SHARP

Worksheet 11 Memorandum: Calculus Part 2

Grade 12 Mathematics

1\. a)
$y = x^2 + 3x - 12$
$\frac{dy}{dx} = 2x + 3$
$\frac{d^2y}{dx^2} = 2$

b)
$f(x) = 4x - 5$
$f'(x) = 4$
$f''(x) = 0$

c)
$y = x^3 - 4x^2 + 8x - 7$
$\frac{dy}{dx} = 3x^2 - 8x + 8$
$\frac{d^2y}{dx^2} = 6x - 8$

d)
$y = \sqrt[3]{x} + \frac{1}{x^2}$
$y = x^{\frac{1}{3}} + x^{-2}$
$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 2x^{-3} = \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{x^3}$
$\frac{d^2y}{dx^2} = -\frac{2}{9}x^{-\frac{5}{3}} + 6x^{-4} = -\frac{2}{9\sqrt[3]{x^5}} + \frac{6}{x^4}$

e)
$f(x) = \frac{5}{x} - 11x^2$
$f(x) = 5x^{-1} - 11x^2$
$f'(x) = -5x^{-2} - 22x = -\frac{5}{x^2} - 22x$
$f''(x) = 10x^{-3} - 22 = \frac{10}{x^3} - 22$

f)
$g(x) = \frac{x^2 - x}{x - 1}$
$g(x) = \frac{x(x-1)}{(x-1)} = x$
$g'(x) = 1$
$g''(x) = 0$

g)
$y = x^5 + 7x^2 - 12$
$\frac{dy}{dx} = 5x^4 + 14x$
$\frac{d^2y}{dx^2} = 20x^3 + 14$

h)
$y = -x^3 + 4x^2 - 18x - 2$
$\frac{dy}{dx} = -3x^2 + 8x - 18$
$\frac{d^2y}{dx^2} = -6x + 8$

i)
$h(x) = \frac{4}{x^3} - \sqrt{x^3}$
$h(x) = 4x^{-3} - x^{\frac{3}{2}}$
$h'(x) = -12x^{-4} - \frac{3}{2}x^{\frac{1}{2}} = -\frac{12}{x^4} - \frac{3}{2}\sqrt{x}$
$h''(x) = 48x^{-5} - \frac{3}{4}x^{-\frac{1}{2}} = \frac{48}{x^5} - \frac{3}{4\sqrt{x}}$

j)
$y = \frac{1}{3}x^3 + \frac{1}{4}x^2 - 3x + 2$
$\frac{dy}{dx} = x^2 + \frac{1}{2}x - 3$
$\frac{d^2y}{dx^2} = 2x + \frac{1}{2}$

2\. a)
$f(x) = x^3 - 5x^2 - 10x + 5$
$f'(x) = 3x^2 - 10x - 10$
$f''(x) = 6x - 10$

b)
$g(x) = -x^3 + x^2 - 3x + 7$
$g'(x) = -3x^2 + 2x - 3$
$g''(x) = -6x + 2$

To find the inflection point set the second derivative to zero:

a)
$f''(x) = 6x - 10 = 0$
$6x = 10$
$x = \frac{10}{6}$ or $\frac{5}{3}$

b)
$g''(x) = -6x + 2 = 0$
$-6x = -2$
$x = \frac{1}{3}$

And $y = \left(\frac{5}{3}\right)^3 - 5\left(\frac{5}{3}\right)^2 - 10\left(\frac{5}{3}\right) + 5$

$y = -20\frac{25}{27}$

∴ inflection point: $\left(\frac{5}{3};\ -20\frac{25}{27}\right)$

Because the graph is a generally increasing graph, the graph is concave down for $x < \frac{5}{3}$, and concave up for $x > \frac{5}{3}$

And $y = -\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - 3\left(\frac{1}{3}\right) + 7$

$y = 6\frac{2}{27}$

∴ inflection point: $\left(\frac{1}{3}; 6\frac{2}{27}\right)$

Because the graph is a generally decreasing graph, the graph is concave up for $x < \frac{1}{3}$, and concave down for $x > \frac{1}{3}$.

c) $h(x) = (x-3)(x+2)(x-1)$

$h(x) = x^3 - 2x^2 - 5x + 6$

$h'(x) = 3x^2 - 4x - 5$

$h''(x) = 6x - 4 = 0$

$\therefore x = \frac{4}{6}\ or\ \frac{2}{3}$

$y = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 - 5\left(\frac{2}{3}\right) + 6$

$y = 2\frac{2}{27}$

∴ inflection point is $\left(\frac{2}{3}; 2\frac{2}{27}\right)$

Because the graph is a generally increasing graph, the graph is concave down for $x < \frac{2}{3}$, and concave up for $x > \frac{2}{3}$

d) $j(x) = x^3 - 2$

$j'(x) = 3x^2$

$j''(x) = 6x = 0$

$\therefore x = 0$

$y = 0^3 - 2$

$y = -2$

∴ inflection point is (0; -2)

Because the graph is a generally increasing graph, the graph is concave down for $x < 0$, and concave up for $x > 0$

e) $k(x) = -3x^3 + 12x^2$

$k'(x) = -9x^2 + 24x$

$k''(x) = -18x + 24 = 0$

$x = \frac{4}{3}$

$\therefore y = -3\left(\frac{4}{3}\right)^3 + 12\left(\frac{4}{3}\right)^2$

$\therefore y = 14\frac{2}{9}$

∴ the inflection point is $\left(\frac{4}{3}; 14\frac{2}{9}\right)$

Because the graph is a generally decreasing graph, the graph is concave up for $x < \frac{4}{3}$, and concave down for $x > \frac{4}{3}$.

f) $m(x) = x^2 - 3x + 12$

$m'(x) = 2x - 3$

$m''(x) = 2$

The graph does not have an inflection point and is concave down over the entire graph.

