Worksheet 10 Memorandum: The Fundamental Counting Principle

Grade 12 Mathematics

1. a) permutation b) combination c) permutation

   d) permutation e) combination

2. a) 2 pants x 3 shirts x 4 ties

   = 24 different outfits.

   b) P (black pants and white shirt and red tie) = $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$

   $= \frac{1}{24}$

   c) P(black shirt) $= \frac{1}{3}$

   d) 4!

   = 24 different arrangements

   e) P(black, blue, grey and red) = $\frac{1}{24}$

   f) $\frac{(4+4-1)!}{(4-1)! \times 4!}$

   = 35 different tie arrangements

3. a) $\frac{(40+4-1)!}{(40-1)! \times 4!}$ n = 40 and r = 4

   = 123 410 possible combinations

   b) P (all marbles evenly distributed) = $\frac{1}{123\ 410}$ (because it is one possible combination out of 123 410 combinations)

4. P R E S T I D I G I T A T I O N

a) 16!

$= 2.09 \times 10^{13}$

b) $16P6 = \frac{16!}{6! \times (16-6)!}$

= 5 765 760

c) 3 T's; 4 I's repeated.

∴ 5 765 760 ÷ (3! x 4!)

= 40 040 words with no repeats

d) 16C5

= 4 368

e) 1 possible combination out of 4 368 possible 6 letter words.

∴ P (word = tastier) = $\frac{1}{4\,368}$

5. a) 7!

= 5 040 possible seating arrangments

b) 12!

= 479 001 600 possible seating arrangments

c) 12C4

= 495 possible groups

d) 12P3

= 1 320 possible ways for the prizes to be handed out.

e) 12C3

= 220

6. P E R P E N D I C U L A R

a) 13!

= 6 227 020 800 possible arrangements

b) 2 Ps, 2 E's, 2 R's;

∴ 2! x 2! x 2!

= 8 possible repeats of each arrangement from above.

c) unique ways: 6 227 020 800 ÷ 8 = 778 377 600.

d) 13C5

= 1 287

e) $\frac{(13+5-1)!}{(13-1)!\times 4!}$

= 30 940

7. a) $\frac{(150+3-1)!}{(150-1)!\times 3!}$ n = 150 voters r = 3 political parties.

On the calculator this will give you an error. So you need to simplify first:

$= \frac{152!}{149!\times 3!}$

$= \frac{152\times 151\times 150\times 149!}{149!\times 3!}$

$= \frac{152\times 151\times 150}{3!}$

= 573 800 possible ways for the voters to vote.

b) i) $\frac{(10+3-1)!}{(10-1)!\times 3!}$

= 220 different ways for the voters to vote.

P (all vote ANC) = 1 way out of 220

$= \frac{1}{220}$

ii) P (3 vote for EFF) = $\frac{1}{220}$

iii) P (7 vote for DA) = $\frac{1}{220}$

8. a) $4^{15} = 1\ 073\ 741\ 824$

b) P (guess all 20 correctly) = $\frac{1}{1\ 073\ 741\ 824}$ (one possible permutation out of more than 1 billion options).

9. a) 5 + 3 + 4 + 7 + 6 = 25 plants

25! = 1.551121 x 10^{25} possible plant combinations

b) 5! = 120

c) 3 + 4 + 7 = 14 plants

14! = 8.717829 x 10^{10}

d) 25C5 = 53 130

e) 25P5 = 6 375 600

10. a) 5 x 7 x 3 = 105 ways to pick the three books

b) 5! = 120 arrangements of the David Baldacci Books

7! = 5 040 arrangements of the Danielle Steel books

3! = 6 arrangements of the John Grisham books

And 3! = 6 ways for the groups of books to be arranged.

∴ 120 + 5 040 + 6 + 6 = 5 172 ways to arrange the books according to author

c) 15C5 = 3 003 different ways to choose 5 books.

d) P (choose all 5 David Baldacci books) = $\frac{1}{3\,003}$

11. a) 16C2 = 120

b) 16C4 = 1 820

c) 16C8 = 12 870