SHARP

Worksheet 7 Memorandum: Functions and Graphs

(Parabolas, Hyperbolas and Exponential Graphs)

Grade 11 Technical Maths

1. Give the standard formula for each of these graphs:

a) parabola → $y = ax^2 + bx + c$

b) exponential graph → $y = a.b^x + q$

c) hyperbola → $y = \frac{a}{x} + q$

d) straight line → $y = mx + c$

2. a) $a(x) = (x-1)^2 + 5$

i) y-intercept

$y = (0-1)^2 + 5$

$y = 6$

∴ (0; 6)

ii) x-intercept(s)

$0 = (x-1)^2 + 5$

$0 = x^2 - 2x + 1 + 5$

$0 = x^2 - 2x + 6$

Not possible to solve. No x-intercepts.

iii) turning point

$x = -\frac{b}{2a}$

$x = \frac{2}{2(1)} = 1$

$y = (1-1)^2 + 5 = 5$

TP = (1; 5)

iv) axes of symmetry

N/A

v) domain → $-\infty < x < \infty$

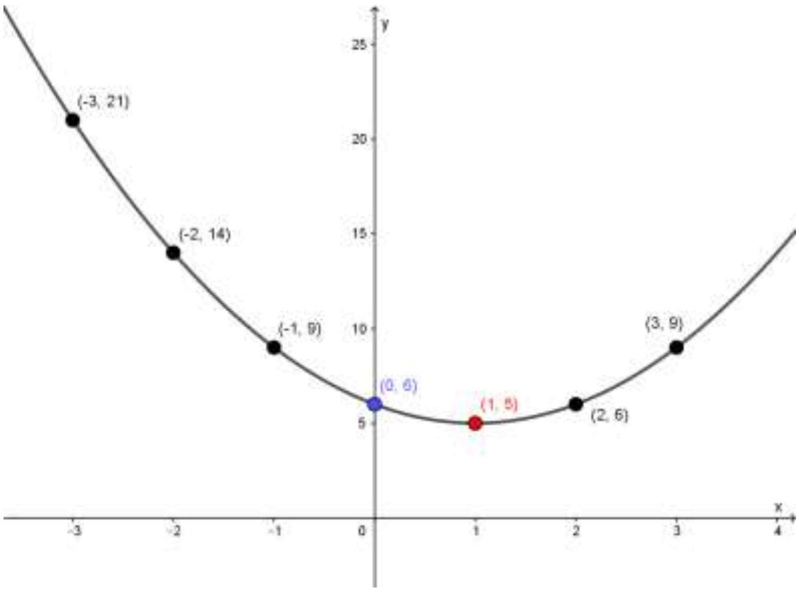
range → $5 < y < \infty$

vi) a table with points for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	21	14	9	6	5	6	9

vii) asymptotes N/A

viii)



b) $b(x) = -2(x-2)^2 + 3$

i) y-intercept

$y = -2(0-2)^2 + 3$

$y = -5$

ii) x-intercept(s)

$0 = -2(x-2)^2 + 3$

$0 = -2(x^2 - 4x + 4) + 3$

$0 = -2x^2 + 8x - 8 + 3$

$0 = -2x^2 + 8x - 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-2)(-5)}}{2(-2)}$

$x = \frac{4 \mp \sqrt{6}}{2}$

$x = 0.78 \quad OR \quad x = 3.22$

(0.78; 0) and (3.22; 0)

iii) turning point

$x = -\frac{b}{2a}$

$x = \frac{-8}{2(-2)}$

$x = 2$

$y = -2(2-2)^2 + 3$

$y = 3$

TP = (2; 3)

