> THERE IS A DIFFERENCE BETWEEN NOT KNOWING, AND NOT KNOWING YET.
>
> SHEILA TOBIAS

Grade 10 Maths

Algebra

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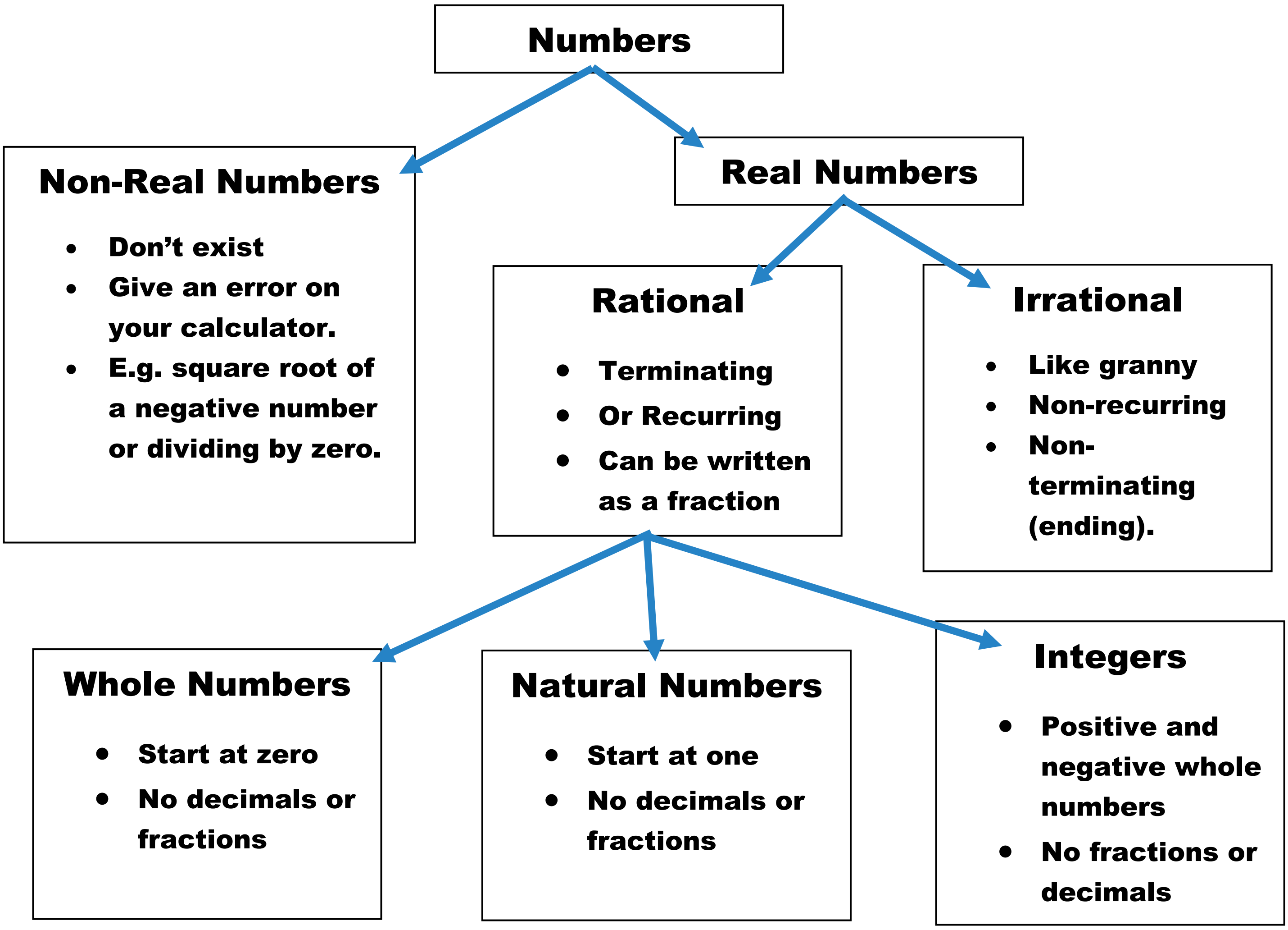
Numbers

