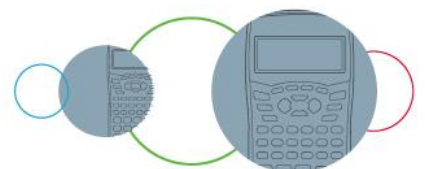


SHARP

Worksheet 10 Memorandum: The Fundamental Counting Principle

Grade 12 Mathematics

1. a) permutation b) combination c) permutation
d) permutation e) combination
2. a) 2 pants x 3 shirts x 4 ties
= 24 different outfits.
- b) $P(\text{black pants and white shirt and red tie}) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$
 $= \frac{1}{24}$
- c) $P(\text{black shirt}) = \frac{1}{3}$
- d) 4!
= 24 different arrangements
- e) $P(\text{black, blue, grey and red}) = \frac{1}{24}$
- f) $\frac{(4+4-1)!}{(4-1)! \times 4!}$
= 35 different tie arrangements
3. a) $\frac{(40+4-1)!}{(40-1)! \times 4!}$ $n = 40$ and $r = 4$
= 123 410 possible combinations
- b) $P(\text{all marbles evenly distributed}) = \frac{1}{123\,410}$ (because it is one possible combination out of 123 410 combinations)



4. PRESTIDIGITATION

a) $16!$

$$= 2.09 \times 10^{13}$$

b) ${}_{16}P_6 = \frac{16!}{6! \times (16-6)!}$

$$= 5\,765\,760$$

c) 3 T's; 4 l's repeated.

$$\therefore 5\,765\,760 \div (3! \times 4!)$$

$$= 40\,040 \quad \text{words with no repeats}$$

d) ${}_{16}C_5$

$$= 4\,368$$

e) 1 possible combination out of 4 368 possible 6 letter words.

$$\therefore P(\text{word} = \text{tastier}) = \frac{1}{4\,368}$$

5. a) $7!$

$$= 5\,040 \text{ possible seating arrangements}$$

b) $12!$

$$= 479\,001\,600 \text{ possible seating arrangements}$$

c) ${}_{12}C_4$

$$= 495 \text{ possible groups}$$

d) ${}_{12}P_3$

$$= 1\,320 \text{ possible ways for the prizes to be handed out.}$$

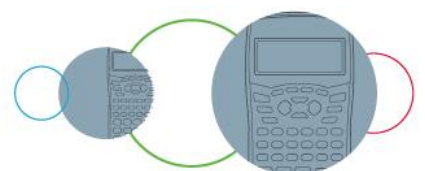
e) ${}_{12}C_3$

$$= 220$$

6. PERPENDICULAR

a) $13!$

$$= 6\,227\,020\,800 \text{ possible arrangements}$$



b) 2 Ps, 2 E's, 2 R's;

$$\therefore 2! \times 2! \times 2!$$

= 8 possible repeats of each arrangement from above.

c) unique ways: $6\,227\,020\,800 \div 8 = 778\,377\,600$.

d) ${}^{13}C_5$

$$= 1\,287$$

e) $\frac{(13+5-1)!}{(13-1)! \times 4!}$

$$= 30\,940$$

7. a) $\frac{(150+3-1)!}{(150-1)! \times 3!}$ $n = 150$ voters $r = 3$ political parties.

On the calculator this will give you an error. So you need to simplify first:

$$= \frac{152!}{149! \times 3!}$$

$$= \frac{152 \times 151 \times 150 \times 149!}{149! \times 3!}$$

$$= \frac{152 \times 151 \times 150}{3!}$$

= 573 800 possible ways for the voters to vote.

b) i) $\frac{(10+3-1)!}{(10-1)! \times 3!}$

= 220 different ways for the voters to vote.

P (all vote ANC) = 1 way out of 220

$$= \frac{1}{220}$$

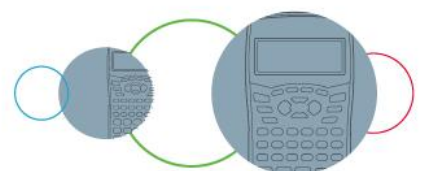
ii) P (3 vote for EFF) = $\frac{1}{220}$

iii) P (7 vote for DA) = $\frac{1}{220}$

8. a) $4^{15} = 1\,073\,741\,824$

b) P (guess all 20 correctly) = $\frac{1}{1\,073\,741\,824}$

(one possible permutation out of more than 1 billion options).



9. a) $5 + 3 + 4 + 7 + 6 = 25$ plants
 $25! = 1.551121 \times 10^{25}$ possible plant combinations
- b) $5! = 120$
- c) $3 + 4 + 7 = 14$ plants
 $14! = 8.717829 \times 10^{10}$
- d) ${}_{25}C_5 = 53\,130$
- e) ${}_{25}P_5 = 6\,375\,600$
10. a) $5 \times 7 \times 3 = 105$ ways to pick the three books
- b) $5! = 120$ arrangements of the David Baldacci Books
 $7! = 5\,040$ arrangements of the Danielle Steel books
 $3! = 6$ arrangements of the John Grisham books
 And $3! = 6$ ways for the groups of books to be arranged.
 $\therefore 120 + 5\,040 + 6 + 6 = 5\,172$ ways to arrange the books according to author
- c) ${}_{15}C_5 = 3\,003$ different ways to choose 5 books.
- d) $P(\text{choose all 5 David Baldacci books}) = \frac{1}{3\,003}$
11. a) ${}_{16}C_2 = 120$
- b) ${}_{16}C_4 = 1\,820$
- c) ${}_{16}C_8 = 12\,870$

