Measurement - Answers

Grade 10 Mathematics

Exercise 1:

1. a) $A = \frac{1}{2}(b \times h)$

$A = \frac{1}{2}(12cm)(5cm)$

$\therefore A = 30cm^2$

Perimeter – need hypotenuse

$\therefore h^2 = a^2 + b^2$

$\therefore h^2 = (12)^2 + (5)^2$

$\therefore h = \sqrt{169}$

$\therefore h = 13\ cm$

$P = l + b + h$

$P = 5cm + 12cm + 13cm$

$P = 30cm$

b) $A = l \times b$

$A = (15cm)(5cm)$

$\therefore A = 75cm^2$

$P = 2(l + b)$

$P = 2(15cm + 5cm)$

$\therefore P = 40cm$

c) $A = \pi r^2$

$A = \pi(3mm)^2$

$A = 28.27mm^2$

$P = 2\pi r$

$P = 2\pi(3)cm$

$P = 18.85cm$

d) $A = \frac{1}{2}(diagonal_1 \times diagonal_2)$

$A = \frac{1}{2}(34mm \times 12mm)$

$A = 204mm^2$

$P = 2(l + b)$

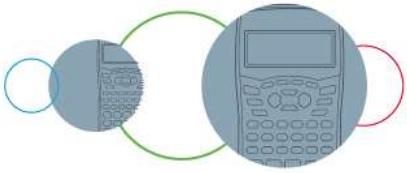
$P = 2(19mm + 21mm)$

$P = 80mm$

2. a) Length of Rectangle = 4 circles times 3m (diameter) = 12m

Breadth of rectangle = 1 diameter of circle = 3m

Radius of circle = $\frac{1}{2}$ of diameter = 1.5m



$$\therefore Shaded\ Area = Area\ of\ Rectangle - 4(Area\ of\ circle)$$
$$= l \times b - 4(\pi r^2)$$
$$= 12m \times 3m - 4(\pi)(1.5)^2$$
$$= 36m^2 - 28.27m^2$$
$$= 7.73m^2$$

b) $Shaded\ Area = Area\ of\ Rectangle - 2(area\ of\ triangle)$
$$= l \times b - 2\left(\frac{1}{2} \times b \times h\right)$$
$$= (34cm \times 12cm) - 2\left(\frac{1}{2} \times 6cm \times 34cm\right)$$
$$= 408cm^2 - 204cm^2$$
$$= 204cm^2$$

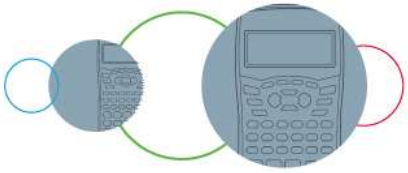
c) $Shaded\ Area = Area\ of\ Triangle - Area\ of\ Circle$
$$= \frac{1}{2} \times b \times h - \pi r^2$$
$$= \frac{1}{2} \times 14m \times 17m - \pi(6m)^2$$
$$= 119m^2 - 113{,}10m^2$$
$$= 5.9m^2$$

d) $Shaded\ Area = Area\ of\ Rectangle - (Area\ of\ arrow\ base + area\ of\ arrow\ head)$
$$= l \times b - ((l \times b) + \left(\frac{1}{2} \times b \times h\right)$$
$$= 28 \times 18 - \left(((28 - 9) \times (4.5 + 4.5)) + \left(\frac{1}{2} \times 18 \times 9\right)\right)$$
$$= 504km^2 - ((19 \times 9) + (81km^2))$$
$$= 504km^2 - (171km^2 + 81km^2)$$
$$= 252km^2$$

e) $Shaded\ Area = Area\ of\ Rectangle - (2 \times area\ of\ semi - circle)$
$$= l \times b - \left(2 \times \left(\frac{1}{2}\pi r^2\right)\right)$$
$$= 75mm \times 21mm - \left(2 \times \left(\frac{1}{2} \times \pi \times (21 \div 2)^2\right)\right)$$
$$= 1\,575mm^2 - 346.36mm^2$$
$$= 1\,228.64\ mm^2$$

3. a) $Volume = l \times b \times h$
$$= 13m \times 5m \times 7m$$
$$= 455m^3$$

$$SA = 2lb + 2lh + 2bh$$
$$SA = 2(13)(5) + 2(13)(7) + 2(5)(7)$$
$$SA = 130m^2 + 182m^2 + 70m^2$$
$$SA = 382m^2$$



b) $Volume = \frac{1}{2} \times b \times h \times l$

$= \frac{1}{2} \times 5cm \times 12cm \times 4cm$

$= 120cm^3$

$r^2 = x^2 + y^2$

$r^2 = 5^2 + 12^2$

$r = \sqrt{169} = 13cm$

$SA = 2Area\ \Delta + area\ rect\ 1 + area\ rect\ 2 + area\ rect\ 3$

$SA = 2\left(\frac{1}{2} \times b \times h\right) + (l \times r) + (l \times b) + (l \times h)$

$SA = 2\left(\frac{1}{2} \times 5 \times 12\right) + (4 \times 13) + (4 \times 5) + (4 \times 12)$

