SHARP

Worksheet 18 Memorandum: Measurement

Grade 10 Mathematics

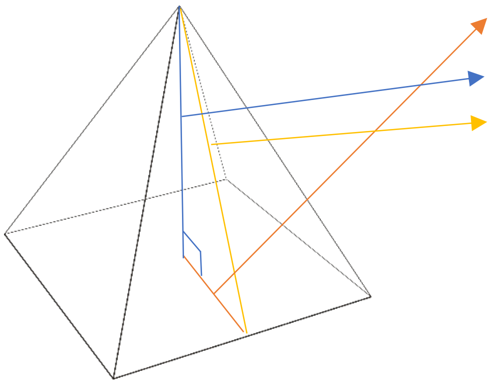
1. a) sphere Volume = $\frac{4}{3}\pi r^3$ Surface Area = $4\pi r^2$

b) pyramid with a square base Volume = $\frac{1}{3}l^2 h$

Surface Area = $area\ of\ square + 4\ \times area\ of\ triangles$

$= l^2 + 4 \times \left(\frac{1}{2}l \times \perp height\right)$

To find the perpendicular height – use Pythagoras:



this is ½ length

this is the height of the pyramid

this is the perpendicular height

$\perp height = \sqrt{\left(\frac{1}{2}l\right)^2 + h^2}$

c) cone Volume = $\pi r^2 \frac{h}{3}$ Surface area = $\pi r^2 + r \times \sqrt{h^2 + r^2}$

d) cylinder Volume = $\pi r^2 h$ Surface area = $2\pi r^2 + 2\pi rh$

e) cube Volume = l^3 Surface area = $6l^2$

f) rectangular prism Volume = lbh Surface area = $2bh + 2bl + 2lh$

g) pyramid with an equilateral triangle base

Volume = $\frac{1}{3}(area\ of\ base) \times height$

$= \frac{1}{3}\left(\frac{1}{2}\ length\ \times perpendicular\ height\right) \times height\ of\ pyramid$

$= \frac{1}{6}\ length\ \times \perp height\ \times height\ of\ pyramid$

Surface area = $area\ of\ base + 3\ \times area\ of\ triangles$

$= \left(\frac{1}{2}\ l\ \times \perp height\right) + 3\ \left(\frac{1}{2}\ l\ \times \perp slant\ height\right)$

2. a) i) Volume = $\frac{4}{3}\pi(2r)^3$

$= \frac{4}{3}\pi(8r^3)$

$= \frac{32}{3}\pi r^3$

Surface Area $= 4\pi(2r)^2$

$= 4\pi . 4r^2$

$= 16\pi r^2$

ii) Volume = $\frac{4}{3}\pi(kr)^3$

$= \frac{4}{3}\pi k^3 r^3$

$= \frac{4k^3}{3}\pi r^3$

Surface Area $= 4\pi(kr)^2$

$= 4\pi k^2 r^2$

$= 4k^2 \pi r^2$

b) i) Volume = $\pi(2r)^2\frac{h}{3}$

$= \pi\, 4r^2\frac{h}{3}$

$= 4\pi\, r^2\frac{h}{3}$

Surface area $= \pi r^2 + r \times \sqrt{h^2 + r^2}$

$= \pi\,(2r)^2 + 2r \times \sqrt{h^2 + (2r)^2}$

$= 4\pi r^2 + 2r \times \sqrt{h^2 + 4r^2}$

ii) Volume = $\pi(kr)^2\frac{h}{3}$

$= \pi\, k^2 r^2\frac{h}{3}$

$= k^2\pi\, r^2\frac{h}{3}$

Surface area $= \pi r^2 + r \times \sqrt{h^2 + r^2}$

$= \pi\,(kr)^2 + kr \times \sqrt{h^2 + (kr)^2}$

$= \pi k^2 r^2 + kr \times \sqrt{h^2 + k^2 r^2}$

c) i) Volume = $\frac{1}{3}(2l)^2 h$

$= \frac{1}{3} \times 4l^2 h$

Surface Area = $(2l)^2 + 4 \times \left(\frac{1}{2} \times 2l \times \perp height\right)$

$= 4l^2 + 4 \times (l \times \perp height)$

ii) Volume = $\frac{1}{3}(kl)^2 h$

$= \frac{1}{3} \times k^2 l^2 h$

Surface Area= $(kl)^2 + 4 \times \left(\frac{1}{2}(kl) \times \perp height\right)$