g) $n(x) = -(x-1)(x-2)(x-4)$

$n(x) = -x^3 + 7x^2 - 14x + 8$

$n'(x) = -3x^2 + 14x - 14$

$n''(x) = -6x + 14 = 0$

$x = \frac{7}{3}$

$y = -\left(\frac{7}{3}\right)^3 + 7\left(\frac{7}{3}\right)^2 - 14\left(\frac{7}{3}\right) + 8$

$y = \frac{20}{27}$

∴ inflection point at $\left(\frac{7}{3}; \frac{20}{27}\right)$

h) $p(x) = -(x^2 - 8)(x + 3)$

$p(x) = -x^3 - 3x^2 + 8x + 24$

$p'(x) = -3x^2 - 6x + 8$

$p''(x) = -6x - 6 = 0$

$x = -1$

$y = -(-1)^3 - 3(-1)^2 + 8(-1) + 24$

$y = 14$

∴ inflection point at $(-1; 14)$

Because the graph is a generally decreasing graph, the graph is concave up for $x < \frac{7}{3}$, and concave down for $x > \frac{7}{3}$

Because the graph is a generally decreasing graph, the graph is concave up for $x < -1$, and concave down for $x > -1$

i) $q(x) = 4x^3 - 6x^2 - 3x + 5$

$q'(x) = 12x^2 - 12x - 3$

$q''(x) = 24x - 12 = 0$

$x = \frac{1}{2}$

$y = 4\left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 5$

$y = 2\frac{1}{2}$

$\therefore$ inflection point $\left(\frac{1}{2}; 2\frac{1}{2}\right)$

Because the graph is a generally increasing graph, the graph is concave down for $x < \frac{1}{2}$, and concave up for $x > \frac{1}{2}$

j) $r(x) = 5x^3 + 3x^2 - 4x + 12$

$r'(x) = 15x^2 + 6x - 4$

$r''(x) = 30x + 6 = 0$

$x = -\frac{1}{5}$

$y = 5\left(-\frac{1}{5}\right)^3 + 3\left(-\frac{1}{5}\right)^2 - 4\left(-\frac{1}{5}\right) + 12$

$y = 12\frac{22}{25}\ or\ \frac{322}{25}$

$\therefore$ inflection point $\left(-\frac{1}{5}; 12\frac{22}{25}\right)$

Because the graph is a generally increasing graph, the graph is concave down for $x < -\frac{1}{5}$, and concave up for $x > -\frac{1}{5}$

3. a) $y = x^3 + 7x^2 + 7x - 15$

i) y-intercept - $(0; -15)$

x-intercepts:

$0 = x^3 + 7x^2 + 7x - 15$ $\quad x = 1$ or $(x - 1)$ is a factor:

$0 = (x - 1)(x^2 + kx + 15)$

To find k:

$-x^2 + kx^2 = 7x^2$

$kx^2 = 8x^2$

$\therefore k = 8$

$0 = (x - 1)(x^2 + 8x + 15)$

$0 = (x - 1)(x + 3)(x + 5)$

$\therefore$ the x-intercepts are (1; 0), (-3; 0) and (-5; 0)

ii) $f'(x) = 3x^2 + 14x + 7 = 0$

$x = \frac{-14 \pm \sqrt{14^2 - 4(3)(7)}}{2(3)}$

$x = \frac{-7 \pm 2\sqrt{7}}{3}$

$x = -0.57 \quad or \quad x = -4.1$

$y = (-0.57)^3 + 7(-0.57)^2 + 7(-0.57) - 15 = -16.9$

And $y = (-4.1)^3 + 7(-4.1)^2 + 7(-4.1) - 15 = 5.05$

So the maximum and minimum points are: (-4.1; 5.05) and (-0.57; -16.9)

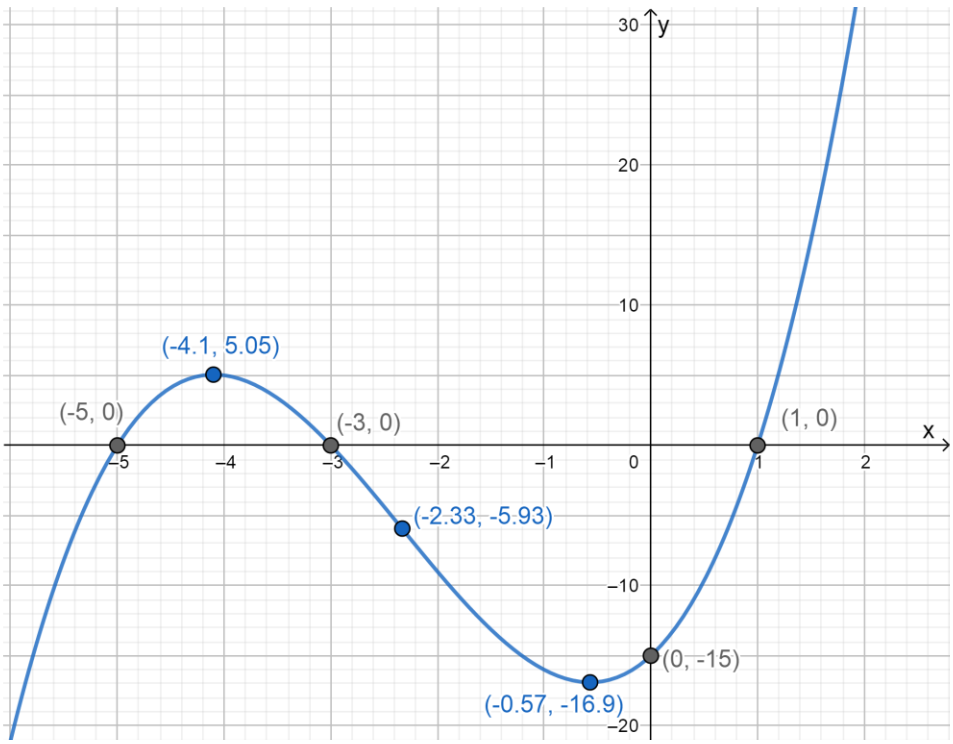
iii) $f''(x) = 6x + 14 = 0$

$\therefore x = -\frac{14}{6}\ or\ -2\frac{1}{3}$

$\therefore y = \left(-2\frac{1}{3}\right)^3 + 7\left(-2\frac{1}{3}\right)^2 + 7\left(-2\frac{1}{3}\right) - 15$

