Worksheet 21 Memorandum: Term 3 Revision

Grade 10 Mathematics

1. a) In ΔAFG and ΔBEG

1.	$A\hat{G}F = E\hat{G}B$	vertically opposite angles
2.	$A\hat{F}G = G\hat{E}B$	Alternating angles, AF || EB
3.	$G\hat{A}F = G\hat{B}E$	Alternating angles, AF || EB or third angle in triangle.

∴ ΔAFG ||| ΔBEG — all 3 angles equal.

b) AEBF is a trapezium, one pair of sides parallel (AF || EB).

c) If FB = EB then

In ΔEBG and ΔBFG

1. BG is common

2. EB = FB given

3. $E\hat{G}B = F\hat{G}B$ — EG ⊥ AB.

∴ ΔEBG ≡ ΔBFG — (SSA)

This means that EG = GF — ΔEBG ≡ ΔBFG proved above.

And then

In ΔAEG and ΔAFG

1. EG = GF — proved above ΔEBG ≡ ΔBFG

2. AG is common

3. $A\hat{G}E = A\hat{G}F$ — EG ⊥ AB

∴ ΔEAG ≡ ΔFAG

This means the AEBF is now a kite, adjacent sides are equal and diagonals intersect at right angles.

2. a) In ΔACF and ΔAIC

1. AC is common

2. $F\hat{A}C = A\hat{C}I$ Alternating angles GF || HK

3. $F\hat{C}A = C\hat{A}I$ Alternating angles FE || AB

∴ ΔACF ≡ ΔCAI (ASA)

b) AFCI is a parallelogram. Both pairs opposite sides equal and parallel.

c) In ΔCFK and ΔACI

1. IA = CF ΔACF ≡ ΔCAI

2. $I\hat{A}C = A\hat{C}F$ ΔACF ≡ ΔCAI

$A\hat{C}F = C\hat{F}K$ alternating angles AJ || FK

$\therefore I\hat{A}C = C\hat{F}K$ both equal to $A\hat{C}F$

3. $A\hat{I}C = K\hat{C}F$ corresponding angles, AB || EF.

∴ ΔCAI ≡ ΔKFC (ASA).

d) AFKI is a trapezium because it has one set of parallel sides.

3. a)

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{3-0}{4-(-2)}$

$m_{AB} = \frac{3}{6}$

$m_{AB} = \frac{1}{2}$

$\therefore m_{DC} \times m_{AB} = -1$

$\therefore m_{DC} = -1 \div \frac{1}{2}$

$\therefore m_{DC} = -2$

$\therefore y = -2x + 2$

b)

$y_{AB} = \frac{1}{2}x + c$ Subs in B (-2; 0)

$0 = \frac{1}{2}(-2) + c$

$c = 1$

$y_{AB} = \frac{1}{2}x + 1$ …1

$y_{DC} = -2x + 2$ … 2

Subs 1 into 2

$\frac{1}{2}x + 1 = -2x + 2$

$2\frac{1}{2}x = 1$

$x = \frac{2}{5}$

$\therefore y = -2\left(\frac{2}{5}\right) + 2 = 1\frac{1}{5}$ E $\left(\frac{2}{5}\,;1\frac{1}{5}\right)$

c) $M_{AB} = \left(\frac{x_1+x_2}{2}\,;\frac{y_1+y_2}{2}\right)$

$M_{AB} = \left(\frac{-2+4}{2};\frac{0+3}{2}\right)$

$M_{AB} = \left(1; 1\frac{1}{2}\right)$

∴ E is not the midpoint of AB.

d) $m_{AC} = \frac{y_2-y_1}{x_2-x_1}$ Use A and F coordinates.

$m_{AC} = \frac{3-0}{4-2.8}$

$m_{AC} = 2\frac{1}{2}$

$y = 2\frac{1}{2}x + c$ Subs in A (4; 3)