$SA = 60cm^2 + 52cm^2 + 20cm^2 + 48cm^2$

$SA = 180cm^2$

c) $Volume = \pi r^2 \times h$

$= \pi(24mm)^2 \times (135mm)$

$= 244\ 290{,}24mm^3$

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(24mm)^2 + 2\pi(24mm)(135mm)$

$SA = 3\ 619{,}11mm^2 + 20\ 257.52mm^2$

$SA = 23\ 976.63\ mm^2$

d) $Volume = \frac{1}{2} \times b \times h \times l$

$= \frac{1}{2} \times 38 \times 84 \times 30$

$= 47\ 880mm^3$

$r^2 = x^2 + y^2$

$r^2 = (30)^2 + (84)^2$

$r = \sqrt{7\ 956}$

$r = 89.2mm$

$SA = 2Area\ \Delta + area\ rect\ 1 + area\ rect\ 2 + area\ rect\ 3$

$SA = 2\left(\frac{1}{2} \times b \times h\right) + (b \times h) + (b \times r) + (b \times l)$

$SA = 2\left(\frac{1}{2} \times 38 \times 30\right) + (38 \times 30) + (38 \times 89.2) + (38 \times 84)$

$SA = 1\ 140mm^2 + 1\ 140mm^2 + 3\ 389.6mm^2 + 3\ 192mm^2$

$SA = 8\ 861.6mm^2$

e) $Volume = l^3$

$= (4cm)^3$

$= 64cm^3$

$SA = 6l^2$

$SA = 6(4cm)^2$

$SA = 96cm^2$

4. a) $V = 168\ 750m^3$

$l = b = x\ m$

$h = 30m$

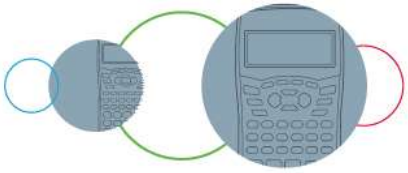
$\therefore Volume = l \times b \times h$

$\therefore 168\ 750 = x \times x \times 30$

$\therefore 5\ 625 = x^2$

$\therefore x = \sqrt{5625}$

$\therefore x = 75m$



b) SA = 4266π mm^2

h = 52mm

$r = ?$

$SA = 2\pi r^2 + 2\pi rh$

$\therefore 4266\pi = 2\pi r^2 + 2\pi r(52) \quad \div 2\pi$

$\therefore 2133 = r^2 + 52r$

$\therefore 0 = r^2 + 52r - 2133$

$\therefore 0 = (r - 27)(r + 79)$

$\therefore r = 27mm \ \ or\ r = -79 \ \left(\frac{N}{A} - length\ cannot\ be\ -\right)$

c) V = 166 375cm^3

$l = ?$

$V = l^3$

$\therefore 166\,375 = l^3$

$\therefore l = \sqrt[3]{166\,375}$

$\therefore l = 55cm$

d) SA = 7508 mm^2

h = 72mm

b = 10 mm

$l = ?$

$SA = 2lb + 2lh + 2bh$

$\therefore 7\,508 = 2l(10) + 2l(72) + 2(10)(72)$

$\therefore 7\,508 = 20l + 144l + 1\,440$

$\therefore 6\,068 = 164l$

$\therefore l = 37mm$

e) V = 19 375π cm^3

r = 25cm

h = ?

$Volume = \pi r^2 h$

$\therefore 19\,375\pi = \pi(25)^2 h$

$h = \frac{19\,375\pi}{625\pi}$

$h = 31cm$

f) V = 26 730mm^3

b = 33mm

h = ?

l = 60mm

$Volume = \frac{1}{2} b \times h \times l$

$\therefore 26\,730 = \frac{1}{2}(33)(h)(60)$

$\therefore h = \frac{26\,730}{990}$

$\therefore h = 27mm$

g) SA = 726 m^2

$l = ?$

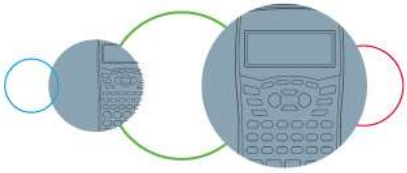
$SA = 6l^2$

$\therefore 726 = 6l^2$

$\therefore l^2 = 121$

$\therefore l = \sqrt{121}$

$\therefore l = 11m$



h) V = 16 864 m^3

l = 16m

b = ?

h = 31m

$Volume = l \times b \times h$

$\therefore 16\,864 = 16 \times b \times 31$

$\therefore b = \frac{16\,864}{496}$

$\therefore b = 34m$

i) SA = 76.1π cm^2

r = 5.2 cm

h = ?