$y = -5\frac{25}{27}$

iv)



b) $y = x^3 + 13x^2 + 34x - 48$

i) y-intercept: (0; -48)

For the x-intercepts:

$0 = x^3 + 13x^2 + 34x - 48$ $\quad\quad$ $(x-1)$ is a factor

$\therefore (x-1)(x^2 + kx + 48)$

$-x^2 + kx^2 = 13x^2$

$kx^2 = 14x^2$

$\therefore k = 14$

$\therefore 0 = (x-1)(x^2 + 14x + 48)$

$0 = (x-1)(x+6)(x+8)$

$\therefore$ The x-intercepts are: (1; 0), (-6; 0) and (-8; 0)

ii) $f'(x) = 3x^2 + 26x + 34 = 0$

$x = \frac{-26 \pm \sqrt{26^2 - 4(3)(34)}}{2(3)}$

$x = \frac{-13 \pm \sqrt{67}}{3}$

$x = -1.6 \quad or \quad x = -7.1$

$\therefore y = (-1.6)^3 + 13(-1.6)^2 + 34(-1.6) - 48 = -73.2$

And $y = (-7.1)^3 + 13(-7.1)^2 + 34(-7.1) - 48 = 8$

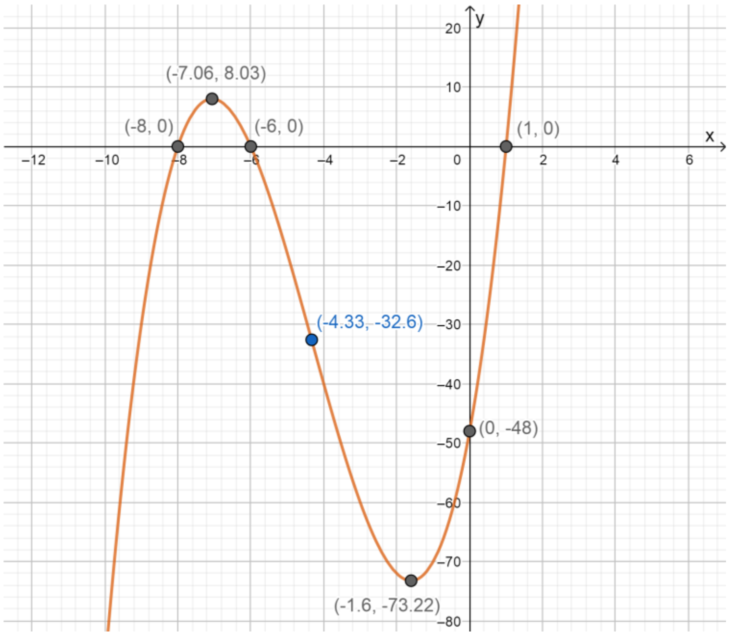
iii) $f''(x) = 6x + 26 = 0$

$\therefore x = -4\frac{1}{3}$

$\therefore y = \left(-4\frac{1}{3}\right)^3 + 13\left(-4\frac{1}{3}\right)^2 + 34\left(-4\frac{1}{3}\right) - 48 = -32\,\frac{16}{27}$

$\therefore$ the inflection point is $\left(-4\frac{1}{3};\ -32\frac{16}{27}\right)$

iv)



c) $y = 3x^3 - 13x^2 - 130x + 336$

i) y-intercept: (0; 336)

x-intercepts:

$0 = 3x^3 - 13x^2 - 130x + 336$ $(x - 8)$ is a factor

$\therefore 0 = (x - 8)(3x^2 + kx - 42)$

$kx^2 - 24x^2 = -13x^2$

$kx^2 = 11x^2$

$\therefore k = 11$

$\therefore 0 = (x - 8)(3x^2 + 11x - 42)$

$\therefore 0 = (x - 8)(3x - 7)(x + 6)$

∴ the x-intercepts are: (8; 0), (-6; 0) and $\left(2\frac{1}{3}; 0\right)$

ii) $\frac{dy}{dx} = 9x^2 - 26x - 130 = 0$

$x = \frac{26 \pm \sqrt{(-26)^2 - 4(9)(-130)}}{2(9)}$

$x = 5.51 \quad or \quad x = -2.62$

$\therefore y = 3(5.51)^3 - 13(5.51)^2 - 130(5.51) + 336 = -273.13$

And $y = 3(-2.62)^3 - 13(-2.62)^2 - 130(-2.62) + 336 = 533.4$

∴ the maximum and minimum are: (-2.62; 533.4) and (5.51; -273.13)

