

# SHARP

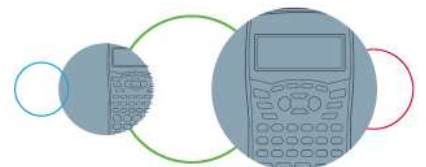
## Worksheet 9 Memorandum: Differential Calculus – First Principles and Differential Calculus

### Grade 12 Mathematics CAPS

1. a)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$   
 $= \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x-3}$   
 $= \lim_{x \rightarrow 3} x + 3$   
 $= 3 + 3$   
 $= 6$
- b)  $\lim_{a \rightarrow -2} \frac{a^2 - 3a - 10}{a + 2}$   
 $= \lim_{a \rightarrow -2} \frac{(a+2)(a-5)}{a+2}$   
 $= \lim_{a \rightarrow -2} a - 5$   
 $= -2 - 5$   
 $= -7$
- c)  $\lim_{b \rightarrow 0} \frac{14b^2}{b}$   
 $= \lim_{b \rightarrow 0} 14b$   
 $= 14(0)$   
 $= 0$
- d)  $\lim_{x \rightarrow 4} (x^2 - 16)$   
 $= (4)^2 - 16$   
 $= 0$
- e)  $\lim_{x \rightarrow -1} (x^3 + x^2 - 1)$   
 $= (-1)^3 + (-1)^2 - 1$   
 $= -1$
- f)  $\lim_{x \rightarrow 3} (-5)$   
 $= -5$
- g)  $\lim_{x \rightarrow c} x^2 - 2c$   
 $= (c)^2 - 2c$   
 $= c^2 - 2c$
- h)  $\lim_{x \rightarrow -3} \frac{x^2 + 7x + 12}{x^2 - x - 12}$   
 $= \lim_{x \rightarrow -3} \frac{(x+3)(x+4)}{(x+3)(x-4)}$   
 $= \lim_{x \rightarrow -3} \frac{x+4}{x-4}$   
 $= \frac{-3+4}{-3-4}$   
 $= \frac{1}{-7}$
- i)  $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x + 1}$   
 $= \frac{(1)^2 + 2(1) - 3}{1 + 1}$   
 $= 0$
- j)  $\lim_{f \rightarrow 0} \frac{f^2 - 3f + 5}{3}$   
 $= \frac{(0)^2 - 3(0) + 5}{3}$   
 $= \frac{5}{3}$

2. The average gradient is the gradient between two points on a line, while gradient at a point is the gradient at the particular point on the graph.

3. a) (3; 4) and (4; 3)  
 $m = \frac{y_2 - y_1}{x_2 - x_1}$   
 $m = \frac{3 - 4}{4 - 3} = -1$
- b) (7; 1) and (9; -3)  
 $m = \frac{y_2 - y_1}{x_2 - x_1}$   
 $m = \frac{-3 - 1}{9 - 7} = -2$



c)  $(-2; 4)$  and  $(5; -6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6 - 3}{5 - (-2)}$$

$$m = \frac{-9}{7}$$

d)  $(5\frac{1}{2}; 6)$  and  $(5\frac{5}{6}; 6\frac{3}{4})$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6\frac{3}{4} - 6}{5\frac{5}{6} - 5\frac{1}{2}}$$

$$m = 2\frac{1}{4}$$

4. a)  $f(x) = 3x^2 - 2$

$$\therefore f(x+h) = 3(x+h)^2 - 2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 2 - (3x^2 - 2)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 2 - 3x^2 + 2}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} 6x + 3h$$

$$\therefore f'(x) = 6x$$

e)  $(1,01; 2)$  and  $(1,02; 2,2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2,2 - 2}{1,02 - 1,01}$$

$$m = 20$$

b)  $f(x) = -3x^3$

$$\therefore f(x+h) = -3(x+h)^3$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-3(x^3 + 3x^2h + 3xh^2 + h^3) - (-3x^3)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-3x^3 - 9x^2h - 9xh^2 - 3h^3 + 3x^3}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-9x^2h - 9xh^2 - 3h^3}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(-9x^2 - 9xh - 3h^2)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} -9x^2 - 9xh - 3h^2$$