$3 = 2\frac{1}{2}(4) + c$

$c = -7$

$y_{AC} = 2\frac{1}{2}x - 7$

e) $y_{AC} = 2\frac{1}{2}x - 7$ …1

$y_{DC} = -2x + 2$ …2

Subs 1 into 2

$2\frac{1}{2}x - 7 = -2x + 2$

$4\frac{1}{2}x = 9$

$x = 2$

$\therefore y = 2\frac{1}{2}(2) - 7 = -2$

$\therefore C\ (2;\ -2)$

f) $m_{EF} = \frac{y_2-y_1}{x_2-x_1}$

$m_{EF} = \frac{1\frac{1}{5}-0}{\frac{2}{5}-2.8}$

$m_{EF} = -\frac{1}{2}$

$m_{BC} = \frac{y_2-y_1}{x_2-x_1}$

$m_{BC} = \frac{0-(-2)}{-2-2}$

$m_{BC} = -\frac{1}{2}$

∴ EF || BC because $m_{BC} = m_{EF}$

g) $m_{AC} \times m_{BC} = 2\frac{1}{2} \times -\frac{1}{2} = -1\frac{1}{4}$

∴ not right-angled.

$d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d_{AB} = \sqrt{\left(4-(-2)\right)^2 + (3-0)^2}$

$d_{AB} = 3\sqrt{5} = 6.71$

$d_{BC} = \sqrt{(-2-2)^2 + \left(0-(-2)\right)^2}$

$d_{BC} = 2\sqrt{5} = 4.47$

$d_{AC} = \sqrt{(4-2)^2 + \left(3-(-2)\right)^2}$

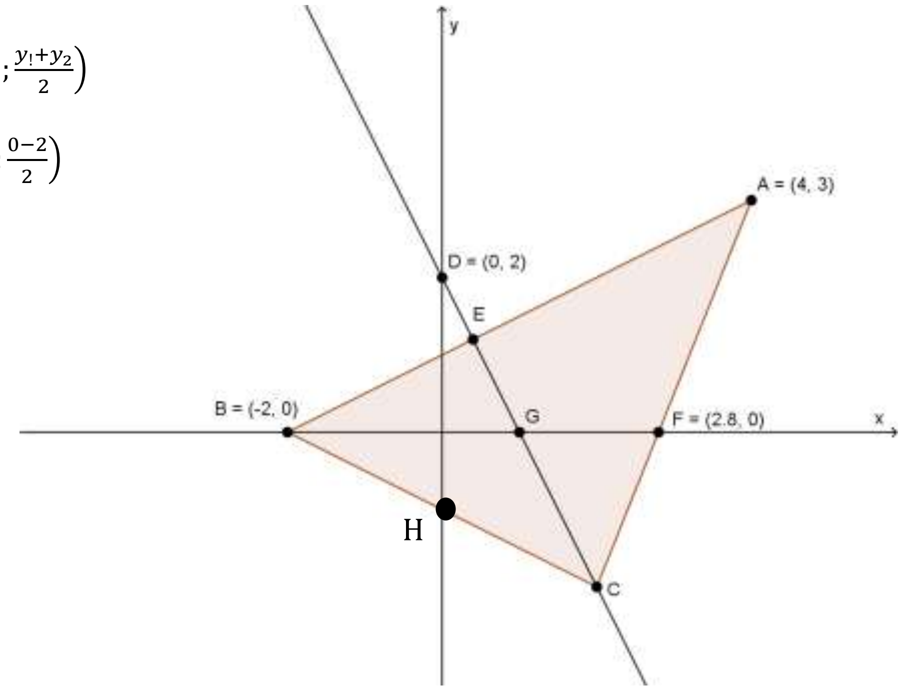
$d_{AC} = \sqrt{29} = 5.39$

∴ ΔABC is a scalene triangle because none of the sides are equal.

h) $M_{BC} = \left(\frac{x_1+x_2}{2}; \frac{y_!+y_2}{2}\right)$

$M_{BC} = \left(\frac{-2+2}{2}; \frac{0-2}{2}\right)$

H = (0; -1)



i) $m_{AH} = \frac{y_2-y_1}{x_2-x_1}$

$m_{AH} = \frac{3-(-1)}{4-0}$

$m_{AH} = 1$

$y_{AH} = 1x + c$ Subs in A (4; 3)

$3 = 1(4) + c$

$c = -1$

$y_{AH} = x - 1$

G is the x-intercept on DC

$\therefore 0 = -2x + 2$

$-2 = -2x$

$x = 1$ ∴ G = (1; 0)

To find whether G is on the line AH substitute in y = 0 and see whether x = 1:

$0 = x - 1$

$x = 1$ ∴ G is on the line of AH.