$SA = 2\pi r^2 + 2\pi rh$

$\therefore 76.1\pi = 2\pi(5.2)^2 + 2\pi(5.2)(h)$

$\therefore 76.1\pi = 54.08\pi + 10.4\pi h$

$\therefore 22.02\pi = 10.4\pi h$

$\therefore h = \frac{22.02\pi}{10.4\pi}$

$\therefore h = 2.12cm$

j) V = 2.795mm^3

b = 4.3mm

h = ?

l = 1mm

$Volume = \frac{1}{2}b \times h \times l$

$\therefore 2.795 = \frac{1}{2}(4.3)(h)(1)$

$\therefore h = \frac{2.795}{2.15}$

$\therefore h = 1.3mm$

Exercise 2

1. Original volume = $l^3 = 4^3 = 64m^3$

a) length $= 2 \times 4$

new volume $= (2 \times 4)^3$

$= 8 \times 64m^3$

The volume is increased by a factor of 8

b) length $= 3 \times 4$

new volume $= (3 \times 4)^3$

$= 27 \times 64m^3$

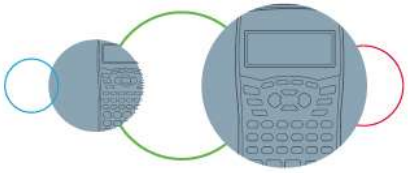
the volume is increased by a factor of 27

c) length $= \frac{1}{2} \times 4$

new volume $= \left(\frac{1}{2} \times 4\right)^3$

$= \frac{1}{8} \times 64m^3$

The volume is decreased by a factor of 8.



2. Original surface area = $4l^2 = 4(3)^2 = 36cm^2$

a) Length $= 2 \times 3$

New surface area $= 4(2 \times 3)^2$

$= 4 \times 36cm^2$

The surface area quadrupled

b) length $= 3 \times 3$

new surface area $= 4(3 \times 3)^2$

$= 9 \times 36cm^2$

The surface area increased by a factor of 9.

c) length $= \frac{1}{2} \times 3$

New surface area $= 4\left(\frac{1}{2} \times 3\right)^2$

$= \frac{1}{4} \times 36cm^2$

The surface area decreased by a factor of 4.

3. a) original volume $= l \times b \times h = 5 \times 6 \times 7 = 210mm^3$

i) length $= 2 \times 5$

New volume $= 2 \times 5 \times 6 \times 7$

$= 2 \times 210mm^3$

The volume was doubled

ii) length $= 2 \times 5$; breadth $= 2 \times 7$

New volume $= 2 \times 5 \times 2 \times 7 \times 6$

$= 4 \times 210mm^3$

The volume was quadrupled

iii) height $= 3 \times 6$

New Volume $= 3 \times 6 \times 5 \times 7$

$= 3 \times 210mm^3$

The volume was tripled

iv) height $= 2 \times 6$; length $= \frac{1}{2} \times 5$

New volume $= 2 \times 6 \times \frac{1}{2} \times 5 \times 7$

$= 210mm^3$

The volume remains the same

b) Original surface area $= 2lb + 2lh + 2bh = 2(5)(7) + 2(5)(6) + 2(6)(7) = 214mm^2$

i) length $= \frac{3}{2} \times 5$

New surface area $= 2\left(\frac{3}{2} \times 5\right)(7) + 2\left(\frac{3}{2} \times 5\right)(6) + 2(6)(7)$

$= 279mm^2$

$\therefore \frac{279}{214} = 1.3$

The surface area is increased by a factor of 1.3

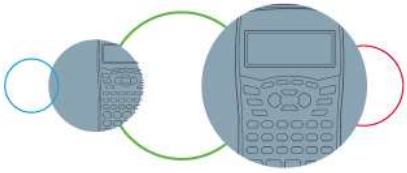
ii) breadth $= 4 \times 7$ height $= 4 \times 6$

New surface area $= 2(5)(4 \times 7) + 2(5)(4 \times 6) + 2(4 \times 6)(4 \times 7)$

$= 1\,864\ mm^2$

$\therefore \frac{1\,864}{214} = 8.71$

The surface area is increased by a factor of 8.71



iii) length $= \frac{1}{2} \times 5$ height $= \frac{1}{2} \times 6$

New surface area $= 2\left(\frac{1}{2} \times 5\right)(7) + 2\left(\frac{1}{2} \times 5\right)\left(\frac{1}{2} \times 6\right) + 2\left(\frac{1}{2} \times 6\right)(7)$

$= 92$

$\therefore \frac{214}{92} = 2.33$

The surface area is decreased by a factor of 2.33

4. Original volume $= \pi r^2 h$

a) height $= 2 \times h$ radius $= \frac{1}{2} \times r$

new volume $= \pi\left(\frac{1}{2} \times r\right)^2 (2 \times h)$

$= \frac{1}{4} \times 2 \times \pi r^2 h$

$= \frac{1}{2} \pi r^2 h$

The new volume is half of the original volume.

b) height $= \frac{1}{2} \times h$ radius $2 \times r$

new volume $= \pi(2 \times r)^2 \left(\frac{1}{2} \times h\right)$

$= 4 \times \frac{1}{2} \times \pi r^2 h$

$2\pi r^2 h$

The new volume is double the original volume.

c) radius $= 3 \times r$

new volume $= \pi(3 \times r)^2 (h)$

$= 9\pi r^2 h$

The new volume is 9 times bigger than the original volume.

