\left(\frac{1}{5}\right) = \left(\frac{1}{5}\right)^3 + 2\left(\frac{1}{5}\right)^2 - 111\left(\frac{1}{5}\right) + 108$

$\therefore f\left(\frac{1}{5}\right) = \frac{1}{125} + \frac{2}{25} - 22\frac{1}{5} + 108$

$\therefore f\left(\frac{1}{5}\right) = 85\frac{111}{125}$

6. Which of the 3 divisors given for each equation is a perfect factor of that equation?

a) A $\rightarrow x + 2$ B $\rightarrow x - 2$ C $\rightarrow 5x - 1$

$g(x) = 6x^2 + 11x - 2$

When $x = -2$ then $g(-2) = 6(-2)^2 + 11(-2) - 2 = 0$

$\therefore$ A is a perfect factor

b) A $\rightarrow 2x + 1$ B $\rightarrow x - 4$ C $\rightarrow x - 2$

$g(x) = x^3 - 7x^2 - 6x + 72$

When $x = -\frac{1}{2}$ then $g\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 6\left(-\frac{1}{2}\right) + 72 = 73\frac{1}{8}$

When $x = 4$ then $g(4) = (4)^3 - 7(4)^2 - 6(4) + 72 = 0$

$\therefore$ B is a perfect factor

c) A $\rightarrow 2x - 1$ B $\rightarrow x + 3$ C $\rightarrow x - 1$

$g(x) = x^3 - 3x^2 - x + 3$

When $x = \frac{1}{2}$ then $g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 3 = 1\frac{7}{8}$

When $x = -3$ then $g(-3) = (-3)^3 - 3(-3)^2 - (-3) + 3 = -48$

When $x = 1$ then $g(1) = (1)^3 - 3(1) - (1) + 3 = 0$

$\therefore$ C is a perfect factor