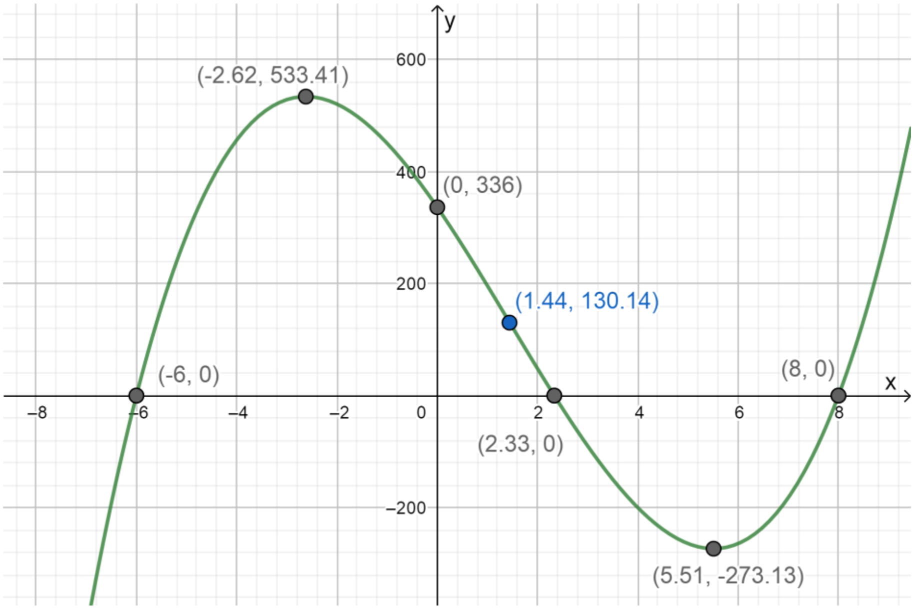
iii) $\frac{d^2y}{dx^2} = 18x - 26 = 0$

$x = 1\frac{4}{9}$

$\therefore y = 3\left(1\frac{4}{9}\right)^3 - 13\left(1\frac{4}{9}\right)^2 - 130\left(1\frac{4}{9}\right) + 336 = 130.14$

∴ point of inflection: $\left(1\frac{4}{9}; 130.14\right)$

iv)



d) $y = -x^3 + 6x^2 + 45x - 162$

i) y-intercept: (0; -162)

For the x-intercepts:

$0 = -(x^3 - 6x^2 - 45x + 162)$ $\therefore (x-3)$ is afactor

$0 = -(x-3)(x^2 + kx - 54)$

$kx^2 - 3x^2 = -6x^2$

$kx^2 = -3x^2$

$k = -3$

$\therefore 0 = -(x-3)(x^2 - 3x - 54)$

$\therefore 0 = -(x-3)(x+6)(x-9)$

∴ The x-intercepts are: (3; 0), (-6; 0) and (9; 0)

ii) $\frac{dy}{dx} = -3x^2 + 12x + 45 = 0$

$x^2 - 4x - 15 = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-15)}}{2}$

$x = 2 \pm \sqrt{19}$

$x = 6.36 \quad or \quad x = -2.36$

$\therefore y = -(6.36)^3 + 6(6.36)^2 + 45(6.36) - 162 = 109.64$

And $y = -(-2.36)^3 + 6(-2.36)^2 + 45(-2.36) - 162 = -221.64$

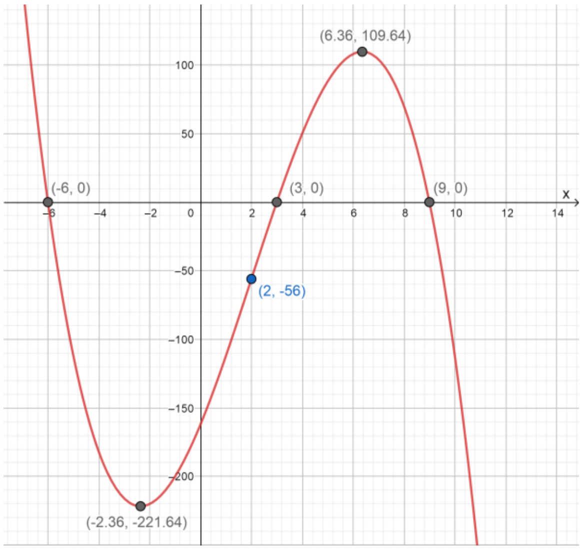
∴ the maximum and minimum are: (6.36; 109.64) and (-2.36; -221.64)

iii) $\frac{d^2y}{dx^2} = -6x + 12 = 0$

$x = 2$

$\therefore y = -(2)^3 + 6(2)^2 + 45(2) - 162 = -56$

∴ the inflection point is (2; -56).

iv) 

e) $y = 3x^3 - 29x^2 + 17x + 9$

i) the y-intercept is (0; 9)

For the x-intercepts:

$0 = 3x^3 - 29x^2 + 17x + 9$ $(x-1)$ is a factor

$\therefore 0 = (x-1)(3x^2 + kx - 9)$

$kx^2 - 3x^2 = -29x^2$

$kx^2 = -26x^2$

$\therefore k = -26$

$\therefore 0 = (x-1)(3x^2 - 26x - 9)$

$\therefore 0 = (x-1)(3x+1)(x-9)$

∴The x-intercepts are: (1; 0), (9; 0) and $\left(-\frac{1}{3}; 0\right)$

ii) $\frac{dy}{dx} = 9x^2 - 58x + 17 = 0$

$x = \frac{58 \pm \sqrt{(-58)^2 - 4(9)(17)}}{2(9)}$

$x = \frac{29 \pm 4\sqrt{43}}{9}$

$x = 6.14 \quad or \quad x = 0.31$

$\therefore y = 3(6.14)^3 - 29(6.14)^2 + 17(6.14) + 9 = -285.48$

And $y = 3(0.31)^3 - 29(0.31)^2 + 17(0.31) + 9 = 11.57$

∴ the maximum and minimum are: (0.31; 11.57) and (6.14; -285.48)