$= k^2 l^2 + 2 \times (kl \times \perp height)$

d) i) Volume = lbh

$= 2lbh$

Surface area = $2bh + 2bl + 2lh$

$= 2(2b)h + 2(2bl) + 2lh$

$= 4bh + 4bl + 2lh$

ii) Volume = lbh

$= klbh$

Surface area = $2bh + 2bl + 2lh$

$= 2(kb)h + 2(kb)l + 2lh$

$= 2kbh + 2kbl + 2lh$

e) i) Volume = l^3

$= (2l)^3$

$= 8l^3$

ii) Volume = l^3

$= (kl)^3$

$= k^3 l^3$

$Surface\ area\ =\ 6l^2$
$= 6(2l)^2$
$= 6\ \times 4l^2$
$= 24l^2$

$Surface\ area\ =\ 6l^2$
$= 6(kl)^2$
$= 6k^2l^2$

f) i) Volume = $\pi(2r)^2h$
$= \pi\ \times 4r^2\ h$
$= 4\pi r^2h$

Surface area = $2\pi(2r)^2 + 2\pi(2r)h$
$= 2\pi\ \times 4r^2 + 4\pi rh$
$= 8\pi\ r^2 + 4\pi rh$

ii) Volume = $\pi(kr)^2h$
$= \pi\ \times k^2r^2h$
$= k^2\ \pi r^2h$

Surface area = $2\pi(kr)^2 + 2\pi(kr)h$
$= 2\pi\ k^2r^2 + 2\pi krh$

g) i) Volume = $\frac{1}{6}\ (2 \times length)\ \times \perp\ height\ \times height\ of\ pyramid$
$= \frac{1}{3}\ length\ \times \perp\ height\ \times height\ of\ pyramid$

Surface area = $\left(\frac{1}{2}\ (2l)\ \times \perp\ height\right) + 3\left(\frac{1}{2}(2l) \times \perp\ slant\ height\right)$
$= l\ \times \perp\ height + 3(l\ \times \perp\ slant\ height)$

ii) Volume = $\frac{1}{6}\ (k \times length)\ \times \perp\ height\ \times height\ of\ pyramid$
$= \frac{k}{6}\ \times length\ \times \perp\ height\ \times height\ of\ pyramid$

Surface area = $\frac{1}{2}\ kl\ \times \perp\ height + 3\ \left(\frac{1}{2}\ kl\ \times \perp\ slant\ height\right)$
$= \frac{1}{2}kl\ \times \perp\ h + \frac{3}{2}\ kl\ \times \perp\ slant\ height$

3. r = 3cm and h = 8cm

a) Surface Area = $4\pi r^2$
$= 4\ \pi\ (3)^2$
$= 4\pi\ \times 9$
$= 36\pi$
$= 113.1\ \ cm^2$

b) Volume = $\frac{4}{3}\pi r^3$
$= \frac{4}{3}\ \pi\ (3)^3$
$= \frac{4}{3}\ \pi\ \times 27$
$= 36\ \pi$
$= 113.1cm^3$

c) Surface area = ½ surface area of ice-cream + surface area of cone – flat surface

$$= \frac{1}{2}\,(4\pi r^2) + \pi r^2 + r \times \sqrt{h^2 + r^2} - \pi r^2$$

$$= 2\pi r^2 + r \times \sqrt{h^2 + r^2}$$

$$= 2\pi(3)^2 + 3 \times \sqrt{8^2 + 3^2}$$

$$= 18\,\pi + 3 \times \sqrt{73}$$

$$= 82.18\ cm^2$$

d) Volume = ½ volume of ice-cream + volume of cone

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3 + \pi r^2 \frac{h}{3}$$

$$= \frac{2}{3} \times \pi\,(3)^3 + \pi\,(3)^2 \times \frac{8}{3}$$

$$= 18\pi + 24\pi$$

$$= 42\,\pi$$

$$= 131.95\ cm^3$$

4. h_h = 50cm   w = l = 70cm h_r = 20cm

a) surface area = surface area of rectangular prism + surface area of pyramid – base

$$= 2lb + 2lh + 2hb - 2lb\ (the\ floor\ and\ \text{"}ceiling\text{"}) + 4l \times slant\ height$$