$$\therefore f'(x) = -9x^2$$

c)  $f(x) = -2x^2 - 3x + 4$

$$\therefore f(x+h) = -2(x+h)^2 - 3(x+h) + 4$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2(x^2 + 2xh + h^2) - 3(x+h) + 4 - (-2x^2 - 3x + 4)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 - 3x - 3h + 4 + 2x^2 + 3x - 4}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2 - 3h}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h - 3)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} -4x - 2h - 3$$

$$\therefore f'(x) = -4x - 3$$

d)  $f(x) = 5x$

$$\therefore f(x+h) = 5(x+h)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

e)  $f(x) = \frac{7}{x}$

$$\therefore f(x+h) = \frac{7}{x+h}$$

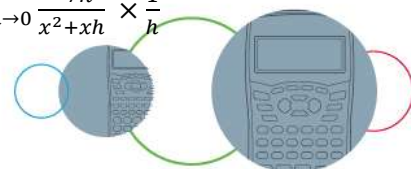
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{7}{x+h} - \frac{7}{x}}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{7(x) - 7(x+h)}{x(x+h)}}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{7x - 7x - 7h}{x^2 + xh} \times \frac{1}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-7h}{x^2 + xh} \times \frac{1}{h}$$



$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{5x+5h-5x}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{5h}{h}$$

$$\therefore f'(x) = 5$$

f)  $f(x) = 9$

$$\therefore f(x+h) = 9$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{9-9}{h}$$

$$\therefore f'(x) = 0$$

h)  $f(x) = 3 + 8x - 2x^2$

$$\therefore f(x+h) = 3 + 8(x+h) - 2(x+h)^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{3+8x+8h-2(x^2+2xh+h^2)-(3+8x-2x^2)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{3+8x+8h-2x^2-4xh-2h^2-3-8x+2x^2}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{8h-4xh-2h^2}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(8-4x-2h)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} 8 - 4x - 2h$$

$$\therefore f'(x) = 8 - 4x$$

i)  $f(x) = \frac{-5}{x}$

$$\therefore f(x+h) = \frac{-5}{x+h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-5}{x+h} - \frac{-5}{x}}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-5x - (-5)(x+h)}{x(x+h)}}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-5x+5x+5h}{x^2+xh} \times \frac{1}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{5h}{x^2+xh} \times \frac{1}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{5}{x^2+xh}$$

$$\therefore f'(x) = \frac{5}{x^2}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-7}{x^2+xh}$$

$$\therefore f'(x) = \frac{-7}{x^2}$$

g)  $f(x) = -5x^2 - 3x$

$$\therefore f(x+h) = -5(x+h)^2 - 3(x+h)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-5(x^2+2xh+h^2)-3x-3h-(-5x^2-3x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-5x^2-10xh-5h^2-3x-3h+5x^2+3x}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-10xh-5h^2-3h}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(-10x-5h-3)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} -10x - 5h - 3$$

$$\therefore f'(x) = -10x - 3$$

j)  $f(x) = 4x^3$

$$\therefore f(x+h) = 4(x+h)^3$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{4(x^3+3x^2h+3xh^2+h^3)-(4x^3)}{h}$$

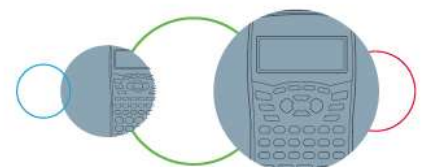
$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{4x^3+12x^2h+12xh^2+4h^3-4x^3}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{12x^2h+12xh^2+4h^3}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(12x^2+12xh+4h^2)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} 12x^2 + 12xh + 4h^2$$

$$\therefore f'(x) = 12x^2$$



5. a)  $f(x) = 3x^3 + 7x^2 - 4x + 2$

$$f'(x) = 9x^2 + 14x - 4$$

c)  $h(x) = (x^2 + 4)^2(x + 1)$

$$h(x) = (x^4 + 8x^2 + 16)(x + 1)$$

$$h(x) = x^5 + 8x^3 + 16x + x^4 + 8x^2 + 16$$

$$\therefore h'(x) = 5x^4 + 24x^2 + 16 + 4x^3 + 16x$$

e)  $k(x) = \frac{x^3 - 3x^2 - 10x + 24}{x + 3}$

$$k(x) = \frac{(x+3)(x^2 - 6x + 8)}{x+3}$$

$$k(x) = x^2 - 6x + 8$$

$$\therefore k'(x) = 2x - 6$$

g)  $n(x) = (x + 3)(x - 2)$

$$n(x) = x^2 + x - 6$$

$$\therefore n'(x) = 2x + 1$$

h)  $p(x) = \frac{x^3 - 5x + 4}{\sqrt[3]{x}}$

$$p(x) = \frac{x^3}{\sqrt[3]{x}} - \frac{5x}{\sqrt[3]{x}} + \frac{4}{\sqrt[3]{x}}$$