Before you can start anything in mathematics - you need to know how numbers work. So here are the basics... (There is a summary on the next page - if you prefer flow diagrams)

Numbers are broken down into Real and Non-Real numbers

- Non-Real ($\mathbb{R}'$) numbers are numbers that don't exist (imaginary or complex numbers) - for example: the square-root of a negative number like $\sqrt{-3}$. These numbers will give an error on the calculator (On the SHARP EL-W535SA or the EL-W506T, the calculator will say - Error 2 – Calculation).
- Real Numbers ($\mathbb{R}$) are numbers that do exist. They are broken down into Rational and Irrational numbers
  - Irrational Numbers ($\mathbb{Q}'$) are numbers that don't make sense. I like to pretend they are like my grandmother - she likes to talk and talk but she doesn't make any sense. In the same way, irrational numbers go on and on (called non-terminating) and they don't have a pattern or make sense (called non-recurring).

    An example of an irrational number is $\sqrt{7} = 2{,}645751311 \ldots$ - do you see that it doesn't have a pattern and it doesn't stop?
  - Rational numbers ($\mathbb{Q}$) on the other hand are either recurring (that means they have a pattern) for example 0,333333... or they are terminating (that means that they end). For example, 2,25 or 3. Rational numbers can be written as a fraction. Rational numbers are also broken down into:
    - Whole numbers ($\mathbb{N}_0$) - these are numbers that start at zero (0) and count up in wholes, e.g. 0, 1, 2, 3... etc. They **DO NOT** have decimals or fractions. A nice way to remember whole numbers start with zero is that there is an **O** in wh**O**le.
    - Natural numbers ($\mathbb{N}$) - these are counting numbers which means that they start at one (1) and count up in wholes, e.g. 1, 2, 3... etc. They **DO NOT** have decimals or fractions. A nice way to remember that natural numbers start with one is that there is a **1** in Natura**1**.
    - Integers ($\mathbb{Z}$) are positive and negative whole numbers, e.g. ... -3, -2, -1, 0, 1, 2, 3... etc. They also **DO NOT** have fractions or decimals.



⁎ Remember to learn both the names and the symbols - because a question can use either the symbols or the names.

Activity 1

1. Tick the correct columns in the table below:

Number	Real	Non-Real	Rational	Irrational	Whole	Natural	Integer
0							
π							
-1							
-1,45							
$\sqrt{3}$							
$\sqrt{-8}$							
$\frac{13}{16}$							
$\left(\sqrt{-5}\right)^2$							

2. Given the equation: $0 = (x + 3)(2x - 5)(x^2 + 6)$

   Solve for x when x is

   a) real

   b) non-real

   c) an integer

Rounding Off

Part of knowing your numbers is remembering the facts about rounding off - so here is some quick revision for you:

When you round off check how many places you want to round off to then look at the number next to the last number you will have once you have rounded off →

e.g. round off to three decimal places: 0, 123456

place rounding off to → (3)

(4) ← number you use to decide whether the number next door goes up or stays the same,

If the number is 5 or bigger than 5 (that is 5, 6, 7, 8, or 9) then the number next door goes up.

If the number is smaller than 5, (that is 4, 3, 2, 1, or 0) the number next door stays the same.

However, remember that when you are working with numbers representing people or things that cannot be a fraction, be careful how you round up - think carefully about what the question is asking you for. For example, if you work out you have to cater for 43,2 people you would have to round UP to 44 because you cannot not cater for the 0,2 part of a person.
Also - when they say round off to the nearest unit - remember that a unit is a whole number.

Activity 2

1. Round off the following numbers to 2 decimal places:

   a) 0,135 b) 2,369

   c) 5,895 d) 4,351

2. Would you round the following up or down?

   a) The average number of dogs per house is 2,3

   b) The average number of people who live in a square meter in china is 11,6

   c) Your bill at the supermarket comes to R11,97

   d) You worked for 2,13 hours

   e) The average number of children in a class is 34,3

Surds between Integers

A surd is a root of a number that cannot be simplified any further, for example $\sqrt{3}$. It will not give you an integer as an answer, but will give you an irrational number.

It is always good to estimate the answer of a surd so that you know that your calculator computation was correct – it's a good way of checking yourself.
The easiest way to check is to figure out where the surd lies on the number line.
$\sqrt{3} = 1{,}732 \ldots$ which shows us that it lies between 1 and 2.