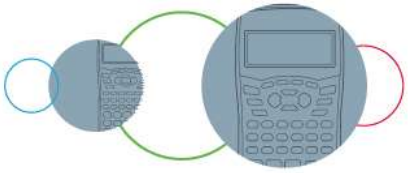
d) height $= 3 \times h$ radius $= \frac{1}{3} \times r$

new volume $= \pi\left(\frac{1}{3} \times r\right)^2 (3 \times h)$

$= \frac{1}{9} \times 3 \times \pi r^2 h$

$= \frac{1}{3} \times \pi r^2 h$

The new volume is one third of the original volume



5. a) Original volume $= \pi r^2 h = 500\pi\ m^3$

i) radius $= 2 \times r$

New volume $= \pi(2 \times r)^2 h$

$= 4 \times \pi r^2 h$

$= 4 \times 500\pi\ m^3$

$= 2\,000\pi\ m^3$

ii) height $= 2 \times h$

New volume $= \pi r^2 (2 \times h)$

$= 2 \times \pi r^2 h$

$= 2 \times 500\pi\ m^3$

$= 1000\pi\ m^3$

iii) radius $= \frac{1}{2} \times r$

New Volume $= \pi\left(\frac{1}{2} \times r\right)^2 h$

$= \frac{1}{4} \times \pi r^2 h$

$= \frac{1}{4} \times 500\pi\ m^3$

$= 125\,\pi\ m^3$

iv) height $= \frac{1}{2} \times h$

New Volume $= \pi r^2 \left(\frac{1}{2} \times h\right)$

$= \frac{1}{2} \times \pi r^2 h$

$= \frac{1}{2} \times 500\pi\ m^3$

$= 250\,\pi\ m^3$

v) radius $= 3 \times r$ height $= \frac{1}{2} \times h$

New Volume $= \pi(3 \times r)^2 \left(\frac{1}{2} \times h\right)$

$= 4\frac{1}{2} \times \pi r^2 h$

$= 4\frac{1}{2} \times 500\,\pi\ m^3$

$= 2\,250\,\pi\ m^3$

b) original volume $= l \times b \times h = 460\ cm^3$

i) length $= 2 \times l$ breadth $= 2 \times b$ height $= 2 \times h$

New volume $= (2 \times l)(2 \times b)(2 \times h)$

$= 8 \times lbh$

$= 8 \times 460\ cm^3$

$= 3\,680\ cm^3$

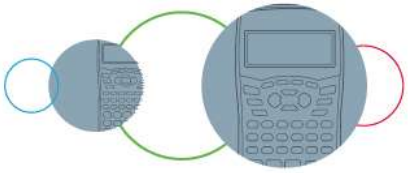
ii) length $= 2 \times l$ breadth $= 2 \times b$ height $= \frac{1}{2} \times h$

New Volume $= (2 \times l)(2 \times b)\left(\frac{1}{2} \times h\right)$

$= 2 \times lbh$

$= 2 \times 460\ cm^3$

$= 920 cm^3$



iii) length $= \frac{1}{2} \times l$ breadth $= \frac{1}{2} \times b$ height $= 2 \times h$

New volume $= \left(\frac{1}{2} \times l\right)\left(\frac{1}{2} \times b\right)(2 \times h)$

$= \frac{1}{2} \times lbh$

$= \frac{1}{2} \times 460\ cm^3$

$= 230\ cm^3$

iv) length $= \frac{1}{2} \times l$ breadth $= \frac{1}{2} \times b$ height $= \frac{1}{2} \times h$

New volume $= \left(\frac{1}{2} \times l\right)\left(\frac{1}{2} \times b\right)\left(\frac{1}{2} \times h\right)$

$= \frac{1}{8} \times lbh$

$= \frac{1}{8} \times 460\ cm^3$

$= 57.5\ cm^3$

v) length $= \frac{5}{3} \times l$ breadth $= \frac{5}{3} \times b$ height $= \frac{5}{3} \times h$

New volume $= \left(\frac{5}{3} \times l\right)\left(\frac{5}{3} \times b\right)\left(\frac{5}{3} \times h\right)$

$= \frac{125}{27} \times lbh$

$= \frac{125}{27} \times 460\ cm^3$

$= 2\ 129.63\ cm^3$

c) Original volume $= l^3 = 125\ mm^3$

i) length $= 2 \times l$

New volume $= (2 \times l)^3$

$= 8 \times l^3$

$= 8 \times 125\ mm^3$

$= 1\ 000\ mm^3$

ii) length $= 3 \times l$

New volume $= (3 \times l)^3$

$= 27 \times l^3$

$= 27 \times 125\ mm^3$

$= 3\ 375\ mm^3$

iii) length $= \frac{1}{2} \times l$

New volume $= \left(\frac{1}{2} \times l\right)^3$

$= \frac{1}{8} \times l^3$

$= \frac{1}{8} \times 125\ mm^3$

$= 15.625\ mm^3$

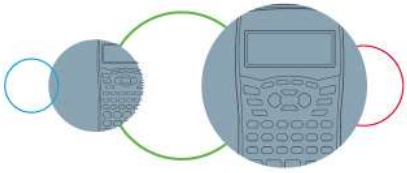
iv) length $= \frac{2}{5} \times l$

New volume $= \left(\frac{2}{5} \times l\right)^3$

$= \frac{8}{125} \times l^3$

$= \frac{8}{125} \times 125\ mm^3$

$= 8\ mm^3$



d) original surface area $= 2lb + 2lh + 2bh = 400\ mm^2$

i) length $= \frac{1}{2} \times l$ breadth $= \frac{1}{2} \times b$ height $= \frac{1}{2} \times h$