$$p(x) = x^{2\frac{2}{3}} - 5x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}$$

$$\therefore p'(x) = \frac{8}{3}x^{\frac{4}{3}} - \frac{10}{3}x^{-\frac{1}{3}} - \frac{4}{3}x^{-\frac{4}{3}}$$

$$\therefore p'(x) = \frac{8}{3}\sqrt[3]{x^4} - \frac{10}{3\sqrt[3]{x}} - \frac{4}{3\sqrt[3]{x^4}}$$

j)  $r(x) = \sqrt{x}(x^3 - \sqrt{x})$

$$r(x) = x^{3\frac{1}{2}} - x$$

$$\therefore r'(x) = 3\frac{1}{2}x^{\frac{5}{2}} - 1$$

$$\therefore r'(x) = 3\frac{1}{2}\sqrt{x^5} - 1$$

6. a)  $y = 3x - 4 @ (3; 5)$

$$\frac{dy}{dx} = 3$$

$$\therefore m = 3$$

b)  $g(x) = \frac{2}{x^2} - \sqrt{x}$

$$g(x) = 2x^{-2} - x^{\frac{1}{2}}$$

$$\therefore g'(x) = -4x^{-3} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore g'(x) = \frac{-4}{x^3} - \frac{1}{2\sqrt{x}}$$

d)  $j(x) = \sqrt{x} + \frac{3x^3}{\sqrt{x^3}}$

$$j(x) = x^{\frac{1}{2}} + 3x^{3-\frac{3}{2}}$$

$$j(x) = x^{\frac{1}{2}} + 3x^{\frac{3}{2}}$$

$$\therefore j'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{9}{2}x^{\frac{1}{2}}$$

$$\therefore j'(x) = \frac{1}{2\sqrt{x}} + 4\frac{1}{2}\sqrt{x}$$

f)  $m(x) = (\sqrt{x} - 3)(\sqrt{x} - 3)$

$$m(x) = x - 3\sqrt{x} - 3\sqrt{x} + 9$$

$$m(x) = x - 6\sqrt{x} + 9$$

$$m(x) = x - 6x^{\frac{1}{2}} + 9$$

$$\therefore m'(x) = 1 - 3x^{-\frac{1}{2}}$$

$$\therefore m'(x) = 1 - \frac{3}{\sqrt{x}}$$

i)  $q(x) = \frac{x+4}{x^2}$

$$q(x) = \frac{x}{x^2} + \frac{4}{x^2}$$

$$q(x) = x^{-1} + 4x^{-2}$$

$$\therefore q'(x) = -x^{-2} - 8x^{-3}$$

$$\therefore q'(x) = \frac{-1}{x^2} - \frac{8}{x^3}$$

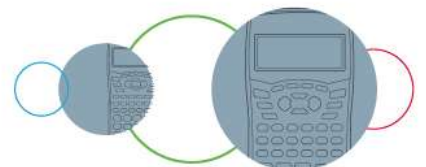
b)  $y = 7x^2 - 4x + 8 @ (-1; 19)$

$$\frac{dy}{dx} = 14x - 4$$

Subs in -1

$$m = 14(-1) - 4$$

$$\therefore m = -18$$



c)  $y = \frac{2}{x} + 3 @ (8; 3\frac{1}{4})$

$$y = 2x^{-1} + 3$$

$$\frac{dy}{dx} = -2x^{-2} \quad \text{Subs in 8}$$

$$\therefore m = -2(8)^{-2}$$

$$\therefore m = -\frac{1}{32}$$

d)  $y = -2x^3 + 3x^2 - 7 @ (-5; 318)$

$$\frac{dy}{dx} = -6x^2 + 6x \quad \text{Subs in -5}$$

$$\therefore m = -6(-5)^2 + 6(-5)$$

$$\therefore m = -180$$

e)  $y = (x + 3)(x - 4) @ (9; 60)$  f)