But, to work out where it lies without using a calculator you do this:
We count up in perfect squares

$$\rightarrow 1^2 = 1; \quad 2^2 = 4; \quad 3^2 = 9; \quad 4^2 = 16; \quad 5^2 = 25 \ \ldots etc.$$

From this we can see that 3 lies between 1 and 4 on the perfect square line. Now we reverse the squares to see that $\sqrt{3}$ lies between 1 and 2 (the square-roots of the perfect square).

Let's practice this again. Between which two integers does $\sqrt{18}$ lie?

Count up in perfect squares: 1, 4, 9, 16, 25, 36...

We can see that 18 falls between 16 and 25.

Now we take the square-root of 16 and 25 to give us 4 and 5, so we know that $\sqrt{18}$ lies between 4 and 5.

We can write this mathematically as: $4 < \sqrt{18} < 5$

Remember that if the question had said between which two numbers does $-\sqrt{18}$ our answer would lie between -5 and -4.

Activity 3

1. Between which two integers do the following surds lie?

a)	$\sqrt{56}$	b)	$-\sqrt{12}$
c)	$-\sqrt{78}$	d)	$\sqrt{15}$
e)	$-\sqrt{43}$	f)	$-\sqrt{29}$
g)	$\sqrt{99}$	h)	$\sqrt{8}$

2*. Between which two integers do the following surds lie?

a)	$\sqrt[3]{6}$	b)	$\sqrt[3]{-45}$
c)	$\sqrt[3]{78}$	d)	$-\sqrt[3]{130}$

Exponents

Base $^{\text{exponent}}$ } power

First, we will revise the basic laws of exponents.

- Law 1: If you have the same bases and you multiply the powers, you add the exponents together
  - $a^m \times a^n = a^{m+n}$

- Law 2: If you have the same bases and you divide the powers, you subtract the exponents.
  - $a^m \div a^n = a^{m-n}$

- Law 3: If you raise an exponent to another exponent you multiply the two exponents together.
  - $(a^n)^m = a^{n \times m}$
  - Remember that all the powers in the bracket are affected by the outside exponent: $(a^n b^m c)^p = a^{np} b^{mp} c^p$

- Law 4: If you take a root of a power the exponent inside is divided by the outside root value.
  - $\sqrt[d]{a^c} = a^{\frac{c}{d}}$
  - A nice way to remember this rule is that cats live inside the house (c), dogs live outside the house (d) and cats are more important that dogs 😉 , so they go on the top of the fraction.

- Law 5: Anything to the power of zero is one.
  - $a^0 = 1$

- Law 6: If your power has a negative exponent the power goes under 1 and the exponent becomes positive (it also works in reverse – if you have a power under 1 with a negative exponent the power goes back on top of the one, and the exponent becomes positive).
    - $a^{-n} = \frac{1}{a^n}$ OR $\frac{1}{a^{-n}} = \frac{a^n}{1}$ (but you can also write it as a^n).
    - A good way to remember this is to think of the exponent as the girl / boyfriend of the base, when the exponent is negative the girl / boyfriend is sad or negative and wants to go downstairs. Once the power is downstairs it makes the girl / boyfriend happy or positive. This also works in reverse, if the power is downstairs and the exponent is negative, then the girl / boyfriend wants to go upstairs to become happy and positive.

You need to learn these exponent laws as they will help you later on with other work that you will cover all the way to matric. Learn them now and you won't be confused later 😉

There are two things that you need to be able to do with exponents:

- Simplify
- And solve

To simplify: use your exponents to get to the simplest possible form of an answer (make sure that all of your exponents are positive).

Here are two examples. Simplify:

1. $\frac{a^2b^3}{c^3} \times \frac{a^3c^5}{b^7}$ First multiply top with top and bottom with bottom (When you multiply same bases you add the exponents)

$= \frac{a^{2+3}b^3c^5}{c^3b^7}$ Then divide the like bases (when you divide the same bases you subtract the exponents.)

$= a^5\, b^{3-7}c^{5-3}$

$= a^5b^{-4}\, c^2$ Finally, write your answer with positive exponents.