New Surface Area $= 2\left(\frac{1}{2}l\right)\left(\frac{1}{2}b\right) + 2\left(\frac{1}{2}l\right)\left(\frac{1}{2}h\right) + 2\left(\frac{1}{2}b\right)\left(\frac{1}{2}h\right)$

$= 2\left(\frac{1}{4}\right)(lb) + 2\left(\frac{1}{4}\right)(lh) + 2\left(\frac{1}{4}\right)(bh)$

$= \frac{1}{4}(2lb + 2lh + 2bh)$

$= \frac{1}{4}(400\ mm^2)$

$= 100\ mm^2$

ii) length $= \frac{3}{2} \times l$ breadth $= \frac{3}{2} \times b$ height $= \frac{3}{2} \times h$

New Surface Area $= 2\left(\frac{3}{2}l\right)\left(\frac{3}{2}b\right) + 2\left(\frac{3}{2}l\right)\left(\frac{3}{2}h\right) + 2\left(\frac{3}{2}b\right)\left(\frac{3}{2}h\right)$

$= 2\left(\frac{9}{4}\right)(lb) + \ 2\left(\frac{9}{4}\right)(lh) + 2\left(\frac{9}{4}\right)(bh)$

$= \frac{9}{4}(2lb + 2lh + 2bh)$

$= \frac{9}{4}(400mm^2)$

$= 900\ mm^2$

e) original surface area $= 2\pi r^2 + 2\pi rh = 51.6\ cm^2$

i) radius $= 3 \times r$ height $= 3 \times h$

new surface area $= 2\pi(3 \times r)^2 + 2\pi(3 \times r)(3 \times h)$

$= 9 \times 2\pi r^2 + 9 \times 2\pi rh$

$= 9(2\pi r^2 + 2\pi rh)$

$= 9(51.6\ cm^2)$

$= \ 464.4\ cm^2$

ii) radius $= 2 \times r$ height $= 2 \times h$

new surface area $= 2\pi(2 \times r)^2 + 2\pi(2 \times r)(2 \times h)$

$= 4 \times 2\pi r^2 + 4 \times 2\pi rh$

$= 4(2\pi r^2 + 2\pi rh)$

$= 4(51.6\ cm^2)$

$= 206.4\ cm^2$

iii) radius $= \frac{1}{2} \times r$ height $= \frac{1}{2} \times h$

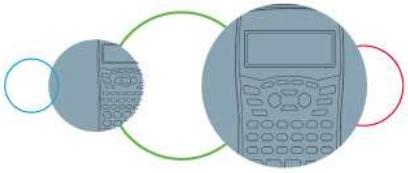
new surface area $= 2\pi\left(\frac{1}{2} \times r\right)^2 + 2\pi\left(\frac{1}{2} \times r\right)\left(\frac{1}{2} \times h\right)$

$= \frac{1}{4} \times 2\pi r^2 + \frac{1}{4} \times 2\pi rh$

$= \frac{1}{4}(2\pi r^2 + 2\pi rh)$

$= \frac{1}{4}(51.6\ cm^2)$

$= 12.9\ cm^2$



iv) radius $= \frac{3}{2} \times r$ height $= \frac{3}{2} \times h$

new surface area $= 2\pi\left(\frac{3}{2} \times r\right)^2 + 2\pi\left(\frac{3}{2} \times r\right)\left(\frac{3}{2} \times h\right)$

$= \frac{9}{4} \times 2\pi r^2 + \frac{9}{4} \times 2\pi rh$

$= \frac{9}{4}(2\pi r^2 + 2\pi rh)$

$= \frac{9}{4}(51.6\ cm^2)$

$= 116.1\ cm^2$

Exercise 3:

1. a) $Volume = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(2cm)^3$

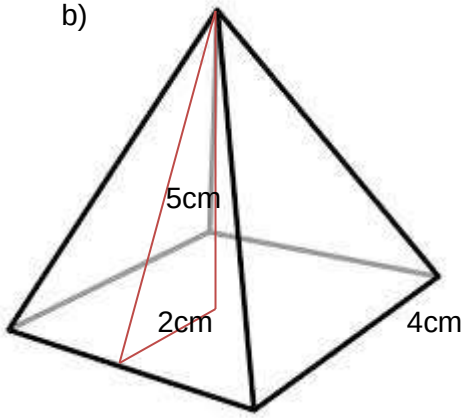
$= 33.51\ cm^3$

$Surface\ Area = 4\pi r^2$

$= 4\pi(2cm)^2$

$= 50.27\ cm^2$

b)