iv) axes of symmetry – N/A

v) domain → $-\infty < x < \infty$

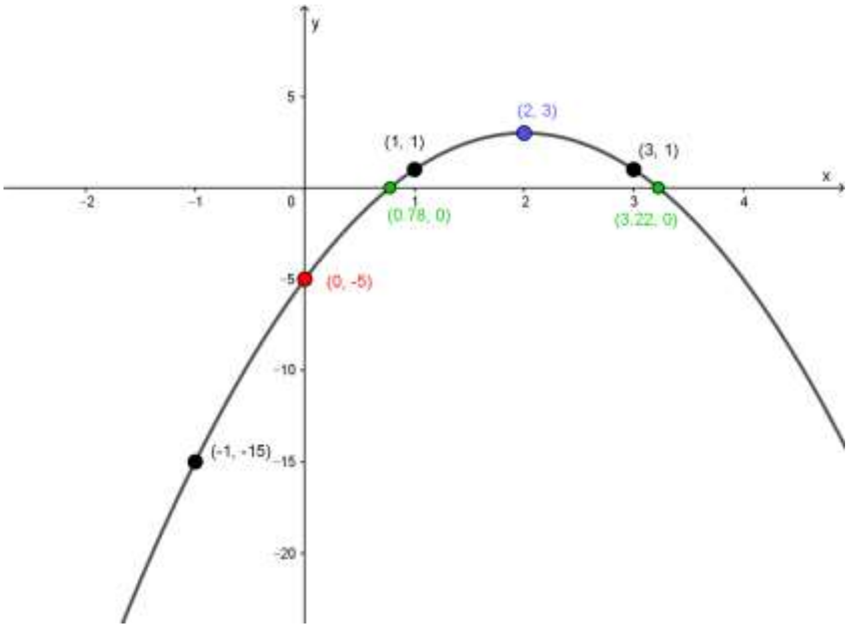
range → $-\infty < y < 3$

vi) a table with points for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
y	-47	-29	-15	-5	1	3	1

vii) asymptotes – N/A

viii)



c) $c(x) = 2x^2 - 12x + 9$

i) y-intercept

$y = 2(0)^2 - 12(0) + 9$

$y = 9$

(0; 9)

ii) x-intercept(s)

$0 = 2x^2 - 12x + 9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(2)(9)}}{2(2)}$

$x = \frac{6 \pm 3\sqrt{2}}{2}$

$x = 5.12 \quad OR \quad x = 0.88$

(5.12; 0) and (0.88; 0)

iii) turning point

$x = -\frac{b}{2a}$

$x = -\frac{-12}{2(2)} = 3$

$y = 2(3)^2 - 12(3) + 9$

$y = -9$

(3; -9)

iv) axes of symmetry – N/A

v) domain → $-\infty < x < \infty$

range → $-9 < y < \infty$

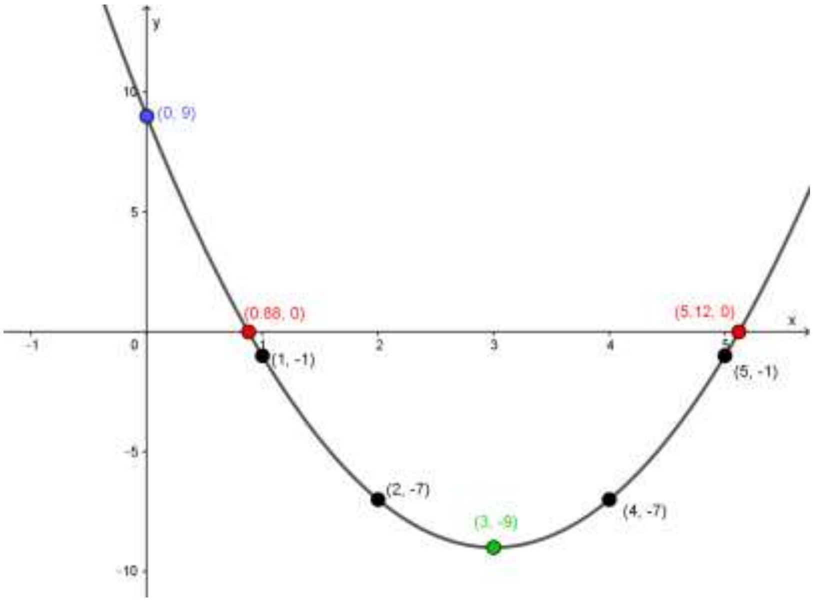
vi)

x	-3	-2	-1	0	1	2	3	4*	5*
y	63	41	23	9	-1	-7	-9	-7	-1

* additional points added to show the symmetry of the graph

vii) asymptotes – N/A

viii)



d) $d(x) = -x^2 + 7x - 6$

i) y-intercept

$y = -(0)^2 + 7(0) - 6$

$y = -6$

∴ (0; 6)

ii) x-intercept(s)

$0 = -x^2 + 7x - 6$

$0 = x^2 - 7x + 6x$

$0 = (x - 1)(x - 6)$

$x = 1 \quad OR \quad x = 6$

(1; 0) and (6; 0)