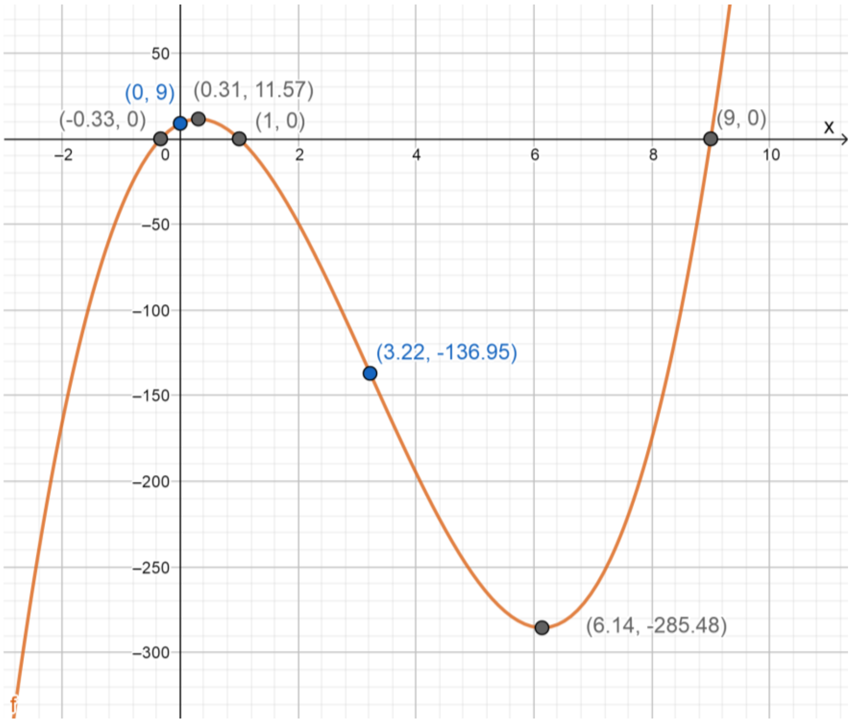
iii) $\frac{d^2y}{dx^2} = 18x - 58 = 0$

$x = 3\frac{2}{9}$

$\therefore y = 3\left(3\frac{2}{9}\right)^3 - 29\left(3\frac{2}{9}\right)^2 + 17\left(3\frac{2}{9}\right) + 9 = -136.95$

∴ the inflection point is $\left(3\frac{2}{9}; -136.95\right)$

iv)



f) $y = -3x^3 + 20x^2 + 69x - 54$

i) y-intercept: (0; -54)

For the x-intercepts:

$0 = -(3x^3 - 20x^2 - 69x + 54)$ $(x-9)$ is a factor

$\therefore 0 = -(x-9)(3x^2 + kx - 6)$

$kx^2 - 27x^2 = -20x^2$

$kx^2 = 7x^2$

$\therefore k = 7$

$\therefore 0 = -(x-9)(3x^2 + 7x - 6)$

$\therefore 0 = -(x-9)(3x-2)(x+3)$

$\therefore$ the x-intercepts are: (9; 0), (-3; 0) and $\left(\frac{2}{3}; 0\right)$

ii) $\frac{dy}{dx} = -9x^2 + 40x + 69 = 0$

$x = \frac{-40 \pm \sqrt{40^2 - 4(-9)(69)}}{2(-9)}$

$x = -1.33$ *or* $x = 5.77$

$\therefore y = -3(-1.33)^3 + 20(-1.33)^2 + 69(-1.33) - 54 = -103.33$

And $y = -3(5.77)^3 + 20(5.77)^2 + 69(5.77) - 54 = 433.69$

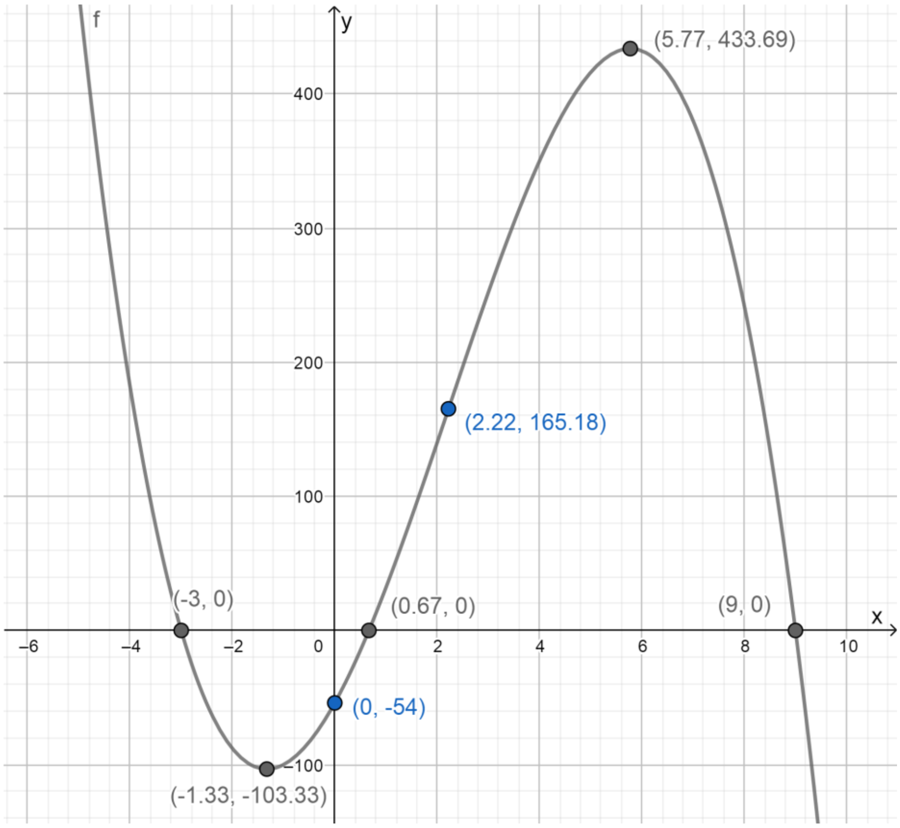
$\therefore$ the maximum and the minimum are (5.77; 433.69) and (-1.33; -103.33)

iii) $\frac{d^2y}{dx^2} = -18x + 40 = 0$

$x = 2\frac{2}{9}$

$\therefore y = -3\left(2\frac{2}{9}\right)^3 + 20\left(2\frac{2}{9}\right)^2 + 69\left(2\frac{2}{9}\right) - 54 = 165.18$

$\therefore$ the inflection point is $\left(2\frac{2}{9}; 165.18\right)$

iv) 

g) $y = 5x^3 - 39x^2 + 78x - 40$

i) y-intercept: (0; -40)

For the x-intercepts:

