

# SHARP

## Worksheet 7 Memorandum: Algebraic Fractions

### Grade 10 Technical Mathematics

1. a)  $\frac{1}{2} + \frac{3}{5}$   
 $= \frac{5}{10} + \frac{6}{10}$   
 $= \frac{11}{10} \text{ or } 1\frac{1}{10}$

b)  $\frac{3}{4} - \frac{7}{11}$   
 $= \frac{33}{44} - \frac{28}{44}$   
 $= \frac{5}{44}$

c)  $\frac{x}{4} + \frac{3}{7}$   
 $= \frac{7x}{28} + \frac{12}{28}$   
 $= \frac{7x+12}{28}$

d)  $\frac{7}{8} \times \frac{4}{21}$   
 $= \frac{1}{2} \times \frac{1}{3}$   
 $= \frac{1}{6}$

e)  $8\frac{1}{3} \div \frac{5}{12}$   
 $= \frac{25}{3} \times \frac{12}{5}$   
 $= \frac{5}{1} \times \frac{4}{1}$   
 $= 20$

f)  $\frac{1}{5} \times 6\frac{2}{5}$   
 $= \frac{1}{5} \times \frac{32}{5}$   
 $= \frac{32}{25}$

2. a)  $\frac{x}{7} + \frac{3x+1}{3}$   
 $= \frac{3x}{21} + \frac{21x+7}{21}$   
 $= \frac{24x+7}{21}$

b)  $\frac{3}{x} + \frac{7}{x+1}$   
 $= \frac{3x+3}{x(x+1)} + \frac{7x}{x(x+1)}$   
 $= \frac{10x+3}{x^2+x}$

c)  $\frac{2x}{4+x} + \frac{x}{3}$   
 $= \frac{6x}{3(4+x)} + \frac{4x+x^2}{3(4+x)}$   
 $= \frac{x^2+10x}{12+3x}$

d)  $\frac{a}{b} - \frac{3a}{4b}$   
 $= \frac{4ab}{4b^2} - \frac{3ab}{4b^2}$   
 $= \frac{ab}{4b^2}$   
 $= \frac{a}{4b}$

e)  $\frac{3a+1}{a} - \frac{4b-1}{b}$   
 $= \frac{3ab+b}{ab} - \frac{4ab-a}{ab}$   
 $= \frac{3ab+b-4ab+a}{ab}$   
 $= \frac{-ab+b+a}{ab}$

f)  $\frac{7a-1}{6b+1} - \frac{b+3}{3a-2}$   
 $= \frac{(7a-1)(3a-2)}{(6b+1)(3a-2)} - \frac{(b+3)(6b+1)}{(6b+1)(3a-2)}$   
 $= \frac{21a^2-17a+2-(6b^2+19b+3)}{(6b+1)(3a-2)}$   
 $= \frac{21a^2-17a-6b^2-19b-1}{18ab-12b+3a-2}$



$$\begin{aligned}
 \text{g)} \quad & \frac{a+1}{b} \times \frac{b^2}{a^2-1} \\
 &= \frac{a+1}{b} \times \frac{b^2}{(a+1)(a-1)} \\
 &= \frac{1}{1} \times \frac{b}{a-1} \\
 &= \frac{b}{a-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{h)} \quad & \frac{a}{9} \times \frac{27}{a^3} \\
 &= \frac{1}{1} \times \frac{3}{a^2} \\
 &= \frac{3}{a^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{i)} \quad & \frac{x^2+x}{3y} \times \frac{6y^2}{x} \times \frac{2}{x+1} \\
 &= \frac{x(x+1)}{3y} \times \frac{6y^2}{x} \times \frac{2}{x+1} \\
 &= \frac{1}{1} \times \frac{2y}{1} \times \frac{2}{1} \\
 &= 4y
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & \frac{a+b}{a} \div \frac{ab+b^2}{a^2+a} \\
 &= \frac{a+b}{a} \times \frac{a(a+1)}{b(a+b)} \\
 &= \frac{1}{1} \times \frac{a+1}{b} \\
 &= \frac{a+1}{b}
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & \frac{x}{4} \div \frac{x^2}{16} \\
 &= \frac{x}{4} \times \frac{16}{x^2} \\
 &= \frac{1}{1} \times \frac{4}{x} \\
 &= \frac{4}{x}
 \end{aligned}$$

$$\begin{aligned}
 \text{l)} \quad & \frac{3(a-1)}{5} \div \frac{9a^2-9}{15} \\
 &= \frac{3(a-1)}{5} \times \frac{15}{9(a^2-1)} \\
 &= \frac{(a-1)}{1} \times \frac{3}{3(a+1)(a-1)} \\
 &= \frac{1}{1} \times \frac{1}{a+1} = \frac{1}{a+1}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \text{a)} \quad & \frac{ab+3b+a^2+3a}{a^2-9} \times \frac{a^2+a-12}{a^2+7ab+6b^2} \\
 &= \frac{b(a+3)+a(a+3)}{(a+3)(a-3)} \times \frac{(a+4)(a-3)}{(a+6b)(a+b)} \\
 &= \frac{(a+b)(a+3)}{(a+3)} \times \frac{(a+4)}{(a+6b)(a+b)} \\
 &= \frac{1}{1} \times \frac{a+4}{a+6b} \\
 &= \frac{a+4}{a+6b}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \frac{y^3-64}{y^2-16} \div \frac{y^2+4y+16}{y^2+9y+20} \\
 &= \frac{(y-4)(y^2+4y+16)}{(y-4)(y+4)} \times \frac{(y+4)(y+5)}{y^2+4y+16} \\
 &= \frac{1}{1} \times \frac{y+5}{1} \\
 &= y + 5
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad & \frac{x^2-9x+14}{x^2+x-56} \times \frac{x^3+512}{x^2-4} \\
 &= \frac{(x-7)(x-2)}{(x+8)(x-7)} \times \frac{(x+8)(x^2-8x+64)}{(x-2)(x+2)} \\
 &= \frac{1}{1} \times \frac{x^2-8x+64}{x+2} \\
 &= \frac{x^2-8x+64}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 \text{d)} \quad & \frac{xy+2y-4x-8}{y^2-12y+32} \times \frac{y^2-y-56}{x^2+9x+14} \\
 &= \frac{y(x+2)-4(x+2)}{(y-4)(y-8)} \times \frac{(y-8)(y+7)}{(x+7)(x+2)} \\
 &= \frac{(y-4)(x+2)}{(y-4)} \times \frac{y+7}{(x+7)(x+2)} \\
 &= \frac{y+7}{x+7}
 \end{aligned}$$

