SHARP

Worksheet 9 Memorandum: Differential Calculus – First Principals and Differential Calculus

Grade 12 Mathematics CAPS

1. a) $\lim_{x\to 3}\frac{x^2-9}{x-3}$

$=\lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}$

$=\lim_{x\to 3} x+3$

$=3+3$

$=6$

b) $\lim_{a\to -2}\frac{a^2-3a-10}{a+2}$

$=\lim_{a\to -2}\frac{(a+2)(a-5)}{a+2}$

$=\lim_{a\to -2} a-5$

$= -2-5$

$= -7$

c) $\lim_{b\to 0}\frac{14b^2}{b}$

$=\lim_{b\to 0} 14b$

$=14(0)$

$=0$

d) $\lim_{x\to 4}(x^2-16)$

$=(4)^2-16$

$=0$

e) $\lim_{x\to -1}(x^3+x^2-1)$

$=(-1)^3+(-1)^2-1$

$= -1$

f) $\lim_{x\to 3}(-5)$

$= -5$

g) $\lim_{x\to c} x^2-2c$

$=(\mathrm{c})^2-2c$

$=c^2-2c$

h) $\lim_{x\to -3}\frac{x^2+7x+12}{x^2-x-12}$

$=\lim_{x\to -3}\frac{(x+3)(x+4)}{(x+3)(x-4)}$

$=\lim_{x\to -3}\frac{x+4}{x-4}$

$=\frac{-3+4}{-3-4}$

$=\frac{1}{-7}$

i) $\lim_{x\to 1}\frac{x^2+2x-3}{x+1}$

$=\frac{(1)^2+2(1)-3}{1+1}$

$=0$

j) $\lim_{f\to 0}\frac{f^2-3f+5}{3}$

$=\frac{(0)^2-3(0)+5}{3}$

$=\frac{5}{3}$

2. The average gradient is the gradient between two points on a line, while gradient at a point is the gradient at the particular point on the graph.

3. a) (3; 4) and (4; 3)

$m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{3-4}{4-3}= -1$

b) (7; 1) and (9; -3)

$m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{-3-1}{9-7}= -2$

c) (-2; 4) and (5; -6)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-6-3}{5-(-2)}$

$m = \frac{-9}{7}$

d) $\left(5\frac{1}{2}; 6\right)$ and $\left(5\frac{5}{6}; 6\frac{3}{4}\right)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{6\frac{3}{4} - 6}{5\frac{5}{6} - 5\frac{1}{2}}$

$m = 2\frac{1}{4}$

e) (1,01; 2) and (1, 02; 2,2)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{2.2-2}{1.02-1.01}$

$m = 20$

4. a) $f(x) = 3x^2 - 2$

$\therefore f(x+h) = 3(x+h)^2 - 2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{3(x^2+2xh+h^2)-2-(3x^2-2)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{3x^2+6xh+3h^2-2-3x^2+2}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{6xh+3h^2}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{h(6x+3h)}{h}$

$\therefore f'(x) = \lim_{h\to 0}\ 6x + 3h$

$\therefore f'(x) = 6x$

b) $f(x) = -3x^3$

$\therefore f(x+h) = -3(x+h)^3$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-3(x^3+3x^2h+3xh^2+h^3)-(-3x^3)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-3x^3-9x^2h-9xh^2-3h^3+3x^3}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-9x^2h-9xh^2-3h^3}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{h(-9x^2-9xh-3h^2)}{h}$

$\therefore f'(x) = \lim_{h\to 0} -9x^2 - 9xh - 3h^2$

$\therefore f'(x) = -9x^2$

c) $f(x) = -2x^2 - 3x + 4$

$\therefore f(x+h) = -2(x+h)^2 - 3(x+h) + 4$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-2(x^2+2xh+h^2)-3(x+h)+4-(-2x^2-3x+4)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-2x^2-4xh-2h^2-3x-3h+4+2x^2+3x-4}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-4xh-2h^2-3h}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{h(-4x-2h-3)}{h}$

$\therefore f'(x) = \lim_{h\to 0} -4x - 2h - 3$

$\therefore f'(x) = -4x - 3$

d) $f(x) = 5x$

$\therefore f(x+h) = 5(x+h)$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

e) $f(x) = \frac{7}{x}$

$\therefore f(x+h) = \frac{7}{x+h}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{\frac{7}{x+h} - \frac{7}{x}}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{\frac{7(x)-7(x+h)}{x(x+h)}}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{7x-7x-7h}{x^2+xh} \times \frac{1}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-7h}{x^2+xh} \times \frac{1}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{5x+5h-5x}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{5h}{h}$