4. a) $M_{NO} = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$(0; 0) = \left(\frac{4+x}{2}; \frac{-1+y}{2}\right)$

$0 = 4 + x$ $0 = -1 + y$

$x = -4$ $y = 1$ ∴ N = (-4; 1)

b) $M_{MN} = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$P = \left(\frac{1+(-4)}{2}; \frac{4+1}{2}\right)$

$P = \left(-1\frac{1}{2}; 2\frac{1}{2}\right)$

$M_{MO} = \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$

$Q = \left(\frac{1+4}{2}; \frac{4+(-1)}{2}\right)$

$Q = \left(2\frac{1}{2}; 1\frac{1}{2}\right)$

c) $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{PQ} = \frac{2\frac{1}{2} - 1\frac{1}{2}}{-1\frac{1}{2} - 2\frac{1}{2}}$

$m_{PQ} = -\frac{1}{4}$

$y_{PQ} = -\frac{1}{4}x + c$ Subs P $\left(-1\frac{1}{2}; 2\frac{1}{2}\right)$

$2\frac{1}{2} = -\frac{1}{4}\left(-1\frac{1}{2}\right) + c$

$c = 2\frac{1}{8}$

$y_{PQ} = -\frac{1}{4}x + 2\frac{1}{8}$ ...1

$m_{MR} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{MR} = \frac{4-0}{1-0}$

$m_{MR} = 4$

$y_{MR} = 4x + c$ Subs R (0; 0)

$0 = 4(0) + c$

$c = 0$

$y_{MR} = 4x$ ...2

Subs 1 into 2:

$-\frac{1}{4}x + 2\frac{1}{8} = 4x$

$4\frac{1}{4}x = 2\frac{1}{8}$

$x = \frac{1}{2}$

$\therefore y = -\frac{1}{4}\left(\frac{1}{2}\right) + 2\frac{1}{8} = 2$ $\therefore S = \left(\frac{1}{2}; 2\right)$

$M_{MR} = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$M_{MR} = \left(\frac{1+0}{2}; \frac{4+0}{2}\right)$

$M_{MR} = \left(\frac{1}{2}; 2\right)$

$\therefore$ yes, S is the midpoint of MR.

d) $d_{NO} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{NO} = \sqrt{(-4-4)^2 + \left(1 - (-1)\right)^2}$

$d_{NO} = 2\sqrt{17}$

$d_{MR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{MR} = \sqrt{(1-0)^2 + (4-0)^2}$

$d_{MR} = \sqrt{17}$

$\therefore Area = \frac{1}{2}b \times \perp h$

$\therefore Area = \frac{1}{2} \times 2\sqrt{17} \times \sqrt{17}$

$\therefore Area = 17\ units^2$

e) **Method 1 of 2:**

$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{MN} = \frac{4-1}{1-(-4)}$

$m_{MN} = \frac{3}{5}$

$m_{MO} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{MO} = \frac{4-(-1)}{1-4}$

$m_{MO} = -1\frac{2}{3}$

$m_{MN} \times m_{MO} = \frac{3}{5} \times -1\frac{2}{3} = -1$

∴ MN ⊥ MO

Method 2 of 2:

$d_{MN} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{MN} = \sqrt{\left(1-(-4)\right)^2 + (4-1)^2}$

$d_{MN} = \sqrt{34}$

$d_{MO} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{MO} = \sqrt{(1-4)^2 + \left(4-(-1)\right)^2}$

$d_{MO} = \sqrt{34}$

And $d_{NO} = 2\sqrt{17}$

$h^2 = a^2 + b^2$ (Theorem of Pythag to prove right angles)

$\left(2\sqrt{17}\right)^2 = \left(\sqrt{34}\right)^2 + \left(\sqrt{34}\right)^2$

68 = 34 + 34

68 = 68 ∴ ΔMNO is a right angled triangle according to Pythag and MN ⊥ MO

f) The theorem is that a line joining two of the midpoints of a triangle will be parallel to and half the length of the third side.

We have already shown that PQ || NO. Now we need to prove that PQ = ½ NO

$d_{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{PQ} = \sqrt{\left(2\frac{1}{2} - \left(-1\frac{1}{2}\right)\right)^2 + \left(1\frac{1}{2} - 2\frac{1}{2}\right)^2}$

$d_{PQ} = \sqrt{17}$

$d_{NO} = 2\sqrt{17}$

$\therefore PQ = \frac{1}{2}NO$

g) $m_{PR} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{MO} = -1\frac{2}{3}$

$m_{PR} = \frac{2\frac{1}{2} - 0}{-1\frac{1}{2} - 0}$

$m_{PR} = -1\frac{2}{3}$ $m_{PR} = m_{MO}$ PR || MO

h) $m_{QR} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{MP} = \frac{3}{5}$

$m_{QR} = \frac{1\frac{1}{2} - 0}{2\frac{1}{2} - 0}$

$m_{QR} = \frac{3}{5}$ QR || MP

∴ Opposite pairs of sides parallel.