NB! Cannot be simplified further!



$$\begin{aligned}
 \text{e)} \quad & \frac{m^2+4m+2mn+8n}{m^2-5m-36} \div \frac{m^2-4n^2}{m^2-14m+45} \\
 &= \frac{m(m+4)+2n(m+4)}{(m-9)(m+4)} \times \frac{(m-9)(m-5)}{(m-2n)(m+2n)} \\
 &= \frac{(m+2n)(m+4)}{(m+4)} \times \frac{(m-5)}{(m-2n)(m+2n)} \\
 &= \frac{m-5}{m-2n}
 \end{aligned}$$

$$\begin{aligned}
 \text{f)} \quad & \frac{a^3-27}{6a^3+18a^2+54a} \times \frac{a^2+8a}{a^2-11a+24} \\
 &= \frac{(a-3)(a^2+3a+9)}{6a(a^2+3a+9)} \times \frac{a(a+8)}{(a-8)(a-3)} \\
 &= \frac{1}{6} \times \frac{(a+8)}{(a-8)} \\
 &= \frac{a+8}{6(a-8)}
 \end{aligned}$$

$$\begin{aligned}
 \text{g)} \quad & \frac{a^2-7a+6}{a^2-a-6} \times \frac{a^2+4a-21}{a^2-5a-6} \\
 &= \frac{(a-6)(a-1)}{(a-3)(a+2)} \times \frac{(a+7)(a-3)}{(a-6)(a+1)} \\
 &= \frac{(a-1)}{(a+2)} \times \frac{(a+7)}{(a+1)} \\
 &= \frac{a^2+6a-7}{a^2+3a+2}
 \end{aligned}$$

$$\begin{aligned}
 \text{h)} \quad & \frac{p^2-7p-60}{p^2+p-20} \div \frac{p^2-6p-72}{p^2+2p-24} \\
 &= \frac{(p-12)(p+5)}{(p+5)(p-4)} \times \frac{(p+6)(p-4)}{(p-12)(p+6)} \\
 &= \frac{1}{1} \times \frac{1}{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{i)} \quad & \frac{a}{a^2-a-12} + \frac{3}{a^2+7a+12} \\
 &= \frac{a}{(a-4)(a+3)} + \frac{3}{(a+3)(a+4)} \\
 &= \frac{a(a+4)}{(a-4)(a+3)(a+4)} + \frac{3(a-4)}{(a+3)(a+4)(a-4)} \\
 &= \frac{a^2+4a+3a-12}{(a-4)(a+3)(a+4)} \\
 &= \frac{a^2+7a-12}{(a-4)(a+3)(a+4)}
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & \frac{a^2-2a}{a^2-4} - \frac{7}{a^2+11a+18} \\
 &= \frac{a(a-2)}{(a-2)(a+2)} - \frac{7}{(a+2)(a+9)} \\
 &= \frac{a}{(a+2)} - \frac{7}{(a+2)(a+9)} \\
 &= \frac{a(a+9)-7}{(a+2)(a+9)} \\
 &= \frac{a^2+9a-7}{(a+2)(a+9)}
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & \frac{8r+tr+8t+t^2}{t^2+16t+63} \div \frac{t^2+17t+72}{t^3+343} \\
 &= \frac{r(8+t)+t(8+t)}{(t+9)(t+7)} \times \frac{(t+7)(t^2-7t+49)}{(t+8)(t+9)} \\
 &= \frac{(8+t)(r+t)}{(t+9)} \times \frac{(t^2-7t+49)}{(t+8)(t+9)} \\
 &= \frac{(r+t)}{(t+9)} \times \frac{(t^2-7t+49)}{(t+9)} \\
 &= \frac{(r+t)(t^2-7t+49)}{(t+9)^2} \text{ OR } = \frac{t^2r-7rt+49r+t^3-7t^2+49t}{(t+9)^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{l)} \quad & \frac{a+1}{a^2+4a-21} + \frac{a^2-36}{a^2+a-42} \\
 &= \frac{a+1}{(a+7)(a-3)} + \frac{(a-6)(a+6)}{(a-6)(a+7)} \\
 &= \frac{a+1}{(a+7)(a-3)} + \frac{a+6}{a+7} \\
 &= \frac{(a+1)+(a+6)(a-3)}{(a+7)(a-3)} \\
 &= \frac{a+1+a^2-3a+6a-18}{(a+7)(a-3)} = \frac{a^2+4a-17}{(a+7)(a-3)}
 \end{aligned}$$