$\therefore f'(x) = 5$

$\therefore f'(x) = \lim_{h\to 0} \frac{-7}{x^2+xh}$

$\therefore f'(x) = \frac{-7}{x^2}$

f) $f(x) = 9$

$\therefore f(x+h) = 9$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{9-9}{h}$

$\therefore f'(x) = 0$

g) $f(x) = -5x^2 - 3x$

$\therefore f(x+h) = -5(x+h)^2 - 3(x+h)$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{-5(x^2+2xh+h^2)-3x-3h-(-5x^2-3x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-5x^2-10xh-5h^2-3x-3h+5x^2+3x}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-10xh-5h^2-3h}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{h(-10x-5h-3)}{h}$

$\therefore f'(x) = \lim_{h\to 0} -10x - 5h - 3$

$\therefore f'(x) = -10x - 3$

h) $f(x) = 3 + 8x - 2x^2$

$\therefore f(x+h) = 3 + 8(x+h) - 2(x+h)^2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{3+8x+8h-2(x^2+2xh+h^2)-(3+8x-2x^2)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{3+8x+8h-2x^2-4xh-2h^2-3-8x+2x^2}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{8h-4xh-2h^2}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{h(8-4x-2h)}{h}$

$\therefore f'(x) = \lim_{h\to 0} 8 - 4x - 2h$

$\therefore f'(x) = 8 - 4x$

i) $f(x) = \frac{-5}{x}$

$\therefore f(x+h) = \frac{-5}{x+h}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{\frac{-5}{x+h} - \frac{-5}{x}}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{\frac{-5x-(-5)(x+h)}{x(x+h)}}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{-5x+5x+5h}{x^2+xh} \times \frac{1}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{5h}{x^2+xh} \times \frac{1}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{5}{x^2+xh}$

$\therefore f'(x) = \frac{5}{x^2}$

j) $f(x) = 4x^3$

$\therefore f(x+h) = 4(x+h)^3$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{4(x^3+3x^2h+3xh^2+h^3)-(4x^3)}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{4x^3+12x^2h+12xh^2+4h^3-4x^3}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{12x^2h+12xh^2+4h^3}{h}$

$\therefore f'(x) = \lim_{h\to 0} \frac{h(12x^2+12xh+4h^2)}{h}$

$\therefore f'(x) = \lim_{h\to 0} 12x^2 + 12xh + 4h^2$

$\therefore f'(x) = 12x^2$

5. a) $f(x) = 3x^3 + 7x^2 - 4x + 2$

$f'(x) = 9x^2 + 14x - 4$

b) $g(x) = \frac{2}{x^2} - \sqrt{x}$

$g(x) = 2x^{-2} - x^{\frac{1}{2}}$

$\therefore g'(x) = -4x^{-3} - \frac{1}{2}x^{-\frac{1}{2}}$

$\therefore g'(x) = \frac{-4}{x^3} - \frac{1}{2\sqrt{x}}$

c) $h(x) = (x^2 + 4)^2(x + 1)$

$h(x) = (x^4 + 8x^2 + 16)(x + 1)$

$h(x) = x^5 + 8x^3 + 16x + x^4 + 8x^2 + 16$

$\therefore h'(x) = 5x^4 + 24x^2 + 16 + 4x^3 + 16x$

d) $j(x) = \sqrt{x} + \frac{3x^3}{\sqrt{x^3}}$

$j(x) = x^{\frac{1}{2}} + 3x^{3-\frac{3}{2}}$

$j(x) = x^{\frac{1}{2}} + 3x^{\frac{3}{2}}$

$\therefore j'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{9}{2}x^{\frac{1}{2}}$

$\therefore j'(x) = \frac{1}{2\sqrt{x}} + 4\frac{1}{2}\sqrt{x}$

e) $k(x) = \frac{x^3 - 3x^2 - 10x + 24}{x + 3}$

$k(x) = \frac{(x+3)(x^2 - 6x + 8)}{x + 3}$

$k(x) = x^2 - 6x + 8$

$\therefore k'(x) = 2x - 6$

f) $m(x) = (\sqrt{x} - 3)(\sqrt{x} - 3)$

$m(x) = x - 3\sqrt{x} - 3\sqrt{x} + 9$

$m(x) = x - 6\sqrt{x} + 9$

$m(x) = x - 6x^{\frac{1}{2}} + 9$

$\therefore m'(x) = 1 - 3x^{-\frac{1}{2}}$

$\therefore m'(x) = 1 - \frac{3}{\sqrt{x}}$

g) $n(x) = (x + 3)(x - 2)$

$n(x) = x^2 + x - 6$

$\therefore n'(x) = 2x + 1$

h) $p(x) = \frac{x^3 - 5x + 4}{\sqrt[3]{x}}$

$p(x) = \frac{x^3}{\sqrt[3]{x}} - \frac{5x}{\sqrt[3]{x}} + \frac{4}{\sqrt[3]{x}}$