$$y = x^2 - x - 12$$

$$\frac{dy}{dx} = 2x - 1 \quad \text{Subs in 9}$$

$$\therefore m = 2(9) - 1$$

$$\therefore m = 17$$

$y = -\frac{3}{x} - 1 @ (-1; 2)$

$$y = -3x^{-1} - 1$$

$$\frac{dy}{dx} = 3x^{-2} \quad \text{Subs in -1}$$

$$\therefore m = 3(-1)^{-2}$$

$$\therefore m = 3$$

g)  $y = x^3 - 2x^2 + 5x + 6 @ (0; 6)$

$$\frac{dy}{dx} = 3x^2 - 4x + 5 \quad \text{Subs in 0}$$

$$\therefore m = 3(0)^2 - 4(0) + 5$$

$$\therefore m = 5$$

h)  $y = -3x^2 - 8x + 9 @ (\frac{1}{2}; 4\frac{1}{4})$

$$\frac{dy}{dx} = -6x - 8 \quad \text{Subs in } \frac{1}{2}$$

$$\therefore m = -6(\frac{1}{2}) - 8$$

$$\therefore m = -11$$

i)  $y = 7x - 8 @ (7; 41)$  j)

$$\frac{dy}{dx} = 7$$

$$\therefore m = 7$$

(the gradient is constant)

$y = 2x^3 + 3x^2 - 36x + 9 @ (-3; 90)$

$$\frac{dy}{dx} = 6x^2 + 6x - 36 \quad \text{Subs in -3}$$

$$\therefore m = 6(-3)^2 + 6(-3) - 36$$

$$\therefore m = 0$$

7. a)  $y = x^2 - 7x + 12$  and  $m = 1$

$$\frac{dy}{dx} = 2x - 7$$

$$\therefore 2x - 7 = 1$$

$$\therefore 2x = 8$$

$$\therefore x = 4$$

Subs in to find y:

$$\therefore y = (4)^2 - 7(4) + 12$$

$$\therefore y = 0$$

$$\therefore (4; 0)$$

b)  $y = x^3 + 4x^2 - 5x - 9$  and  $m = 86$

$$\frac{dy}{dx} = 3x^2 + 8x - 5$$

$$\therefore 86 = 3x^2 + 8x - 5$$

$$\therefore 0 = 3x^2 + 8x - 91$$

$$\therefore 0 = (3x - 13)(x + 7)$$

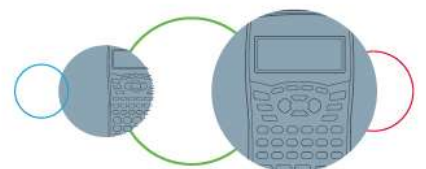
$$\therefore x = \frac{13}{3} \quad \text{or} \quad x = -7$$

Subs in to find y:

$$\therefore y = (\frac{13}{3})^3 + 4(\frac{13}{3})^2 - 5(\frac{13}{3}) - 9 = 125\frac{22}{27}$$

OR  $y = (-7)^3 + 4(-7)^2 - 5(-7) - 9 = -121$

$$\therefore (\frac{13}{3}; 125\frac{22}{27}) \quad \text{or} \quad (-7; -121)$$



c)  $y = \sqrt{x} - 3$  and  $m = \frac{1}{2}$

$$y = x^{\frac{1}{2}} - 3$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{1}{2} = \frac{1}{2\sqrt{x}}$$

$$2\sqrt{x} = 2$$

$$\sqrt{x} = 1$$

$$x = 1$$

Subs in to find y:

$$y = \sqrt{1} - 3$$

$$\therefore y = -2$$

$$\therefore (1; -2)$$

e)  $y = x^4 + 3x^2 - 4x - 9$  and  $m = -14$

$$\frac{dy}{dx} = 4x^3 + 6x - 4$$

$$\therefore -14 = 4x^3 + 6x - 4$$

$$\therefore 0 = 4x^3 + 6x + 10$$

$$\therefore 0 = (x + 1)(x^2 + kx + 10)$$

$$x^2 + kx^2 = 0$$

$$kx^2 = -x^2$$

$$\therefore k = -1$$

$$\therefore 0 = (x + 1)(x^2 - x + 10)$$

$$\therefore x = -1 \text{ and cant factorise}$$

second quadratic equation

Subs in to find y:

$$y = (-1)^4 + 3(-1)^2 - 4(-1) - 9 = -1$$

$$\therefore (-1; -1)$$

h)  $y = 3x^2 + 4x - 18$  and  $m = -8$

$$\frac{dy}{dx} = 6x + 4$$

$$\therefore -8 = 6x + 4$$

$$\therefore -12 = 6x$$

$$\therefore x = -2$$

Subs in to find y:

$$y = 3(-2)^2 + 4(-2) - 18 = -14 \quad \therefore (-2; -14)$$

d)  $y = -x^2 + 11x - 15$  and  $m = 0$

$$\frac{dy}{dx} = -2x + 11$$

$$0 = -2x + 11$$

$$2x = 11$$

$$x = 5\frac{1}{2}$$

Subs in to find y:

$$\therefore y = -\left(5\frac{1}{2}\right)^2 + 11\left(5\frac{1}{2}\right) - 15$$

$$\therefore y = 15\frac{1}{4}$$

$$\therefore \left(5\frac{1}{2}; 15\frac{1}{4}\right)$$

f)  $y = \frac{4}{x} + 3$  and  $m = -1$  ( $x > 0$ )

$$y = 4x^{-1} + 3$$

$$\frac{dy}{dx} = -4x^{-2}$$

$$\therefore -1 = -\frac{4}{x^2}$$

$$\therefore -x^2 = -4$$

$$\therefore x^2 = 4$$

$$\therefore x = \pm 2 \quad \text{but } x > 0 \therefore x = 2$$

Subs in to find y:

$$y = \frac{4}{2} + 3 = 5$$

$$\therefore (2; 5)$$

g)  $y = -x^3 + 7x^2 - 9x - 16$  and  $m = -113$

$$\frac{dy}{dx} = -3x^2 + 14x - 9$$

$$\therefore -113 = -3x^2 + 14x - 9$$

$$\therefore 3x^2 - 14x - 104 = 0$$

$$\therefore (3x - 26)(x + 4) = 0$$

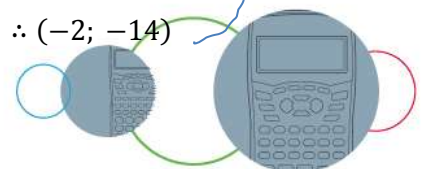
$$\therefore x = \frac{26}{3} \quad \text{or } x = -4$$

Subs in to find y:

$$y = -\left(\frac{26}{3}\right)^3 + 7\left(\frac{26}{3}\right)^2 - 9\left(\frac{26}{3}\right) - 16 = -219\frac{5}{27}$$

$$y = -(-4)^3 + 7(-4)^2 - 9(-4) - 16 = 196$$

$$\therefore \left(\frac{26}{3}; -219\frac{5}{27}\right) \text{ or } (-4; 196)$$



i)  $y = \frac{-3}{x^2} + 2$  and  $m = -6$

$$y = -3x^{-2} + 2$$

$$\frac{dy}{dx} = 6x^{-3}$$

$$\therefore -6 = \frac{6}{x^3}$$

$$\therefore -6x^3 = 6$$

$$\therefore x^3 = -1$$

$$\therefore x = -1$$

Subs in to find y:

$$y = \frac{-3}{(-1)^2} + 2$$

$$y = -1$$

$$\therefore (-1; -1)$$

j)  $y = x^3 + 6x^2 + 18x - 20$  and  $m = 9$

$$\frac{dy}{dx} = 3x^2 + 12x + 18$$

$$\therefore 9 = 3x^2 + 12x + 18$$

$$\therefore 0 = 3x^2 + 12x + 9$$

$$\therefore 0 = x^2 + 4x + 3$$

$$\therefore 0 = (x + 3)(x + 1)$$

$$\therefore x = -3 \text{ or } x = -1$$

Subs in to find y:

$$y = (-3)^3 + 6(-3)^2 + 18(-3) - 20 = -47$$

$$y = (-1)^3 + 6(-1)^2 + 18(-1) - 20 = -33$$

$$\therefore (-3; -47) \text{ and } (-1; -33)$$

8. a) The gradient of  $y = ax^2 + bx - 6$  at  $(2; -8)$  is  $m = -5$ .