$r^2 = x^2 + y^2$

$r^2 = (4 \div 2)^2 + (5)^2$

$r = \sqrt{29} = 5.39\ cm$

$Volume = \frac{1}{3}(l^2) \times h$

$= \frac{1}{3}(4^2)(5)$

$= 26.67\ cm^3$

$Surface\ Area = (l^2) + \frac{1}{2}(4l)(s)$

$= (4)^2 + \frac{1}{2}(4)(4)(5.39)$

$= 59.12\ cm^2$

c) $Volume = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(\pi)(55)^2(70)$

$= 221\ 744.08\ mm^3$

$r^2 = x^2 + y^2$

$r^2 = (55)^2 + (70)^2$

$r = \sqrt{7\ 925} = 89.02\ mm$

$Surface\ Area = \pi r^2 + \pi rs$

$= \pi(55)^2 + \pi(55)(89.02)$

$= 24\ 884.87\ mm^2$

d) $Volume = \frac{4}{3}\pi r^3$

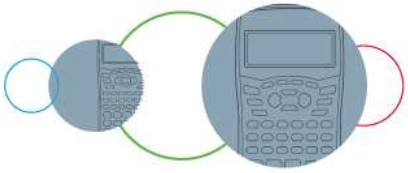
$= \frac{4}{3}\pi(16mm)^3$

$= 17\ 157.28\ mm^3$

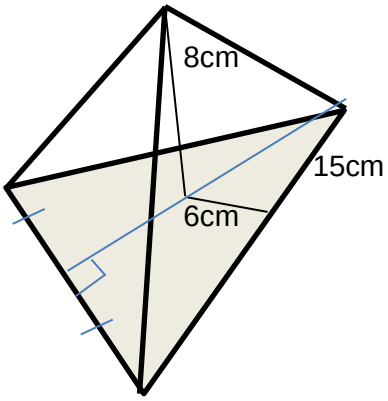
$Surface\ Area = 4\pi r^2$

$= 4\pi(16mm)^2$

$= 3\ 216.99\ mm^2$



e)



$r^2 = x^2 + y^2$

$h^2 = (15)^2 - (7.5)^2$

$h = \sqrt{168.75} = 12.99cm$

$Volume = \frac{1}{3}\left(\frac{1}{2}b \times h\right) \times height$

$= \frac{1}{3}\left(\frac{1}{2}(15)(12.99)\right) \times (8)$

$= 259.8\ cm^3$

$r^2 = x^2 + y^2$

$r^2 = (6)^2 + (8)^2$

$r = \sqrt{100} = 10$

$Surface\ Area = \left(\frac{1}{2}b \times h\right) + 3\left(\frac{1}{2}b \times h\right)$

$= \frac{1}{2}(15 \times 12.99) + 3\left(\frac{1}{2} \times 15 \times 10\right)$

$= 322.425cm^2$

f) $Volume = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3.4)^2(11)$

$= 133.16\ cm^3$

$r^2 = x^2 + y^2$

$r^2 = (3.4)^2 + (11)^2$

$r = \sqrt{132.56} = 11.51\ cm$

$Surface\ Area = \pi r^2 + \pi rs$

$= \pi(3.4)^2 + \pi(3.4)(11.51)$

$= 159.26\ cm^2$

g) Note: A right circular pyramid is a right cone

$Volume = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(14)^2(17)$

$= 3\,489.26mm^3$

$r^2 = x^2 + y^2$

$r^2 = (14)^2 + (17)^2$

$r = \sqrt{485} = 22.02$

$Surface\ Area = \pi r^2 + \pi rs$

$= \pi(14)^2 + \pi(14)(17)$

$= 1\,363.45mm^2$

h) $Volume = \frac{4}{3}\pi r^3$

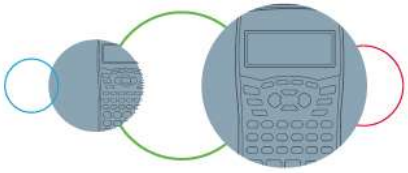
$= \frac{4}{3}\pi(59)^3$

$= 860\,289.54\ mm^3$

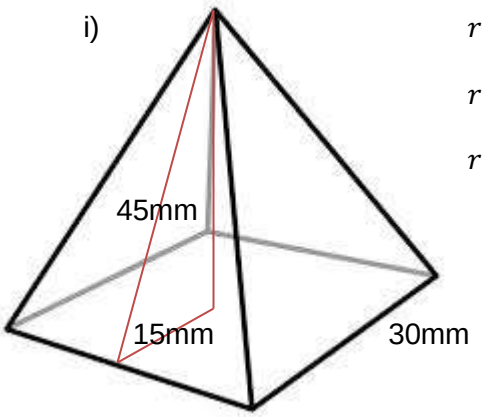
$Surface\ Area = 4\pi r^2$

$= 4\pi(59)^2$

$= 43\,743.54\ mm^2$



i)