$0 = 5x^3 - 39x^2 + 78x - 40$ $(x-2)$ is a factor

$\therefore 0 = (x-2)(5x^2 + kx + 20)$

$kx^2 - 10x^2 = -39x^2$

$kx^2 = -29x^2$

$\therefore k = -29$

$\therefore 0 = (x-2)(5x^2 - 29 + 20)$

$\therefore 0 = (x-2)(5x-4)(x-5)$

$\therefore$ the x-intercepts are: (2; 0), (5; 0) and $\left(\frac{4}{5}; 0\right)$

ii) $\frac{dy}{dx} = 15x^2 - 78x + 78 = 0$

$x = \frac{78 \pm \sqrt{(-78)^2 - 4(15)(78)}}{2(15)}$

$x = \frac{13 \pm \sqrt{39}}{5}$

$x = 3.85 \quad or \quad x = 1.35$

$\therefore y = 5(3.85)^3 - 39(3.85)^2 + 78(3.85) - 40 = -32.44$

And $y = 5(1.35)^3 - 39(1.35)^2 + 78(1.35) - 40 = 6.52$

$\therefore$ the maximum and minimum are (1.35; 6.52) and (3.85; -32.44)

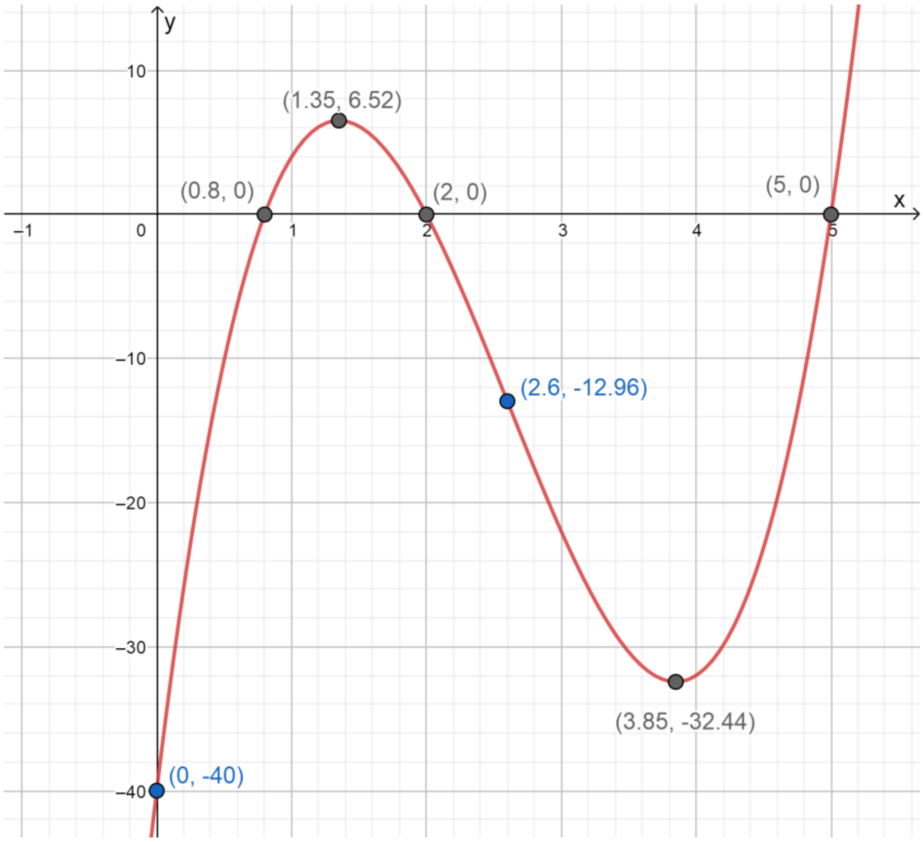
iii) $\frac{d^2y}{dx^2} = 30x - 78 = 0$

$x = 2\frac{3}{5}$

$\therefore y = 5\left(2\frac{3}{5}\right)^3 - 39\left(2\frac{3}{5}\right)^2 + 78\left(2\frac{3}{5}\right) - 40 = -12.96$

$\therefore$ the inflection point is $\left(2\frac{3}{5}; -12.96\right)$

iv)



h) $y = -x^3 + 6x^2 + 67x - 360$

i) y-intercept: (0; -360)

For the x-intercepts:

$0 = -(x^3 - 6x^2 - 67x + 360)$ $\quad$ $(x-5)$ is a factor

$\therefore 0 = -(x-5)(x^2 + kx - 72)$

$kx^2 - 5x^2 = -6x^2$

$kx^2 = -x^2$

$\therefore k = -1$

$\therefore 0 = -(x-5)(x - x - 72)$

$\therefore 0 = -(x-5)(x-9)(x+8)$

$\therefore$ the x-intercepts are (5; 0), (9; 0) and (-8; 0)

ii) $\frac{dy}{dx} = -3x^2 + 12x + 67 = 0$

$x = \frac{-12 \pm \sqrt{12^2 - 4(-3)(67)}}{2(-3)}$

$x = \frac{6 \mp \sqrt{237}}{3}$

$x = -3.13 \quad or \quad x = 7.13$

$\therefore y = -(-3.13)^3 + 6(-3.13)^2 + 67(-3.13) - 360 = -480.26$

And $y = -(7.13)^3 + 6(7.13)^2 + 67(7.13) - 360 = 60.26$

$\therefore$ the maximum and minimum are: (7.13; 60.26) and (-3.13; -480.26)