iii) turning points

$x = -\frac{b}{2a}$

$x = -\frac{7}{2(-1)}$

$x = 3\frac{1}{2}$

$y = -\left(3\frac{1}{2}\right)^2 + 7\left(3\frac{1}{2}\right) - 6$

$y = 6\frac{1}{4}$

iv) axes of symmetry – N/A

v) domain → $-\infty < x < \infty$

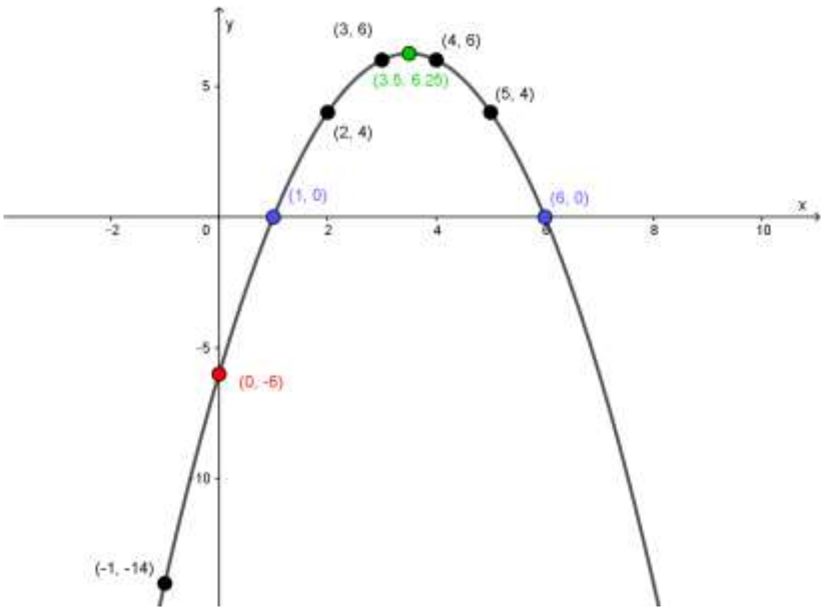
range →

vii) asymptotes – N/A

vi)

X	-3	-2	-1	0	1	2	3	4*	5*
y	-36	-24	-14	-6	0	4	6	6	4

viii)



e) $e(x) = \frac{-3}{x} + 4$

i) y-intercept

$y = -\frac{3}{0} + 4$

Not possible, x = 0 is an

Asymptote

ii) x-intercept(s)

$0 = -\frac{3}{x} + 4$

$-4 = -\frac{3}{x}$

$-4x = -3 \qquad \therefore x = \frac{3}{4} \qquad \left(\frac{3}{4}; 0\right)$

iii) turning points

N/A

iv) axes of symmetry

$m = +or - 1$ Subs in (0; 4)

$4 = +(0) + c$ OR $4 = -(0) + c$

$c = 4$ $c = 4$

$\therefore y = x + 4$ $\therefore y = -x + 4$

v) domain $\rightarrow x < 0 \; and \; x > 0$

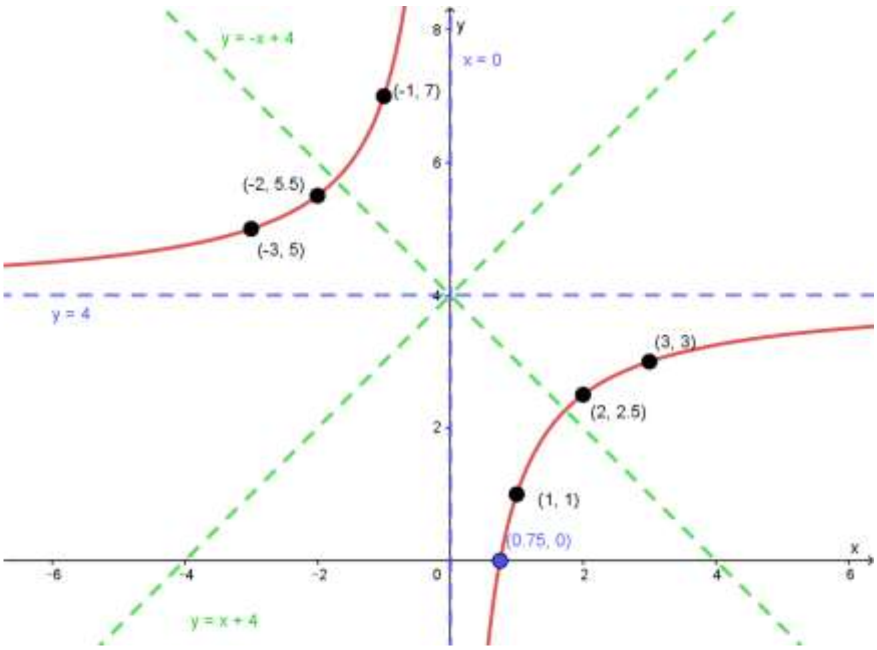
range $\rightarrow y < 4 \; and \; y > 4$

vi)

x	-3	-2	-1	0	1	2	3
y	5	5.5	7	asymptote	1	2.5	3

vii) asymptotes $\rightarrow x = 0, and \; y = 4$

viii)



f) $f(x) = \frac{5}{x} - 9$

i) y-intercept

$y = \frac{5}{0} - 9$

No y-intercept

X = 0 is an asymptote

ii) x-intercept(s)

$0 = \frac{5}{x} - 9$

$9 = \frac{5}{x}$

$9x = 5$

$x = \frac{5}{9}$

iii) turning points

N/A

iv) axes of symmetry

$m = +or - 1$

$-9 = +(0) + c$

$c = -9$

$\therefore y = x - 9$

Subs in (0; -9)