$d_{MP} = \sqrt{\left(1 - \left(-1\frac{1}{2}\right)\right)^2 + \left(4 - 2\frac{1}{2}\right)^2}$ $d_{MQ} = \sqrt{\left(1 - 2\frac{1}{2}\right)^2 + \left(4 - 1\frac{1}{2}\right)^2}$

$d_{MP} = \frac{1}{2}\sqrt{34}$ $d_{MQ} = \frac{1}{2}\sqrt{34}$

$d_{PR} = \sqrt{\left(-1\frac{1}{2} - 0\right)^2 + \left(2\frac{1}{2} - 0\right)^2}$ $d_{QR} = \sqrt{\left(2\frac{1}{2} - 0\right)^2 + \left(1\frac{1}{2} - 0\right)^2}$

$d_{PR} = \frac{1}{2}\sqrt{34}$ $d_{QR} = \frac{1}{2}\sqrt{34}$

∴ All four sides equal.

MPRQ is a square – opposite sides parallel and all four sides equal.

5.

Final Amount	Principal Amount	Interest	Number of Years
R500 000	372 300.82	4.9%	7 years
120 862	67 900	12%	6.5 years
137 700	47 520	12.65%	15 years
34 560	19 740	7%	10.73 years
42 770	18 200	15%	9 years

Working out on the following page.

i.

A = 500 000 $A = P(1 + in)$

P = ? $500\,000 = P\,(1 + 0.049 \times 7)$

$i = 4.9\% = 0.049$ $P = \frac{500\,000}{1.343}$

n = 7 years $P = 372\,300.82$

ii.

A = ? $A = P(1 + in)$

P = 67 900 $A = 67\,900(1 + 0.12 \times 6.5)$

$i = 12\% = 0.12$ $A = 120\,862$

n = 6.5 years

iii.

A = 137 700 $A = P(1 + in)$

P = 47 520 $137\,700 = 47\,520(1 + i \times 15)$

$i = ?$ $2\frac{79}{88} = 1 + i \times 15$

n = 15 years $1\frac{79}{88} = i \times 15$

$i = 0.1265 = 12.65\%$

iv.

A = 34 560 $A = P(1 + in)$

P = 19 740 $34\,560 = 19\,740(1 + 0.07 \times n)$

$i = 7\% = 0.07$ $1\frac{247}{329} = 1 + 0.07n$

n = ? $\frac{247}{329} = 0.07\,n$

$n = 10.73\ years$

Or n = 10 years and 9 months

v.

A = ? $A = P(1 + in)$

P = 18 200 $A = 18\,200\,(1 + 0.15 \times 9)$

$i = 15\% = 0.15$ $A = 42\,770$

n = 9 years

6. .

Final Amount	Principal Amount	Interest	Number of Years
803 813.72	142 720	2.5%	70
72 000	24 466.51	4.6%	24
36 100	22 680	2.95%	16
948 480	270 160	28.55%	5
232 735.27	203 280	7%	2

i.

A = ? $A = P(1 + i)^n$

P = 142 720 $A = 142\,720(1 + 0.025)^{70}$

$i = 2.5\% = 0.025$ $A = 803\,813.72$

n = 70 years

ii.

A = 72 000 $A = P(1 + i)^n$

P = ? $72\,000 = P(1 + 0.046)^{24}$

$i = 4.6\% = 0.046$ $P = \frac{72\,000}{(1+0.046)^{24}}$

n = 24 $P = 24\,466.51$

iii.

A = 36 100

P = 22 680

$i = ?$

n = 16 years

$A = P(1+i)^n$

$36\,100 = 22680(1+i)^{16}$

$1\frac{671}{1134} = (1+i)^{16}$

$\sqrt[16]{1\frac{671}{1134}} = 1+i$

$1.0295 = 1+i$

$i = 0.0295 = 2.95\%$

iv.