$$\frac{dy}{dx} = 2ax + b$$

and  $-8 = a(2)^2 + b(2) - 6$

$$\therefore -5 = 2a(2) + b$$

$$0 = 4a + 2b + 2 \dots 2$$

$$\therefore b = -5 - 4a \dots 1$$

Subs 1 into 2:

$$\therefore 0 = 4a + 2(-5 - 4a) + 2$$

$$\therefore 0 = 4a - 10 - 8a + 2$$

$$\therefore 4a = -8$$

$$\therefore a = -2$$

Subs back into 1:

$$\therefore b = -5 - 4(-2)$$

$$\therefore b = 3$$

b) For the graph of  $y = x^3 + 5x^2 - ax + b$  the gradient at the point  $(-4; 50)$  is  $m = -1$ .

$$\frac{dy}{dx} = 3x^2 + 10x - a$$

and  $50 = (-4)^3 + 5(-4)^2 - a(-4) + b \dots 2$

$$\therefore -1 = 3x^2 + 10x - a \quad (\text{subs in } x = -4)$$

$$\therefore -1 = 3(-4)^2 + 10(-4) - a$$

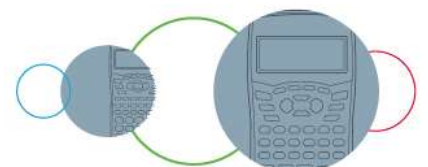
$$\therefore -9 = -a$$

$$\therefore a = 9$$

Subs back into 2 to find b:

$$50 = -64 + 80 + 4(9) + b$$

$$\therefore b = -2$$



- c) The gradient of  $y = ax^2 - 14x - 10$  at the point  $(b; 182)$  is  $m = -50$

$$\frac{dy}{dx} = 2ax - 14 \quad \text{and} \quad 182 = ab^2 - 14b - 10 \dots 2$$

$$-50 = 2ab - 14$$

$$-36 = 2ab$$

$$\therefore -\frac{18}{b} = a \dots 1 \text{ Subs back into 2 to find } b:$$

$$182 = \left(\frac{-18}{b}\right)b^2 - 14b - 10$$

$$192 = -18b - 14b$$

$$\therefore 192 = -32b$$

$$\therefore b = -6$$

Subs back into 1 to find  $a$ :

$$a = \frac{-18}{-6}$$

$$\therefore a = 3$$

- d) The graph  $y = 2x^3 + ax^2 + bx + 21$  has the gradient  $m = 43$  at the point  $(-2; -13)$ .

$$\frac{dy}{dx} = 6x^2 + 2ax + b \quad \text{and} \quad -13 = -2(-2)^3 + a(-2)^2 + b(-2) + 21 \dots 2$$

$$\therefore 43 = 6(-2)^2 + 2a(-2) + b$$

$$\therefore 43 = 24 - 4a + b$$

$$\therefore b = 19 + 4a \dots 1$$

Subs 1 into 2 and solve for  $a$ :

$$-13 = 16 + 4a - 2(19 + 4a) + 21$$

$$-50 = 4a - 38 - 8a$$

$$-12 = -4a$$

$$a = 3$$

Subs back into 1 and solve for  $b$ :

$$\therefore b = 19 + 4(3)$$

$$\therefore b = 31$$

- e) At the point  $(-5; 695)$  the gradient of  $y = -2x^3 + ax^2 - 9x + b$  is  $m = -319$ .

$$\frac{dy}{dx} = -6x^2 + 2ax - 9 \quad \text{and} \quad 695 = -2(-5)^3 + a(-5)^2 - 9(-5) + b \dots 2$$

$$-319 = -6(-5)^2 + 2a(-5) - 9$$

$$-319 = -150 - 10a - 9$$

$$\therefore 10a = 160$$

$$\therefore a = 16$$

Subs back into 2 to solve for  $b$ :

$$695 = 250 + 25a + 45 + b$$

$$400 = 25(16) + b$$

$$\therefore b = 0$$