$p(x) = x^{2\frac{2}{3}} - 5x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}$

$\therefore p'(x) = \frac{8}{3}x^{\frac{4}{3}} - \frac{10}{3}x^{-\frac{1}{3}} - \frac{4}{3}x^{-\frac{4}{3}}$

$\therefore p'(x) = \frac{8}{3}\sqrt[3]{x^4} - \frac{10}{3\sqrt[3]{x}} - \frac{4}{3\sqrt[3]{x^4}}$

i) $q(x) = \frac{x + 4}{x^2}$

$q(x) = \frac{x}{x^2} + \frac{4}{x^2}$

$q(x) = x^{-1} + 4x^{-2}$

$\therefore q'(x) = -x^{-2} - 8x^{-3}$

$\therefore q'(x) = \frac{-1}{x^2} - \frac{8}{x^3}$

j) $r(x) = \sqrt{x}(x^3 - \sqrt{x})$

$r(x) = x^{3\frac{1}{2}} - x$

$\therefore r'(x) = 3\frac{1}{2}x^{\frac{5}{2}} - 1$

$\therefore r'(x) = 3\frac{1}{2}\sqrt{x^5} - 1$

6. a) $y = 3x - 4$ @ (3; 5)

$\frac{dy}{dx} = 3$

$\therefore m = 3$

b) $y = 7x^2 - 4x + 8$ @ (-1; 19)

$\frac{dy}{dx} = 14x - 4$ Subs in -1

$m = 14(-1) - 4$

$\therefore m = -18$

c) $y = \frac{2}{x} + 3$ @ $\left(8;\ 3\frac{1}{4}\right)$

$y = 2x^{-1} + 3$

$\frac{dy}{dx} = -2x^{-2}$ Subs in 8

$\therefore m = -2(8)^{-2}$

$\therefore m = -\frac{1}{32}$

d) $y = -2x^3 + 3x^2 - 7$ @ (-5; 318)

$\frac{dy}{dx} = -6x^2 + 6x$ Subs in -5

$\therefore m = -6(-5)^2 + 6(-5)$

$\therefore m = -180$

e) $y = (x+3)(x-4)$ @ (9; 60)

$y = x^2 - x - 12$

$\frac{dy}{dx} = 2x - 1$ Subs in 9

$\therefore m = 2(9) - 1$

$\therefore m = 17$

f) $y = -\frac{3}{x} - 1$ @ (-1; 2)

$y = -3x^{-1} - 1$

$\frac{dy}{dx} = 3x^{-2}$ Subs in -1

$\therefore m = 3(-1)^{-2}$

$\therefore m = 3$

g) $y = x^3 - 2x^2 + 5x + 6$ @ (0; 6)

$\frac{dy}{dx} = 3x^2 - 4x + 5$ Subs in 0

$\therefore m = 3(0)^2 - 4(0) + 5$

$\therefore m = 5$

h) $y = -3x^2 - 8x + 9$ @ $\left(\frac{1}{2}; 4\frac{1}{4}\right)$

$\frac{dy}{dx} = -6x - 8$ Subs in $\frac{1}{2}$

$\therefore m = -6\left(\frac{1}{2}\right) - 8$

$\therefore m = -11$

i) $y = 7x - 8$ @ (7; 41)

$\frac{dy}{dx} = 7$

$\therefore m = 7$

(the gradient is constant)

j) $y = 2x^3 + 3x^2 - 36x + 9$ @ (-3; 90)

$\frac{dy}{dx} = 6x^2 + 6x - 36$ Subs in -3

$\therefore m = 6(-3)^2 + 6(-3) - 36$

$\therefore m = 0$

7. a) $y = x^2 - 7x + 12$ and m = 1

$\frac{dy}{dx} = 2x - 7$

$\therefore 2x - 7 = 1$

$\therefore 2x = 8$

$\therefore x = 4$

Subs in to find y:

$\therefore y = (4)^2 - 7(4) + 12$

$\therefore y = 0$

$\therefore$ (4; 0)

b) $y = x^3 + 4x^2 - 5x - 9$ and m = 86

$\frac{dy}{dx} = 3x^2 + 8x - 5$

$\therefore 86 = 3x^2 + 8x - 5$

$\therefore 0 = 3x^2 + 8x - 91$

$\therefore 0 = (3x - 13)(x + 7)$

$\therefore\ x = \frac{13}{3} \quad or \quad x = -7$

Subs in to find y:

$\therefore y = \left(\frac{13}{3}\right)^3 + 4\left(\frac{13}{3}\right)^2 - 5\left(\frac{13}{3}\right) - 9 = 125\frac{22}{27}$

OR $y = (-7)^3 + 4(-7)^2 - 5(-7) - 9 = -121$

$\therefore \left(\frac{13}{3}; 125\frac{22}{27}\right) \quad or \quad (-7;\ -121)$

c) $y = \sqrt{x} - 3$ and $m = \frac{1}{2}$

$y = x^{\frac{1}{2}} - 3$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\frac{1}{2} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x} = 2$

$\sqrt{x} = 1$

$x = 1$

Subs in to find y:

$y = \sqrt{1} - 3$

$\therefore y = -2$

$\therefore (1; -2)$

d) $y = -x^2 + 11x - 15$ and m = 0

$\frac{dy}{dx} = -2x + 11$

$0 = -2x + 11$

$2x = 11$

$x = 5\frac{1}{2}$

Subs in to find y:

$\therefore y = -\left(5\frac{1}{2}\right)^2 + 11\left(5\frac{1}{2}\right) - 15$

$\therefore y = 15\frac{1}{4}$

$\therefore \left(5\frac{1}{2}; 15\frac{1}{4}\right)$

e) $y = x^4 + 3x^2 - 4x - 9$ and m = -14

$\frac{dy}{dx} = 4x^3 + 6x - 4$

$\therefore -14 = 4x^3 + 6x - 4$

$\therefore 0 = 4x^3 + 6x + 10$

$\therefore 0 = (x + 1)(x^2 + kx + 10)$

$x^2 + kx^2 = 0$

$kx^2 = -x^2$

$\therefore k = -1$

$\therefore 0 = (x + 1)(x^2 - x + 10)$

$\therefore x = -1$ and cant factorise second quadratic equation

Subs in to find y:

$y = (-1)^4 + 3(-1)^2 - 4(-1) - 9 = -1$

$\therefore (-1; -1)$

f) $y = \frac{4}{x} + 3$ and m = -1 $(x > 0)$

$y = 4x^{-1} + 3$

$\frac{dy}{dx} = -4x^{-2}$

$\therefore -1 = -\frac{4}{x^2}$

$\therefore -x^2 = -4$

$\therefore x^2 = 4$

$\therefore x = \pm 2$ but $x > 0 \therefore x = 2$

Subs in to find y:

$y = \frac{4}{2} + 3 = 5$

$\therefore (2; 5)$

g) $y = -x^3 + 7x^2 - 9x - 16$ and m = -113

$\frac{dy}{dx} = -3x^2 + 14x - 9$

$\therefore -113 = -3x^2 + 14x - 9$

$\therefore 3x^2 - 14x - 104 = 0$

$\therefore (3x - 26)(x + 4) = 0$

$\therefore x = \frac{26}{3}$ or $x = -4$

Subs in to find y:

$y = -\left(\frac{26}{3}\right)^3 + 7\left(\frac{26}{3}\right)^2 - 9\left(\frac{26}{3}\right) - 16 = -219\frac{5}{27}$

$y = -(-4)^3 + 7(-4)^2 - 9(-4) - 16 = 196$

$\therefore \left(\frac{26}{3}; -219\frac{5}{27}\right)$ or $(-4; 196)$

h) $y = 3x^2 + 4x - 18$ and m = -8

$\frac{dy}{dx} = 6x + 4$

$\therefore -8 = 6x + 4$

$\therefore -12 = 6x$

$\therefore x = -2$

Subs in to find y: $y = 3(-2)^2 + 4(-2) - 18 = -14$ $\therefore (-2; -14)$

i) $y = \frac{-3}{x^2} + 2$ and m = -6

$y = -3x^{-2} + 2$

$\frac{dy}{dx} = 6x^{-3}$

$\therefore\ -6 = \frac{6}{x^3}$

$\therefore\ -6x^3 = 6$

$\therefore x^3 = -1$

$\therefore x = -1$

Subs in to find y:

$y = \frac{-3}{(-1)^2} + 2$

$y = -1$

$\therefore (-1;\ -1)$

j) $y = x^3 + 6x^2 + 18x - 20$ and m = 9

$\frac{dy}{dx} = 3x^2 + 12x + 18$

$\therefore 9 = 3x^2 + 12x + 18$

$\therefore 0 = 3x^2 + 12x + 9$

$\therefore 0 = x^2 + 4x + 3$

$\therefore 0 = (x+3)(x+1)$

$\therefore x = -3 \quad or \quad x = -1$

Subs in to find y:

$y = (-3)^3 + 6(-3)^2 + 18(-3) - 20 = -47$

$y = (-1)^3 + 6(-1)^2 + 18(-1) - 20 = -33$

$\therefore (-3;\ -47)\ and\ (-1;\ -33)$

8. a) The gradient of $y = ax^2 + bx - 6$ at (2; -8) is m = -5.

$\frac{dy}{dx} = 2ax + b$ and $-8 = a(2)^2 + b(2) - 6$

$\therefore\ -5 = 2a(2) + b$ $\qquad 0 = 4a + 2b + 2$ …2

$\therefore b = -5 - 4a$ …1

Subs 1 into 2:

$\therefore 0 = 4a + 2(-5 - 4a) + 2$

$\therefore 0 = 4a - 10 - 8a + 2$

$\therefore 4a = -8$

$\therefore a = -2$

Subs back into 1:

$\therefore b = -5 - 4(-2)$

$\therefore b = 3$

b) For the graph of $y = x^3 + 5x^2 - ax + b$ the gradient at the point (-4; 50) is m = -1.

$\frac{dy}{dx} = 3x^2 + 10x - a$ and $50 = (-4)^3 + 5(-4)^2 - a(-4) + b$ …2

$\therefore\ -1 = 3x^2 + 10x - a$ (subs in $x - 4$)

$\therefore\ -1 = 3(-4)^2 + 10(-4) - a$

$\therefore\ -9 = -a$

$\therefore a = 9$

Subs back into 2 to find b:

$50 = -64 + 80 + 4(9) + b$

$\therefore b = -2$

c) The gradient of $y = ax^2 - 14x - 10$ at the point (b; 182) is m = -50

$\frac{dy}{dx} = 2ax - 14$ and $182 = ab^2 - 14b - 10 \ldots 2$

$-50 = 2ab - 14$

$-36 = 2ab$

$\therefore -\frac{18}{b} = a \ldots 1$ Subs back into 2 to find b:

$$182 = \left(\frac{-18}{b}\right)b^2 - 14b - 10$$

$$192 = -18b - 14b$$

$$\therefore 192 = -32b$$

$$\therefore b = -6$$

Subs back into 1 to find a:

$a = \frac{-18}{-6}$

$\therefore a = 3$

d) The graph $y = 2x^3 + ax^2 + bx + 21$ has the gradient m = 43 at the point (-2; -13).

$\frac{dy}{dx} = 6x^2 + 2ax + b$ and $-13 = -2(-2)^3 + a(-2)^2 + b(-2) + 21$ ..2

$\therefore 43 = 6(-2)^2 + 2a(-2) + b$

$\therefore 43 = 24 - 4a + b$

$\therefore b = 19 + 4a \ldots 1$ Subs 1 into 2 and solve for a:

$$-13 = 16 + 4a - 2(19 + 4a) + 21$$

$$-50 = 4a - 38 - 8a$$

$$-12 = -4a$$

$$a = 3$$

Subs back into 1 and solve for b:

$\therefore b = 19 + 4(3)$

$\therefore b = 31$

e) At the point (-5; 695) the gradient of $y = -2x^3 + ax^2 - 9x + b$ is m = -319.

$\frac{dy}{dx} = -6x^2 + 2ax - 9$ and $695 = -2(-5)^3 + a(-5)^2 - 9(-5) + b \ldots 2$

$-319 = -6(-5)^2 + 2a(-5) - 9$

$-319 = -150 - 10a - 9$

$\therefore 10a = 160$

$\therefore a = 16$ Subs back into 2 to solve for b:

$$695 = 250 + 25a + 45 + b$$

$$400 = 25(16) + b$$

$$\therefore b = 0$$