$-9 = -(0) + c$

$c = -9$

$\therefore y = -x - 9$

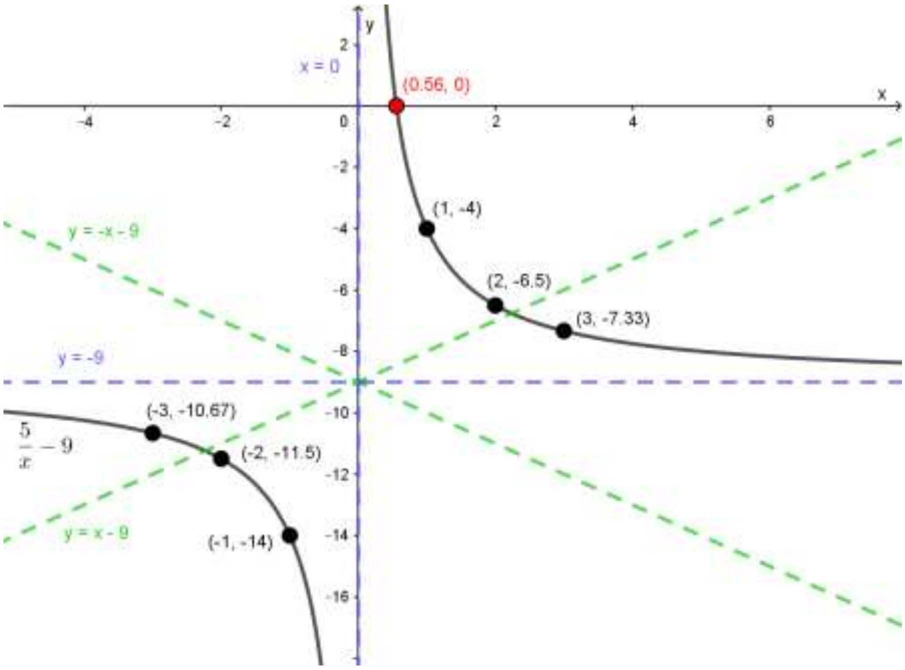
v) domain $\rightarrow x < 0, and\ x > 0$

range $\rightarrow y < -9\ and\ y > -9$

vi)

x	-3	-2	-1	0	1	2	3
y	$-10\frac{2}{3}$	$-11\frac{1}{2}$	-14	Asymptote	-4	$-6\frac{1}{2}$	$-7\frac{1}{3}$

vii) asymptotes $\rightarrow x = 0 \quad and\ y = -9$



viii)

g) $g(x) = \frac{1}{x} + 2$

i) y-intercept

$y = \frac{1}{0} + 2$

No y-intercept

x = 0 is an asymptote

ii) x-intercept(s)

$0 = \frac{1}{x} + 2$

$-2 = \frac{1}{x}$

$-2x = 1$

$x = -\frac{1}{2}$ $\left(-\frac{1}{2}; 0\right)$

iii) turning points

N/A

iv) axes of symmetry

$m = +or - 1$ | Subs in (0; 2)

$2 = +(0) + c$ | $2 = -(0) + c$

$c = 2$ | $c = 2$

$\therefore y = x + 2$ | $\therefore y = -x + 2$

v) domain $\rightarrow x < 0\ and\ x > 0$

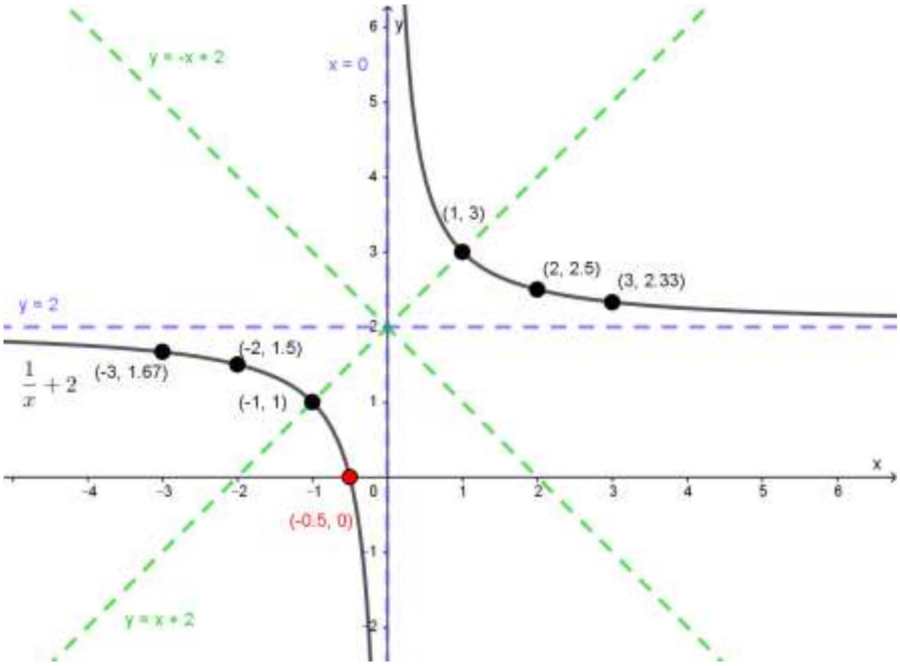
range $\rightarrow y < 2\ and\ y > 2$

vi)