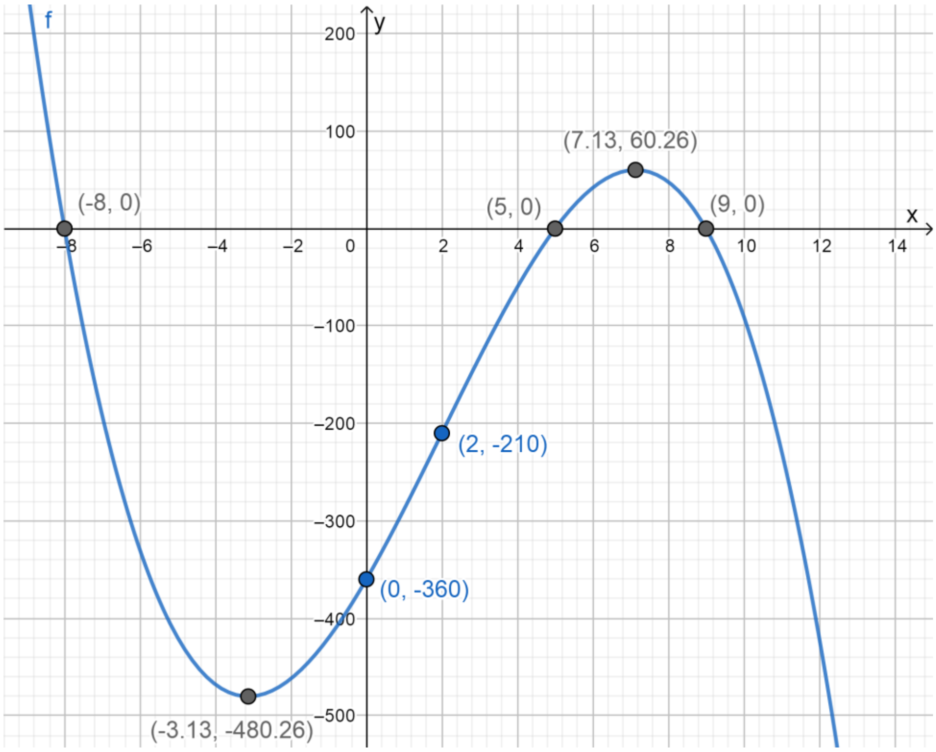
iii) $\frac{d^2y}{dx^2} = -6x + 12 = 0$

$x = 2$

$\therefore y = -(2)^3 + 6(2)^2 + 67(2) - 360 = -210$

$\therefore$ the inflection point is (2; -210)

iv)



i) $y = 6x^3 + 7x^2 - 84x - 160$

i) y-intercept: (0; -160)

For the x-intercepts:

$0 = 6x^3 + 7x^2 - 84x - 160$ $\quad$ $(x-4)$ is a factor

$\therefore 0 = (x-4)(6x^2 + kx + 40)$

$kx^2 - 24x^2 = 7x^2$

$kx^2 = 31x^2$

$\therefore k = 31$

$\therefore 0 = (x-4)(6x^2 + 31x + 40)$

$\therefore 0 = (x-4)(3x + 8)(2x + 5)$

$\therefore$ the x-intercepts are: (4; 0), $\left(-2\frac{2}{3}; 0\right)$ and $\left(-2\frac{1}{2}; \ 0\right)$

ii) $\frac{dy}{dx} = 18x^2 + 14x - 84 = 0$

$x = \frac{-14 \pm \sqrt{14^2 - 4(18)(-84)}}{2(18)}$

$x = 1.81 \quad or \quad x = -2.58$

$\therefore y = 6(1.81)^3 + 7(1.81)^2 - 84(1.81) - 160 = -253.5$
And $y = 6(-2.58)^3 + 7(-2.58)^2 - 84(-2.58) - 160 = 0.27$
$\therefore$ the maximum and minimum are: (-2.58; 0.27) and (1.81; -253.5)

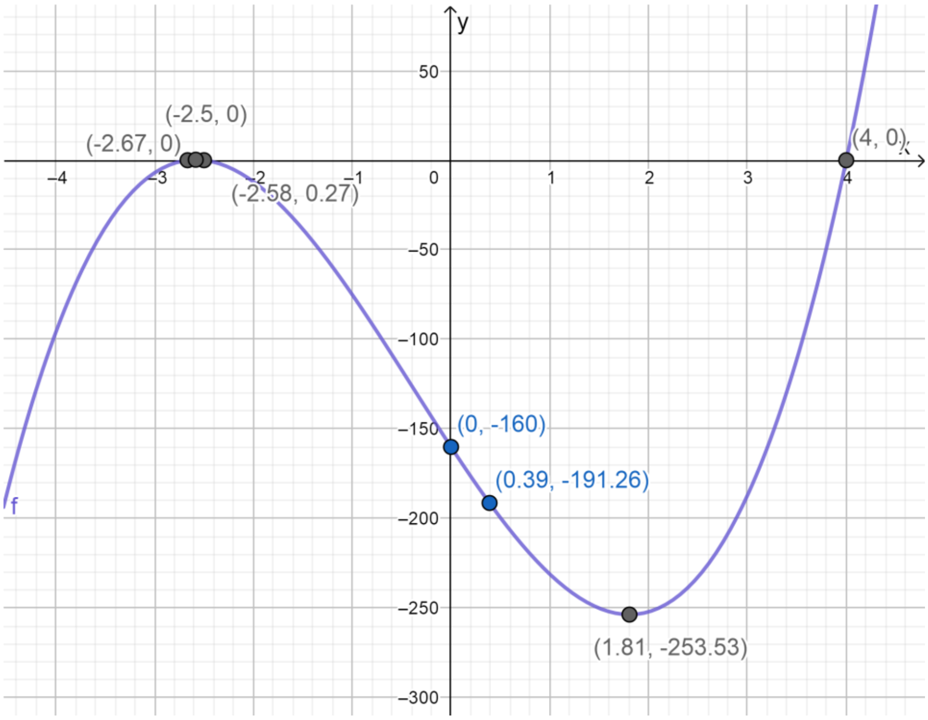
iii) $\frac{d^2y}{dx^2} = 36x + 14 = 0$

$x = \frac{7}{18}$

$\therefore y = 6\left(\frac{7}{18}\right)^3 + 7\left(\frac{7}{18}\right)^2 - 84\left(\frac{7}{18}\right) - 160 = -191.26$