$r^2 = x^2 + y^2$

$r^2 = (30 \div 2)^2 + (45)^2$

$r = \sqrt{2\,250} = 47.43mm$

$Volume = \frac{1}{3}(l^2) \times h$

$= \frac{1}{3}(30)^2(45)$

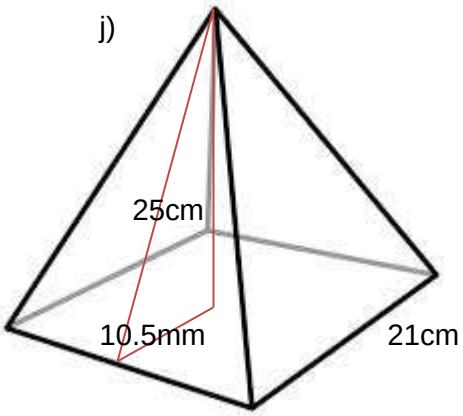
$= 13\,500\ mm^3$

$Surface\ Area = (l^2) + \frac{1}{2}(4l)(s)$

$= (30)^2 + \frac{1}{2}\big(4(30)\big)(47.43)$

$= 3\,745.8\ mm^2$

j)



$r^2 = x^2 + y^2$

$r^2 = (21 \div 2)^2 + (25)^2$

$r = \sqrt{735.25} = 27.12cm$

$Volume = \frac{1}{3}(l^2) \times h$

$= \frac{1}{3}(21)^2(25)$

$= 3\,675\ cm^3$

$Surface\ Area = (l^2) + \frac{1}{2}(4l)(s)$

$= (21)^2 + \frac{1}{2}(4)(21)(27.12)$

$= 1\,580.04\ cm^2$

2. a) $Volume = \frac{1}{3}\pi r^2 h$

$\therefore 8008\frac{1}{3}\pi\ cm^3 = \frac{1}{3}\pi(31cm)^2 h$

$\therefore h = \frac{8008\frac{1}{3}\pi}{320\frac{1}{3}\pi}$

$\therefore h = 8\frac{1}{3} = 25\ cm$

$r^2 = x^2 + y^2$

$r^2 = (31)^2 + (25)^2$

$r = \sqrt{1\,586} = 39.82\ cm$

$SA = \pi r^2 + \pi rs$

$SA = \pi(31)^2 + \pi(31)(39.82)$

$SA = 6\,897.12\ cm^2$

b) $Volume = \frac{4}{3}\pi r^3$

$\therefore 3\,305.97\ mm^3 = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{3305.97}{\frac{4}{3}}$

$\therefore r = \sqrt[3]{2479.4775} = 13.53mm$

$SA = 4\pi r^2$

$SA = 4\pi(13.53)^2$

$SA = 2\,300.41\ mm^2$

c) $Volume = \frac{1}{3}(l^2) \times h$

$\therefore 14.583333 = \frac{1}{3}(l^2) \times 7$

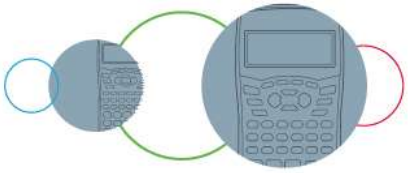
d) $Volume = \frac{1}{3}\pi r^2 h + \frac{4}{3}\pi r^3 \times \frac{1}{2}$

$\therefore 74\,666\frac{2}{3}\pi = \frac{1}{3}\pi(40)^2 h + \frac{2}{3}\pi(40)^3$

$\therefore 74\,666\frac{2}{3}\pi - 42\,666\frac{2}{3}\pi = 533\frac{1}{3}h$

$\therefore h = \frac{32\,000}{533\frac{1}{3}}$

$\therefore h = 60\ mm$



$\therefore l^2 = \frac{14.58333}{\frac{1}{3}\times 7}$

$\therefore l = \sqrt{6.249999} = 2.5\ cm$

$r^2 = x^2 + y^2$

$r^2 = (2.5 \div 2)^2 + (7)^2$

$r = \sqrt{50.5625} = 7.11\ cm$

$SA = l^2 + \frac{1}{2}(4l)(h)$

$SA = (2.5)^2 + 2(2.5)(7.11)$

$SA = 41.8\ cm^2$

$SA = 5l^2 + \frac{1}{2}(4 \times l)(h)$

$SA = 5(19.72)^2 + 2(19.72)^2$

$SA = 2\ 722.15\ cm^2$

$r^2 = x^2 + y^2$

$r^2 = (40)^2 + (60)^2$

$r = \sqrt{5\ 200} = 72.11\ mm$

$SA = \pi rs + \frac{1}{2}\times 4\pi r^2$

$SA = \pi(40)(72.11) + 2\pi(40)^2$

$SA = 19\ 114.71\ mm^2$

e) $Volume = l^3 + \frac{1}{3}(l^2) \times h$

$\therefore 518.689333 = l^3 + \frac{1}{3}(l)^2(l)$

$\therefore 518.689333 = \frac{4}{3}l^3$

$\therefore l^3 = 389.0169998$

$\therefore l = \sqrt[3]{389.0169998} = 19.72cm$

3. a) $Volume = vol\ of\ cylinder + vol\ of\ cone + vol\ of\ \frac{1}{2}sphere$

$= \pi r^2 h + \frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi(8)^2(8 \times 2) + \frac{1}{3}\pi(8)\left(\frac{3}{2}\times 8\right) + \frac{2}{3}\pi(8)^3$