x	-3	-2	-1	0	1	2	3
y	$1\frac{2}{3}$	$1\frac{1}{2}$	1	asymptote	3	$2\frac{1}{2}$	$2\frac{1}{3}$

vii) asymptotes $\rightarrow x = 0 \quad and\ y = 2$

viii)



h) $h(x) = \frac{-4}{x} - 3$

i) y-intercept

$y = \frac{-4}{0} - 3$

No y-intercept

x = 0 is an asymptote

(can't divide by zero)

ii) x-intercept(s)

$0 = -\frac{4}{x} - 3$

$3 = -\frac{4}{x}$

$3x = -4$

$x = -\frac{4}{3}$ $\left(-\frac{4}{3}, 0\right)$

iii) turning points

N/A

iv) axes of symmetry

$m = +or - 1$ | Subs in (0; -3)

$-3 = +(0) + c$ | $-3 = -(0) + c$

$c = -3$ | $c = -3$

$\therefore y = x - 3$ | $\therefore y = -x - 3$

v) domain $\rightarrow x < 0 \; and \; x > 0$

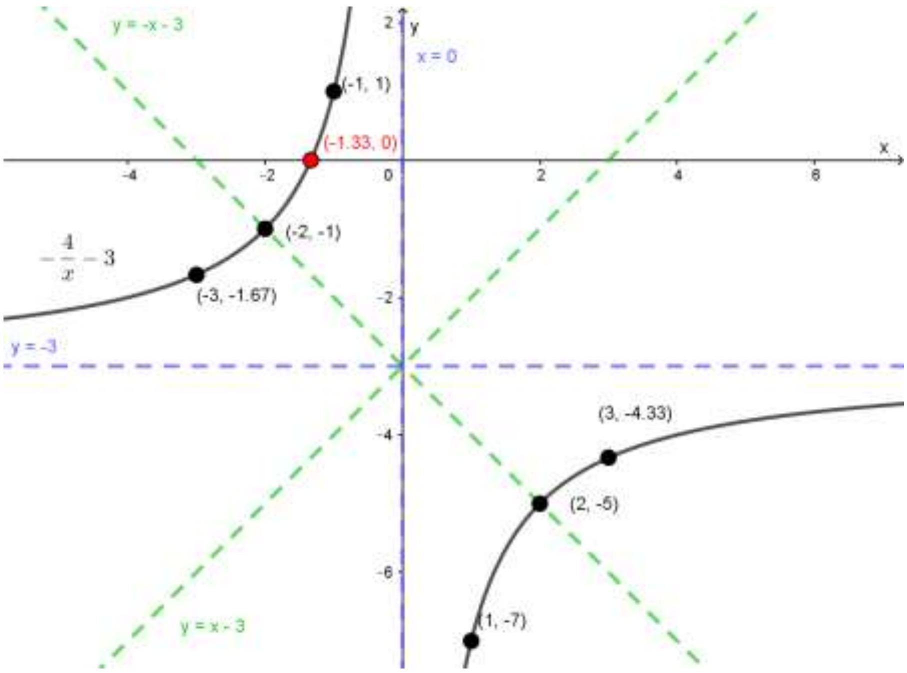
range $\rightarrow y < -3 \quad and \; y > -3$

vi)

x	-3	-2	-1	0	1	2	3
y	$-1\frac{2}{3}$	-1	1	asymptote	-7	-5	$-4\frac{1}{3}$

vii) asymptotes $x = 0 \quad and \; y = -3$

viii)



i) $i(x) = 3.2^x + 4$

i) y-intercept

$y = 3.2^0 + 4$

$y = 7$

ii) x-intercept(s)

$0 = 3.2^x + 4$

$-4 = 3.2^x$

A positive base cannot have a negative answer, therefore there is no x-intercept.