A = 948 480

P = 270 160

$i = ?$

n = 5 years

$A = P(1+i)^n$

$948\,480 = 270\,160(1+i)^5$

$\frac{11856}{3377} = (1+i)^5$

$\sqrt[5]{\frac{11856}{3377}} = 1+i$

$1.2855 = 1+i$

$i = 0.2855 = 28.55\%$

v.

A = ?

P = 203 280

$i = 7\% = 0.07$

n = 2 years

$A = P(1+i)^n$

$A = 203\,280(1+0.07)^2$

$A = 232\,735.27$

7.

Fridge A = LG
R12 499.00
Deposit = 10%
Contract Term
= 30 months
Contract Total
= R19 502.66
House and Home

Fridge B = Defy
R11 299.00
Deposit = 8%
Contract Term
= 30 months
Contract Total
= R17 962.80
OK Furniture

a) Fridge A Deposit = R12 499 x 10% = R 1 249.90

Fridge B Deposit = R 11 299 x 8% = R903.92

b) Fridge A = (R19 502.66 – R 1 249,90) ÷ 30 = R608.43

Fridge B = (R17 962.80 – R903.92) ÷ 30 = R568.63

c) House and Home interest: A = R19 502.66 – R1 249.90 = R18 252.76

P = R11 249.10 $A = P(1 + in)$

$i = ?$ $18\ 252.76 = 11\ 249.10(1 + i \times 2.5)$

n = 30 months ÷12 = 2.5 $1.622597363 = 1 + i \times 2.5$

$0.622597363 = i \times 2.5$

$i = 0.249 = 24.9\%$

OK Furniture interest: A = R17 962.80 – R903.92 = R17 058.88

P = R10 395.08 $A = P(1 + in)$

$i = ?$ $17\ 058.88 = 10\ 395.08\ (1 + i \times 2.5)$

n = 30 months ÷ 12 = 2.5 $1.641053268 = 1 + i \times 2.5$

$0.641053268 = i \times 2.5$

$i = 0.2564 = 25.64\%$

∴ House and Home charges a lower interest rate.

d) This is an opinion question – either fridge will do, as long as they have a valid reason for choosing that fridge.

8. a) A = 249 600 $A = P\ (1 + i)^n$

P = 96 000 $249\ 600 = 96\ 000\ (1 + i)^{15}$

$i = ?$ $2\frac{3}{5} = (1 + i)^{15}$

n = 15 years $\sqrt[15]{2\frac{3}{5}} = 1 + i$

$1.065773432 = 1 + i$

$i = 0.0658 = 6.58\%$

b) A = ? $A = P\ (1 + i)^n$

P = 96 000 $A = 96\ 000(1 + 0.13)^{30}$

$i = 13\% = 0.13$ $A = 3\ 755\ 126\ people$ (Remember to round off because

n = 30 years. You cannot get 0.2 of a person).

9. R1 = 0.0862 CHF (Swiss Franc)

1 GBP = R17.60 (Great British Pound)

a) R10 ÷ R17.60 = 0.57 GBP

b) R6000 x 0.0862 = 517.20 CHF

c) 312 CHF ÷ 0.0862 = R3 619.49

d) R6000 – R3 619.49 = R2 380.51

R2 380.51 ÷ 17.60 = 135.26 GBP

e) 135.26 – 120 = 15.26 GBP

15.26 x 17.60 = R268.58

10. 1 048, 1 244, 1 370, 1400, 1 500, 1 550, 1 650, 1 780, 1 780, 1800, 1 900, 2 000, 2 680, 3 685

a) The day she ate 3 685 calories (Or the third day)

b) Range = 3 685 – 1 048 = 2 637

c) $\bar{x} = \frac{sum\ of\ all\ observations}{number\ of\ observations}$

$\bar{x} = \frac{1\,048+1\,244+1\,370+1\,400+1\,500+1\,550+1\,650+1\,780+1\,780+1\,800+1\,900+2\,000+2\,680+3\,685}{14}$