$= 4\ 389.85\ cm^3$

$r^2 = x^2 + y^2$

$r^2 = (8)^2 + \left(\frac{3}{2}\times 8\right)^2$

$r = \sqrt{208} = 14.42\ cm$

$Surface\ Area = \pi rs + 2\pi rh + \frac{1}{2}(4\pi r^2)$

$= \pi(8)(14.42) + 2\pi(8)(2 \times 8) + 2\pi(8)^2$

$= 1\ 568.79\ cm^2$

b) $Surface\ area = 5l^2 + \frac{1}{2}(4l)(s)$

$\therefore 1\ 575 = 5x^2 + \frac{1}{2}(4)(x)(1.12x)$

$\therefore 1\ 575 = 7.24x^2$

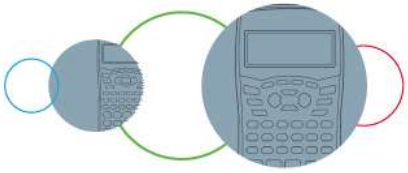
$\therefore x^2 = 217.5414\ldots$

$\therefore x = \sqrt{217.5414\ldots} = 14.75\ cm$

$r^2 = x^2 + y^2$

$r^2 = x^2 + \left(\frac{1}{2}x\right)^2$

$r = \sqrt{\frac{5}{4}x^2} = 1.12x$



$$Volume = l^3 + \frac{1}{3}(l^2)(h)$$
$$= (14.75)^3 + \frac{1}{3}(14.75)^2(14.75)$$
$$= 4\,278.73cm^3$$

c) $r_1^2 = x^2 + y^2$ $r_2^2 = x^2 + y^2$ $r_3^2 = x^2 + y^2$

$r_1^2 = (4.5)^2 + (3)^2$ $r_2^2 = (4.5)^2 + (6)^2$ $r_3^2 = (4.5)^2 + (9)^2$

$r_1 = \sqrt{29.25} = 5.41cm$ $r_2 = \sqrt{56.25} = 7.5cm$ $r_3 = \sqrt{101.25} = 10.06$

$$Surface\ Area = lb + 2lh + 2bh + \frac{1}{2}(4l)(s) + \frac{1}{2}(4l)(s) + \frac{1}{2}(4l)(s)$$
$$= (27)(9) + 2(27)(3) + 2(9)(3) + \frac{1}{2}\big(4(9)(5.41)\big) + \frac{1}{2}(4)(9)(7.5) + \frac{1}{2}(4)(9)(10.06)$$
$$= 459 + 97.38 + 135 + 181.08$$
$$= 872.46cm^2$$

$$Volume = lbh + \frac{1}{3}(l^2)(h) + \frac{1}{3}(l^2)(h) + \frac{1}{3}(l^2)(h)$$
$$= (27)(9)(3) + \frac{1}{3}(9^2)(3) + \frac{1}{3}(9)^2(6) + \frac{1}{3}(9^2)(9)$$
$$= 729 + 81 + 162 + 243$$
$$= 1\,215\ cm^3$$

4. a) Shaded Volume = Cylinder Volume – Volume of Sphere

$$= \pi r^2 h - \frac{4}{3}\pi r^3$$
$$= \pi(4)^2(12) - \frac{4}{3}\pi(4)^3$$
$$= 192\pi - 85\frac{1}{3}\pi$$
$$= 335.10\ cm^3$$

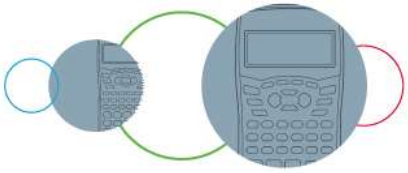
b) Shaded Volume = Volume of rectangular prism – volume of pyramid

$$= lbh - \frac{1}{3}(lb)(h)$$
$$= (15)(15)(20) - \frac{1}{3}(15)(15)(20)$$
$$= 3\,000cm^3$$

c) $r^2 = x^2 + y^2$

$y^2 = (8)^2 - (4)^2$

$y = \sqrt{48} = 6.93\ mm$



Shaded Volume = Volume of cylinder – volume of equilateral pyramid

$= \pi r^2 h - \frac{1}{3}\left(\frac{1}{2} b \times y\right)(h)$

$= \pi(5)^2(16) - \frac{1}{3}\left(\frac{1}{2} \times 8 \times 6.93\right)(16)$

$= 1\,256.637 - 148.48$

$= 1\,108.16\, mm^3$

d) Shaded Volume = Volume of rectangular prism – volume of sphere

$= lbh - \frac{4}{3}\pi r^3$

$= (44)(79)(40) - \frac{4}{3}\,\pi(20)^3$

$= 139\,040 - 33\,510.32$

$= 105\,529.68\, mm^3$

e) Shaded Volume = Volume of Cylinder – (2x volume of ½ spheres)

$= \pi r^2 h - 2\left(\frac{1}{2} \times \frac{4}{3}\pi r^3\right)$

$= \pi(90)^2(188) - \frac{4}{3}(\pi \times 90^3)$

$= 4\,784\,017.293 - 3\,053\,628.059$

$= 1\,730\,389.234\, mm^3$