iii) turning points

N/A

iv) axes of symmetry

N/A

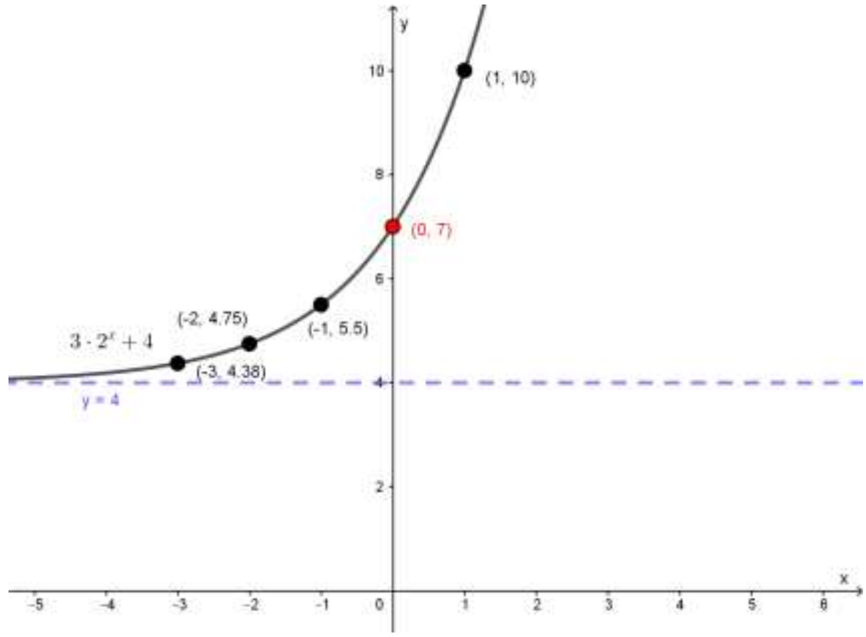
v) domain → $-\infty < x < \infty$ range → $y > 4$

vi)

x	-3	-2	-1	0	1	2	3
y	4.375	4.75	5.5	7	10	16	28

vii) asymptotes → $y = 4$

viii)



j) $j(x) = -2.5^x - 1$

i) y-intercept

$y = -2.5^0 - 1$

$y = -3$

(0; -3)

ii) x-intercept(s)

$0 = -2.5^x - 1$

$1 = -2.5^x$

No x-intercept (as in i) above).

iii) turning points

N/A

iv) axes of symmetry

N/A

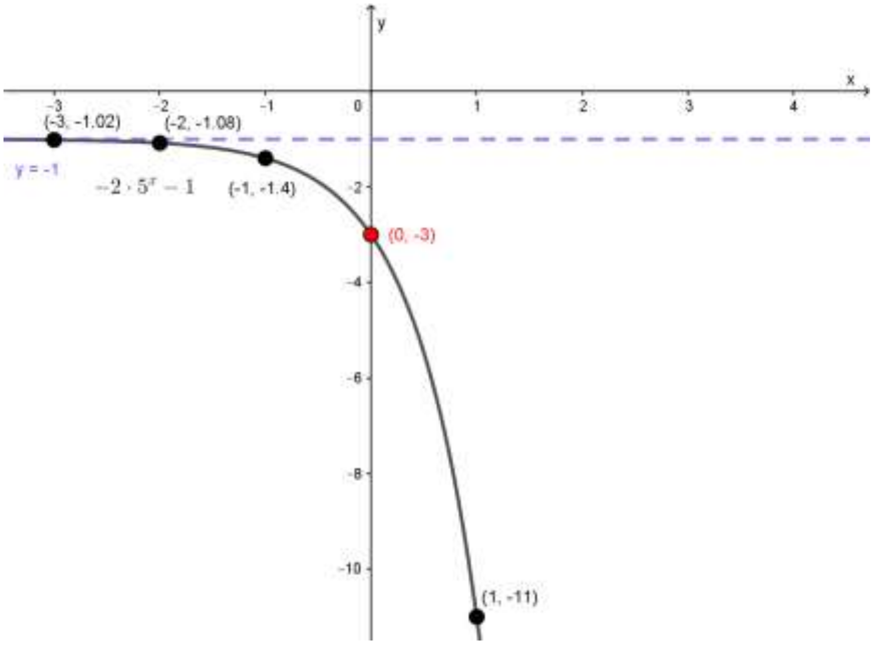
v) domain → $-\infty < x < \infty$ range → $y < -1$

vi)

x	-3	-2	-1	0	1	2	3
y	-1.016	-1.08	-1.4	-3	-11	-51	-251

vii) asymptotes $\rightarrow y = -1$

viii)



3. a) A parabola with turning point (1; 5) and intersecting the origin.

(1; 5) = (p; q) in $y = a(x-p)^2 + q$

$\therefore y = a(x-1)^2 + 5$ Subs in the second point (0; 0)

$\therefore 0 = a(0-1)^2 + 5$

$\therefore -5 = a$

$\therefore$ The equation is: $y = -5(x-1)^2 + 5$

b) A parabola with x-intercepts at x = 3 and x = -4 and y-intercept at y = 5.