$= \frac{a^5c^2}{b^4}$ And now we can't simplify anymore so this is our final answer.

2. $\frac{49^{x+2}875^{x}}{35^{3x-1}}$

First break down the numbers into their prime factors.

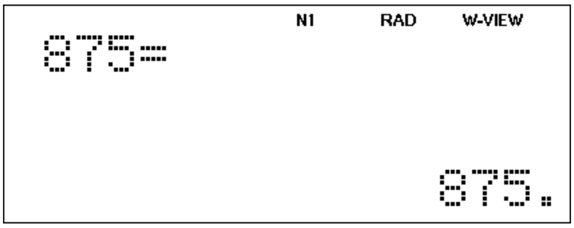
$49 = 7 \times 7 = 7^2$

$875 = 125 \times 7 = 5 \times 5 \times 5 \times 7 = 5^3 \times 7$

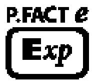
$35 = 5 \times 7$

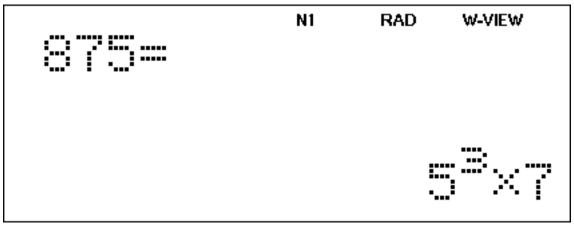
There is a short way to do this:

Type the number into your Sharp scientific calculator (the EL-W535SA or the EL-W506T) then press 



Then press 





$$= \frac{(7^2)^{x+2}\,(5^3 \times 7)^x}{(5 \times 7)^{3x-1}}$$

Multiply out the exponents

$$= \frac{7^{2x+4}\,5^{3x}7^{x}}{5^{3x-1}7^{3x-1}}$$

Simplify by adding (when you multiply the bases) and subtract (when dividing).

$$= 7^{2x+4+x-(3x-1)}\;5^{3x-(3x-1)}$$

$$= 7^{3x+4-3x+1}\;5^{3x-3x+1}$$

$$= 7^5\;5^1$$

$$= 84\;035$$

Pay attention to how the question is asked.

If they ask for positive exponents, you don't need to give the multiplied answer out.

To solve: you will be given an equation and you need to work out what the value of x is.

There are three different ways of asking for x:

- x as an exponent
- x as a base
- x as an answer

Here are some examples:

Solve for x:

1. $3.2^x - 5 = 7$ — First take the number that is not attached to the power to the other side.

$3.2^x - 5 + 5 = 7 + 5$

$3.2^x = 12$ — Then divide by 3 (remember that 3.2^x means 3×2^x).

$2^x = 4$ — Now find the prime factors of 4 (you can do this on your calculator again).

$2^x = 2^2$ — Because the bases are the same, we know that the exponents must also be equal for the equals sign to be true, so:

$x = 2$

2. $5^{x+1} - \frac{5^{x+1}}{2} = 312\frac{1}{2}$ — Separate the 5^{x+1} into two separate bases.

$5^x5^1 - \frac{5^x5^1}{2} = 312\frac{1}{2}$ — Now take out the 5^x as a common factor

$5^x\left(5 - \frac{5}{2}\right) = 312\frac{1}{2}$ — Simplify inside the brackets.

$5^x\left(2\frac{1}{2}\right) = 312\frac{1}{2}$ — Divide both sides by $2\frac{1}{2}$

$5^x = 125$ — Prime factorise 125.

$5^x = 5^3$ — Drop the bases on both sides, because same bases means that the exponents are equal.

$\therefore x = 3$

3. $\frac{x^3}{4} - 4 = 2\frac{3}{4}$ First take the number that is not attached to the power across.

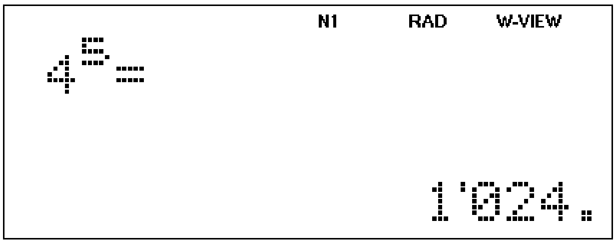
$\frac{x^3}{4} = 6\frac{3}{4}\ or\ \frac{27}{4}$ Now multiply out by the 4

$x^3 = 27$ Take the cube-root of both sides.