Slant height = $\perp\ height = \sqrt{\left(\frac{1}{2}l\right)^2 + h^2}$

$\therefore h =$

$$= \sqrt{\left(\frac{1}{2} \times 70\right)^2 + (20)^2}$$

$$= \sqrt{1625}$$

$$= 5\sqrt{65} = 40.31\ cm$$

Surface area continued $= 2lh + 2hb + 4l \times slant\ height$

$$= 2(70)(50) + 2(50)(70) + 4(70)\left(5\sqrt{65}\right)$$

$$= 14\,000 + 11\,287.16$$

$$= 25\,287.16\ cm^2$$

b) i) Volume = volume of rectangular prism + volume of pyramid

$$= lbh + \frac{1}{3}l^2h$$

$$= 70 \times 70 \times 50 + \frac{1}{3}\,(70)^2 \times 20$$

$$= 245\,000 + 32\,666\frac{2}{3} = 277\,666\frac{2}{3}\ cm^3$$

ii) volume = $\pi\, r^2 h$

$= \pi\,(10)^2 \times 10$

$= 1000\,\pi$

$= 3\ 141.59\ cm^3$

iii) number of pots = volume of house ÷ volume of pot

$= 277\ 666\frac{2}{3} \div 3\ 141.59$

$= 88.38\ or\ 89\ pots$

c) i) volume of one kernel = volume of sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\,\pi\,(2)^3$

$= 1\frac{1}{3}\,\pi$

$= 4.19\ cm^3$

Total pieces of popcorn = volume of house ÷ size of one kernel

$= 277\ 666\frac{2}{3} \div 4.19$

$= 66\ 268.89\ pieces\ of\ popcorn\ or\ 66\ 269$

ii) Length = 2 x total pieces of popcorn

$= 2 \times 66\ 269$

$= 132\ 538\ cm\ or\ 1325.38\ m$

Please note that this is really not a very realistic question. It might be fun to have a discussion about what could have been changed to make the question more realistic. You can even redo the calculations so that you can see whether your changes make them more realistic.

5. a) Convert liters into cm^3

$= 2.1\ l \times 1\ 000$

$= 2\ 100\ ml$

This means the volume of the milk carton is 2 100cm^3

Volume = lbh Let height = x and length = width = $\frac{1}{3}x$

$\therefore 2\,100 = \left(\frac{1}{3}x\right)\left(\frac{1}{3}x\right)(x)$

$\therefore 2\,100 = \frac{1}{9}x^3$

$\therefore 18\,900 = x^3$

$\therefore \sqrt[3]{18\,900} = x$

$\therefore x = 26.64\ cm$

Therefore the height is 26.64cm and the width and length is 8.88cm.

b) Volume $= lbh$

$= (2\ \times 8.88)(2\ \times 8.88)(26.64)$

$= 8\,402.7\ cm^3$

Therefore the milk container can now hold 8.4 liters of milk.

c) If only the height of the container is halved and everything else stays the same, then the milk carton will also be halved, and half of the milk will be available. This means that there will be 2.1 liters ÷ 2 = 1.05 liters of milk.

d) Volume = $\pi r^2 h$

$\therefore 2\,100 = \pi\, r^2\ \times 26.64$

$\therefore 78.8288 = \pi\, r^2$

$\therefore 25.09 = r^2$

$\therefore \sqrt{25.09} = r$

$\therefore r = 5.009\ \approx 5\ cm$

6. length = 21cm height = 42cm volume = 55 566 cm^3

a) Volume $= lbh$

$\therefore 55\,566 = 21\ \times 42\ \times w$

$\therefore 55\,566 = 882\ \times w$

$\therefore w = 63\ cm$

b) Please note that it doesn't matter in which way the boxes are stacked, they will take up the same volume of space.

∴ 55 566 x 4 = 222 264 cm^3

c) Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times \left(\frac{7}{2}\right)^3$ Remember radius = ½ diameter.

$= 57\frac{1}{6}\pi$

$= 179.59\ cm^3$

d) 7 goes into length 21cm = 3 times

7 goes into width 63cm = 9 times

7 goes into height 42cm = 6 times

Therefore number of oranges = 3 x 9 x 6 = 162 oranges.

e) Volume of oranges = 162 x 179.59cm^3

= 29 093.58

Therefore amount of air in the box between the oranges = 55 566 – 29 093.58

= 26 472.42 cm^3

f) Volume = 179.59 – 20%

= 179.59 x 0.8

= 143.67 cm^3

$\therefore 143.67 = \frac{4}{3}\pi r^3$

$\therefore 107.754 = \pi r^3$

$\therefore 34.299 = r^3$

$\therefore \sqrt[3]{34.299} = r$

$\therefore r = 3.25\ cm$

Therefore the diameter = 2 x 3.25 = 6.5 cm