$x_1 = 3$ *and* $x_2 = -4$ in $y = a(x - x_1)(x - x_2)$

$\therefore y = a(x-3)(x+4)$ Subs in the second point (0; 5)

$\therefore 5 = a(0-3)(0+4)$

$\therefore 5 = -12a$

$\therefore a = -\frac{5}{12}$

$\therefore$ The equation is: $y = -\frac{5}{12}(x-3)(x+4) = -\frac{5}{12}(x^2 + x - 12) = -\frac{5}{12}x^2 - \frac{5}{12}x + 5$

c) A hyperbola with an asymptote at y = 2 and point (-1; 3) on the graph

$y = 2$ means that $q = 2$ in $y = \frac{a}{x} + q$

$\therefore y = \frac{a}{x} + 2$ Subs in (-1; 3)

$\therefore 3 = \frac{a}{-1} + 2$

$\therefore 1 = \frac{a}{-1}$

$\therefore a = -1$

$\therefore$ The equation is: $y = \frac{-1}{x} + 2$

d) An exponential graph with the points (1; 13), (3; 685) and (4; 4801)

Subs in the various points:

$13 = ab^1 + q$ $685 = ab^3 + q$ $4801 = ab^4 + q$

$q = 13 - ab$

$685 - 13 = ab^3 - ab$ $4801 - 685 = ab^4 - ab^3$

$672 = ab(b^2 - 1)$ …(1) $4116 = ab^3(b - 1)$ …(2)

Divide (2) by (1)

$\therefore \frac{4116}{672} = \frac{ab^3(b-1)}{ab(b-1)(b+1)}$

$\therefore \frac{49}{8} = \frac{b^2}{b+1}$

$\therefore 49(b + 1) = 8b^2$

$\therefore 8b^2 - 49b - 49 = 0$

$\therefore (8b + 7)(b - 7) = 0$

$\therefore b = -\frac{7}{8}$ $or\ b = 7$

Since an exponential graph cannot have a negative base $b = -\frac{7}{8}$ is N/A

Now $q = 13 - a(7)$ and $685 = a(7)^3 + q$

$\therefore q = 685 - 343\ a$

Let q = q then

$13 - 7a = 685 - 343a$

$336a = 672$

$\therefore a = 2$

And finally, $q = 13 - (2)(7) = -1$

The final equation is $\therefore y = 2.7^x - 1$

e) We have the two x-intercepts: (-4; 0) and (2; 0) and we substitute these into

$y = a(x - x_1)(x - x_2)$

$\therefore y = a(x + 4)(x - 2)$ Subs in the other point (1; -10)

$\therefore -10 = a(1 + 4)(1 - 2)$

$\therefore -10 = -5a$

$\therefore a = 2$

Subs back in:

$\therefore y = 2(x + 4)(x - 2)$

$\therefore y = 2(x^2 + 2x - 8)$

$\therefore y = 2x^2 + 4x - 16$

f) We have a parabola with the y-intercept (0; 9) and the two points (-2; 5) and (1; 2)

Substitute in the y-intercept:

$y = ax^2 + bx + c$ c = 9

$y = ax^2 + bx + 9$

Subs in (-2; 5)

$5 = a(-2)^2 + b(-2) + 9$

$\therefore -4 = 4a - 2b$

$\therefore -2 = 2a - b$

$\therefore b = 2a + 2$ ... (1)

Subs in (1;2)

$\therefore 2 = a(1)^2 + b(1) + 9$

$\therefore\ -7 = a + b$ ... (2)

Subs (1) into (2)

$\therefore\ -7 = a + 2a + 2$

$\therefore\ -9 = 3a$

$\therefore a = \ -3$

Subs back into (1)

$\therefore b = 2(-3) + 2$

$\therefore b = \ -4$

$\therefore$ The equation is: $y = \ -3x^2 - 4x + 9$

g) The graph is a hyperbola with an asymptote at y = -3 and the point (2; -2)

This means q = -3

Subs into the formula $y = \frac{a}{x} + q$

$\therefore y = \frac{a}{x} - 3$

Subs in the point (2; -2)

$\therefore\ -2 = \frac{a}{2} - 3$

$\therefore 1 = \frac{a}{2}$

$\therefore a = 2$

$\therefore$ The equation is: $y = \frac{2}{x} - 3$

h) The turning point is (3; 8) and one of the x-intercepts is (1; 0)

Subs into the turning point parabola formula: $y = a(x - p)^2 + q$

$\therefore y = a(x - 3)^2 + 8$

Subs in the second point (1; 0)

$\therefore 0 = a(1 - 3)^2 + 8$

$\therefore -8 = 4a$

$\therefore a = -2$

Subs back in:

$\therefore y = -2(x - 3)^2 + 8$ And simplify

$\therefore y = -2(x^2 - 6x + 9) + 8$

$\therefore y = -2x^2 + 12x - 18 + 8$

$\therefore$ The equation is: $y = -2x^2 + 12x - 10$

i) The graph is a hyperbola with asymptote at y = 2 and a point (-1; 4)

This means q = 2

Subs into the formula $y = \frac{a}{x} + q$

$\therefore y = \frac{a}{x} + 2$

Subs in the point (-1; 4)