$\sqrt[3]{x^3} = \sqrt[3]{27}$ A cube-root of a cube cancels each other out.

$\therefore x = 3$

4. $4^5 = x$ Simply put the power into your calculator by pressing:



$\therefore x = 1\ 024$

Activity 4

1. Simplify the following:

a) $\frac{c^4d^2e}{f^2d^{-2}} \times \frac{(e^3f)^{-1}}{\sqrt[3]{d^{-3}c^6}}$

b) $\frac{1}{a^{-2}} \times \left(\frac{b}{a^3}\right)^{-1} \div \frac{b^2}{a^2}$

c) $\frac{x^{-\frac{1}{4}}y^3}{z^3} \times \frac{(yz)^{-3}}{\sqrt[4]{x^5}}$

d) $\frac{18^x.24^{x+1}}{36^{2x-1}}$

e) $\frac{21^{x+1}.63^{x-1}}{9^x.49^{2x}}$

f) $\frac{a^{-1}b^6}{c^{-6}d^4} \times \frac{ab^4c^5}{d^{-7}} \div \frac{(a^3b^5)^2}{c^{-1}d^3}$

g) $\frac{(ef^{-1})^4}{g^{-2}h^3} \times \frac{e^6f^4g^2}{h} \times \left(\frac{e}{f^{-2}}\right)^{-1}$

h) $\left(\frac{1}{x} + \frac{x}{y}\right)^{-2}$

i) $\left(\left(\frac{x}{y}\right)^{-2} + \left(\frac{x}{y} - \frac{y}{x}\right)^{-3}\right)^0$

j) $\frac{7^{x+1} \times 2^{2x-3}}{14^{x-5}}$

2. Solve for x:

a) $2^x - \frac{2^{x+1}}{5} = 0{,}3$

b) $2^{x+1} - \frac{2^{x+1}}{3} = 21\frac{1}{3}$

c) $10x^3 + 50 = 1\,300$

d) $\frac{4x^4}{3} - 5 = 103$

e) $6^{2\frac{1}{2}} = x$

f) $x = 4^{1\frac{1}{3}}$

g) $4^{x+2} - 4^x = \frac{15}{16}$

h) $4^x - \frac{4^x}{2} = 1$

i) $5^x + 3.5^x = 4$

j) $-\frac{1}{5}x^3 + \frac{3}{5} = -1$

Multiplying Algebraic Expressions

When you multiply a single term with a bracket (where there could be many terms) each term in the bracket is multiplied by that single term.

When you have a bracket with more than one term, being multiplied with a bracket that has more than one term you need to make sure that each term in the first bracket is multiplied by each term in the second bracket. An easy way to check that you are doing this is to draw arrows in one colour to each term in the second bracket, then do the next term in another colour and so on...

For example, multiply $2x - y$ by $3x^2 - 4xy + y^2$

$(2x - y)(3x^2 - 4xy + y^2)$

Start with the $2x$ and multiply each term in the second bracket with the $2x$ (arrows in blue). Then do the second term, $-y$ (arrows in green). Don't forget to multiply out the signs as well.

$= 6x^3 - 8x^2y + 2xy^2 - 3x^2y + 4xy^2 - y^3$

Now simplify your answer by adding all the like terms together.

$= 6x^3 - 11x^2y + 6xy^2 - y^3$

Activity 5

Multiply out and simplify the following expressions

a) $(3x-5)(x^2+4x-6)$

b) $\left(\frac{1}{2}y+2\right)(4y^2-6y+3)$

c) $\left(\frac{1}{4}x-1\right)\left(-x^2-\frac{1}{3}x+4\right)$

d) $(6x+4y)(2x^2+4xy+3)$

e) $(-4x+3)\left(\frac{1}{5}x^2+11x-6\right)$

f) $(-3a-2b)(-7a^2+8ab-b^2)$

g) $\left(\frac{3}{4}c^2-3d\right)\left(\frac{1}{2}c^2+cd+6d^2\right)$

h) $(5e+2f)\left(e^2-8ef+\frac{1}{3}f^2\right)$

i) $(-2g+5h)(6g-6gh+h)$

j) $\left(8k-\frac{4}{5}m\right)\left(2k^3+\frac{1}{4}k^2m-km\right)$

Factorising

Factorising is to take many terms in an expression and turn them into one term. Factorising will be used throughout the rest of your maths school career so make sure that you understand this section and that you spend time practicing it as well.

