

SHARP

Worksheet 9: Differential Calculus – First Principles and Differentiation

Grade 12 Mathematics CAPS

1. Find the limits for the following:

a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

b) $\lim_{a \rightarrow -2} \frac{a^2 - 3a - 10}{a + 2}$

c) $\lim_{b \rightarrow 0} \frac{14b^2}{b}$

d) $\lim_{x \rightarrow 4} (x^2 - 16)$

e) $\lim_{x \rightarrow -1} (x^3 + x^2 - 1)$

f) $\lim_{x \rightarrow 3} (-5)$

g) $\lim_{x \rightarrow c} x^2 - 2c$

h) $\lim_{x \rightarrow -3} \frac{x^2 + 7x + 12}{x^2 - x - 12}$

i) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x + 1}$

j) $\lim_{f \rightarrow 0} \frac{f^2 - 3f + 5}{3}$

2. What is the difference between the average gradient and gradient at a point?

3. Find the average gradient between each of these points:

a) (3; 4) and (4; 3)

b) (7; 1) and (9; -3)

c) (-2; 4) and (5; -6)

d) $(5\frac{1}{2}; 6)$ and $(5\frac{5}{6}; 6\frac{3}{4})$

e) (1,01; 2) and (1, 02; 2,2)

4. Find the value of $f'(x)$ from first principles for each of these:

a) $f(x) = 3x^2 - 2$

b) $f(x) = -3x^3$

c) $f(x) = -2x^2 - 3x + 4$

d) $f(x) = 5x$

e) $f(x) = \frac{7}{x}$

f) $f(x) = 9$

g) $f(x) = -5x^2 - 3x$

h) $f(x) = 3 + 8x - 2x^2$

i) $f(x) = \frac{-5}{x}$

j) $f(x) = 4x^3$

5. Find the derivatives for each of these using the short-rules learnt:

a) $f(x) = 3x^3 + 7x^2 - 4x + 2$

b) $g(x) = \frac{2}{x^2} - \sqrt{x}$

c) $h(x) = (x^2 + 4)^2(x + 1)$

d) $j(x) = \sqrt{x} + \frac{3x^3}{\sqrt{x^3}}$

e) $k(x) = \frac{x^3 - 3x^2 - 10x + 24}{x + 3}$

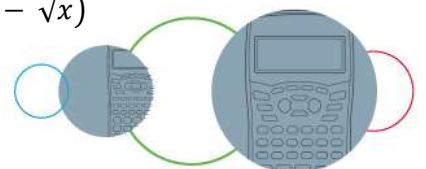
f) $m(x) = (\sqrt{x} - 3)(\sqrt{x} - 3)$

g) $n(x) = (x + 3)(x - 2)$

h) $p(x) = \frac{x^3 - 5x + 4}{\sqrt[3]{x}}$

i) $q(x) = \frac{x + 4}{x^2}$

j) $r(x) = \sqrt{x}(x^3 - \sqrt{x})$



6. Find the gradient at the point for each graph and coordinate given:

a) $y = 3x - 4$ @ (3; 5)

b) $y = 7x^2 - 4x + 8$ @ (-1; 19)

c) $y = \frac{2}{x} + 3$ @ $(8; 3\frac{1}{4})$

d) $y = -2x^3 + 3x^2 - 7$ @ (-5; 318)

e) $y = (x + 3)(x - 4)$ @ (9; 60)

f) $y = -\frac{3}{x} - 1$ @ (-1; 2)

g) $y = x^3 - 2x^2 + 5x + 6$ @ (0; 6)

h) $y = -3x^2 - 8x + 9$ @ $(\frac{1}{2}; 4\frac{1}{4})$

i) $y = 7x - 8$ @ (7; 41)

j) $y = 2x^3 + 3x^2 - 36x + 9$ @ (-3; 90)

7. Given the gradient of the graph at a point, find the coordinate of that point for each of these graphs:

a) $y = x^2 - 7x + 12$ and $m = 1$

b) $y = x^3 + 4x^2 - 5x - 9$ and $m = 86$

c) $y = \sqrt{x} - 3$ and $m = \frac{1}{2}$

d) $y = -x^2 + 11x - 15$ and $m = 0$

e) $y = x^4 + 3x^2 - 4x - 9$ and $m = -14$

f) $y = \frac{4}{x} + 3$ and $m = -1$ ($x > 0$)

g) $y = -x^3 + 7x^2 - 9x - 16$ and $m = -113$

h) $y = 3x^2 + 4x - 18$ and $m = -8$

i) $y = \frac{-3}{x^2} + 2$ and $m = -6$

j) $y = x^3 + 6x^2 + 18x - 20$ and $m = 9$

8. Find the value of a and b for each of these:

a) The gradient of $y = ax^2 + bx - 6$ at (2; -8) is $m = -5$.

b) For the graph of $y = x^3 + 5x^2 - ax + b$ the gradient at the point (-4; 50) is $m = -1$.

c) The gradient of $y = ax^2 - 14x - 10$ at the point (b; 182) is $m = -50$

d) The graph $y = 2x^3 + ax^2 + bx + 21$ has the gradient $m = 43$ at the point (-2; -13).

e) At the point (-5; 695) the gradient of $y = -2x^3 + ax^2 - 9x + b$ is $m = -319$.