$\therefore$ the inflection point is $\left(\frac{7}{18}; -191.26\right)$

iv)



j) $y = -x^3 - 7x^2 + 84x + 288$

i) y-intercept: (0; 288)
For the x-intercepts:
$0 = -(x^3 + 7x^2 - 84x - 288)$ $(x - 8)$ is a factor
$\therefore 0 = -(x - 8)(x^2 + kx + 36)$
$kx^2 - 8x^2 = 7x^2$
$kx^2 = 15x^2$
$\therefore k = 15$
$\therefore 0 = -(x - 8)(x^2 + 15x + 36)$
$\therefore 0 = -(x - 8)(x + 12)(x + 3)$
$\therefore$ the x-intercepts are: (8; 0), (-12; 0) and (-3; 0)

ii) $\frac{dy}{dx} = -3x^2 - 14x + 84 = 0$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(-3)(84)}}{2(-3)}$$

$$x = \frac{-7 \mp \sqrt{301}}{3}$$

$x = -8.12 \quad or \quad x = 3.45$

$\therefore y = -(-8.12)^3 - 7(-8.12)^2 + 84(-8.12) + 288 = -320.23$

And $y = -(3.45)^3 - 7(3.45)^2 + 84(3.45) + 288 = 453.42$

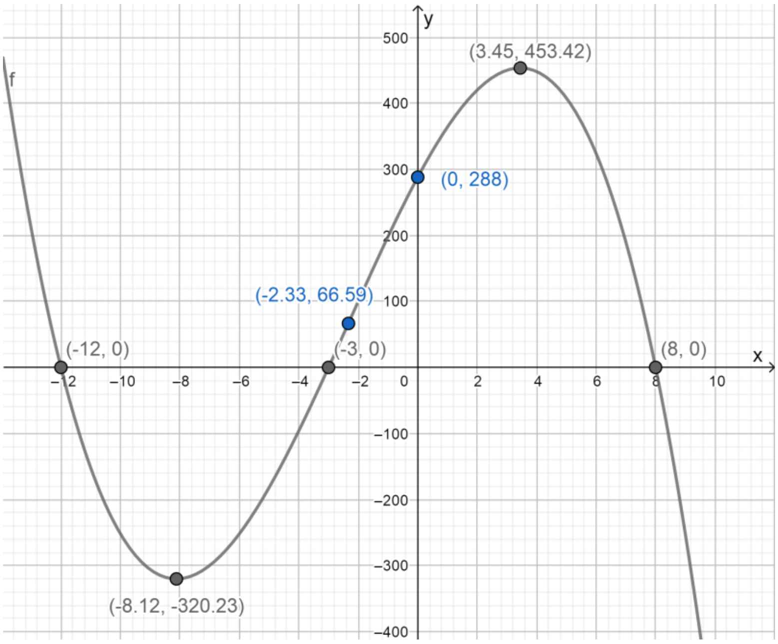
∴ the maximum and minimum are: (3.45; 453.42) and (-8.12; -320.23)

iii) $\frac{d^2y}{dx^2} = -6x - 14 = 0$

$$x = -2\tfrac{1}{3}$$

$$\therefore y = -\left(-2\tfrac{1}{3}\right)^3 - 7\left(-2\tfrac{1}{3}\right)^2 + 84\left(-2\tfrac{1}{3}\right) + 288 = 66.59$$

iv)



4. a) $v(t) = \frac{1}{3}t^3 - 3t^2 + 6t = 0$

$$\frac{1}{3}t(t^2 - 9t + 18) = 0$$

$$\frac{1}{3}t(t-3)(t-6) = 0$$

$\therefore t = 0, \quad t = 3 \quad or\ t = 6$

The molecule cannot get through the cell wall when the graph is negative, and this is between t = 3 and t = 6

b) $v'(t) = t^2 - 6t + 6 = 0$

$$t = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(6)}}{2}$$

$$t = 3 \pm \sqrt{3}$$

$t = 4.73 \quad or \quad t = 1.27$

$\therefore v(4.73) = \frac{1}{3}(4.73)^3 - 3(4.73)^2 + 6(4.73) = -3.46$

And $v(1.27) = \frac{1}{3}(1.27)^3 - 3(1.27)^2 + 6(1.27) = 3.46$

Therefore, the maximum point is at (1.27; 3.46)

5. Volume of original box $= x^2\, y = 3$

$\therefore y = \frac{3}{x^2}$

Surface area of the box $= 2x^2 + 2xy + 2xy$

SA $= 2x^2 + 4xy$ Substitute volume for y:

$SA = 2x^2 + 4x\left(\frac{3}{x^2}\right)$

$SA = 2x^2 + \frac{12}{x} = 2x^2 + 12x^{-1}$

Differentiate the SA and set equal to zero:

$SA' = 4x - 12x^{-2} = 0$

$4x - \frac{12}{x^2} = 0$

$4x^3 - 12 = 0$

$4x^3 = 12$

$x^3 = 3$

$x = \sqrt[3]{3} \approx 1.44$

$\therefore$ the maximum surface area is $SA = 2\,(1.44)^2 + \frac{12}{1.44} = 12.48\, units^2$

6. a) $P = 300m = 2x + 2y$

$2y = 300 - 2x$

$y = 150 - x$

b) $A = xy$

$A = x(150 - x)$

$A = 150x - x^2$

$\therefore A' = 150 - 2x = 0$

$x = 75m$

$\therefore$ the maximum area is $A = 150(75) - (75)^2 = 5\,625m^2$

c) $\therefore$ new perimeter = 380m

$2y = 380 - 2x$

$y = 190 - x$

d) $A = x(190 - x)$

$A = 190x - x^2$

$\therefore A' = 190 - 2x = 0$

$x = 95m$

$\therefore$ the new maximum area is $A = 190(95) - (95)^2 = 9\,025m^2$