$\therefore 4 = \frac{a}{-1} + 2$

$\therefore 2 = \frac{a}{-1}$

$\therefore a = -2$

$\therefore$ The equation is: $y = \frac{-2}{x} + 2$

j) The graph is an exponential graph with asymptote at y = -1, y-intercept at (0; -3) and a second point at (-1; -1.4)

This means that q = -1

Subs into the formula $y = a.b^x + q$

$\therefore y = a.b^x - 1$

Subs in the y-intercept:

$\therefore\ -3 = a.b^0 - 1$

$\therefore\ -2 = a(1)$

$\therefore a = \ -2$

$\therefore y = -2.b^x - 1$

Subs in the second point (-1; -1.4):

$\therefore\ -1.4 = \ -2b^{-1} - 1$

$\therefore -0.4 = \frac{-2}{5} = \ -\frac{2}{b}$

$\therefore b = 5$

$\therefore$ The equation is: $y = \ -2.5^x - 1$

4. a) Straight Line:

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Subs in (-1; 0) and (2; -6)

$\therefore m = \frac{0-(-6)}{-1-2}$

$\therefore m = \ -2$

Now we subs back into: $y = mx + c$

$\therefore y = \ -2x + c$ Subs in point (-1; 0)

$\therefore 0 = \ -2(-1) + c$

$\therefore c = \ -2$

$\therefore y = \ -2x - 2$

For the parabola $f(x) = ax^2 + bx + c$ we have the y-intercept at (0; -8)

So $y = ax^2 + bx - 8$

Subs in (-1; 0)

$\therefore 0 = a(-1)^2 + b(-1) - 8$

$\therefore 8 = a - b$

$\therefore a = 8 + b$ ... (1)

Subs in (2; -6)

$\therefore -6 = a(2)^2 + b(2) - 8$

$\therefore 2 = 4a + 2b$ ... (2)

Subs (1) into (2)

$\therefore 2 = 4(8 + b) + 2b$

$\therefore 2 = 32 + 4b + 2b$

$\therefore -30 = 6b$

$\therefore b = -5$

Subs back into (1)

$\therefore a = 8 + (-5)$

$\therefore a = 3$

$\therefore$ The equation for the parabola is: $y = 3x^2 - 5x - 8$

b) $y = 3x^2 - 5x - 8$ Let y = 0

$\therefore 0 = 3x^2 - 5x - 8$

$\therefore 0 = (3x - 8)(x + 1)$

$\therefore 3x - 8 = 0 \quad OR \quad x + 1 = 0$

$\therefore x = \frac{8}{3} = 2\frac{2}{3}$ OR $x = -1$ $\therefore \left(2\frac{2}{3}; 0\right)$ is the second x-intercept

c) The y-intercept of the straight line is (0; -2)

The y-intercept of the parabola is (0; -8)

$\therefore$ the distance between the two is 6 units. (8 – 2)

d) Find the turning point of the parabola.

$x = -\frac{b}{2a}$

$\therefore x = \frac{-(-5)}{2(3)}$

$\therefore x = \frac{5}{6}$

Subs back in to find y:

$\therefore y = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) - 8$

$\therefore y = -10\frac{1}{12}$

The turning point is $\therefore \left(\frac{5}{6};\ -10\frac{1}{12}\right)$

e) If a second straight line is drawn from the point (0; -8) and the point (2; -6), determine the equation of this straight line, $h(x)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Subs in (0; -8) and (2; -6)

$\therefore m = \frac{-8-(-6)}{0-2}$

$\therefore m = 1$

Subs back into the equation $y = mx + c$:

$\therefore y = 1x + c$

$c = -8$ (the y-intercept)

$\therefore$ The equation of the line is: $y = x - 8$

5. a) $f(x) = \frac{a}{x} + q$ and y = 2 is an asymptote so that q = 2

$\therefore y = \frac{a}{x} + 2$ Subs in the point A (-1; -1)

$\therefore\ -1 = \frac{a}{-1} + 2$

$\therefore\ -3 = \frac{a}{-1}$

$\therefore a = 3$ The equation for f is $\therefore f(x) = \frac{3}{x} + 2$

For $h(x) = b^x + q$ Subs in C (0; 2)

$\therefore 2 = b^0 + q$

$\therefore 2 = 1 + q$

$\therefore q = 1$

$\therefore y = b^x + 1$ Subs in F (1; 4)

$\therefore 4 = b^1 + 1$

$\therefore 3 = b$ $\therefore$ The equation of h(x) is given by $h(x) = 3^x + 1$

b) $\therefore m = + or - 1$ Subs in (0; 2)

$\therefore 2 = +1(0) + c$ OR $\therefore 2 = -1(0) + c$

$\therefore c = 2$ $\therefore c = 2$

The equations for the axes of symmetry are: $\therefore y = x + 2$ and $y = -x + 2$

c) f(x) domain: $x < 0 \; and \; x > 0$

range: $y < 2 \; and \; y > 2$

h(x) domain: $-\infty < x < \infty$

range: $y > 1$