$\bar{x} = \frac{25\,387}{14}$

$\bar{x} = 1\,813.36$

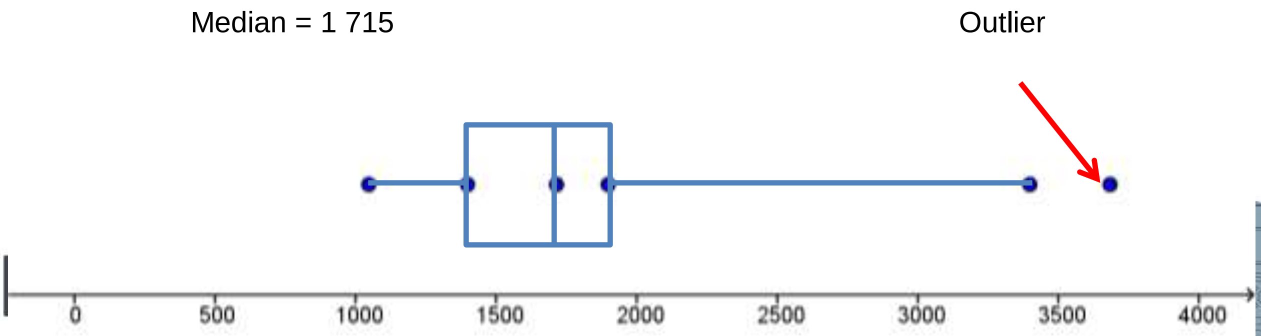
d) Mode = 1 780

Median = $\frac{1650+1780}{2} = \frac{3\,430}{2} = 1\,715$

e) Min = 1 048 Max = 3 685

Q1 = 1 400 Q3 = 1 900

Median = 1 715



f) Wanda eats less than 1 900 calories 75% of the time, and then splurges and eats over 2 000 calories for 25% of the time. There is one outlier for the data = 3 685.

g) 60^{th} percentile = $\frac{6}{10}(14+1) = 9th\ position$

∴ 60^{th} percentile = 1 780

11. a) $\cot\gamma = \frac{BC}{AB}$

$BC = AB\cot\gamma$

$BC = 6 \times \frac{1}{\tan 37.78°}$

BC = 7.74

b) $h^2 = a^2 + b^2$

$AF^2 = (2.62)^2 + (3.38)^2$

$AF = \sqrt{18.2888}$

AF = 4.28

c) $\tan F = \frac{AE}{EF}$

$\tan F = \frac{2.62}{3.38}$

$A\hat{F}E = 37.38°$

d) ED = BC = 7.74 (EBCD is a rectangle)

$h^2 = a^2 + b^2$

$AD^2 = (2.62)^2 + (7.74)^2$

$AD = \sqrt{66.772}$

$AD = 8.17$

e) $\tan D = \frac{AE}{ED}$

$\tan D = \frac{2.62}{7.74}$

$A\hat{D}E = 18.7°$

f) $D\hat{A}F$ = 180° - 18.7° - 90° = 71.3°

g) BE = 6 – 2.62 = 3.38 = DC EBCD is a rectangle.

FD = 7.74 – 3.38 = 4.36

$h^2 = a^2 + b^2$

$FC^2 = (3.38)^2 + (4.36)^2$

$FC = \sqrt{30.434}$

$FC = 5.52$

12. a) $\sin C = \frac{AB}{AC}$

$\sin C = \frac{61.58}{67.25}$

$A\hat{C}B = 66.3°$

b) $h^2 = a^2 + b^2$

$(67.25)^2 = (61.58)^2 + CB^2$

$CB = \sqrt{730.4661}$

$CB = 27.03$

c) $D\hat{C}B = 66{,}3° - 20.92 = 45.38°$

$\tan C = \frac{DB}{CB}$

$DB = CB \tan C$

$DB = 27.03 \tan 45.38°$

$DB = 27.39$

d) $\tan E = \frac{AB}{BE}$

$\tan E = \frac{61.58}{27}$

$B\hat{E}A = 66.32°$

$\therefore F\hat{E}B = 66.32° - 8.38° = 57.94°$

$\tan E = \frac{FB}{EB}$

$FB = EB \tan E$

$FB = 27 \tan 57.94°$

$FB = 43.11$

e) FD = FB – BD

FD = 43.11 – 27.39

FD = 15.72

AF = AB – FB

AF = 61.58 – 43.11

AF = 18.47

f) $Area = \frac{1}{2}\, b \times h$

$Area = \frac{1}{2}\,(27.03 + 27) \times 61.58$

$Area = 1\,663.58$

g) $AE^2 = (61.58)^2 + (27)^2$

$AE = \sqrt{4\,521.0964}$

$AE = 67.24$

$\therefore$ AE $\approx$ AC and thus ΔACE is an isosceles triangle.