There are four steps to factorisation:

1. Look for the highest common factor (if there is one take it out).

   e.g. $2x^2-8$ → the value 2 is the highest common factor because it goes into both terms. When you take it out (in other words, you divide each term by 2 and leave the 2 on the outside of the brackets and the answers inside the brackets) and put it in the front you get $2(x^2-4)$.

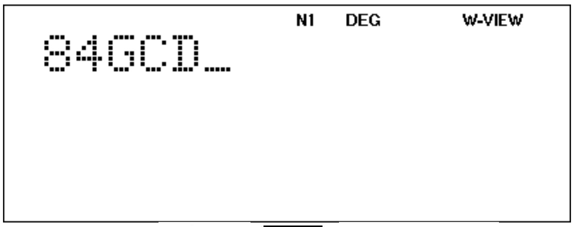
   Remember that if you multiply out the brackets you have just factorised it should give you your original equation or expression.

   On your Sharp EL-W535SA and EL-W506T there is a function that helps you find the highest common factor of any set of numbers.

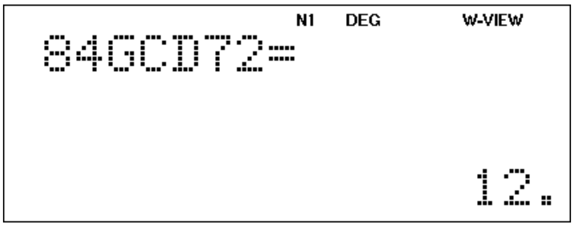
   Press HOME.

   Say for example we want to find the highest common factor of 84 and 72.

   Press 8 4 2nd F 2 (GCD ENG>)



Then press 



Our highest common factor is 12. (*GCD stands for Greatest Common Divisor which is the same as Highest Common Factor).

2. Now look at how many terms the expression has

a) If there are 2 terms it could be:

i) A difference of squares

To factorise $a^2 - b^2$ we say

$$\left(\sqrt{first\ term} - \sqrt{second\ term}\right)\left(\sqrt{first\ term} + \sqrt{second\ term}\right)$$

ii) A difference or sum of cubes

To factorise $a^3 \pm b^3$ we say:

$$\left(\sqrt[3]{first\ term} \pm \sqrt[3]{second\ term}\right)\left(\left(\sqrt[3]{first\ term}\right)^2 \mp \sqrt[3]{first\ term} \times \sqrt[3]{second\ term} + \left(\sqrt[3]{second\ term}\right)^2\right)$$

OR $(a \pm b)(a^2 \mp ab + b^2)$

iii) Note that you cannot factorise a sum of squares, i.e. something like this: $a^2 + b^2$. So you leave the expression as is, or make a note that it cannot be factorised. (Remember to check for that highest common factor though).

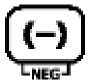
b) If there are three terms it could be a trinomial.

The easiest way to factorise a trinomial is to use your Sharp scientific calculator (both the EL-W535SA and the EL-W506T will work).

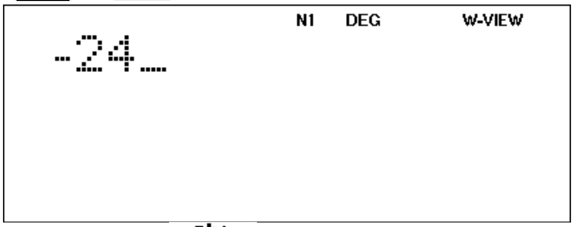
We will use the example: $x^2 - 5x - 24$

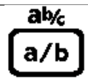
**Some theory before we work through the method:* *Remember after you have factorised you will have two brackets – if you multiply them out you will get two* x *terms which you add (whether the sign is a minus or a plus) together to give you the middle term. We are going to use this fact in our method.*

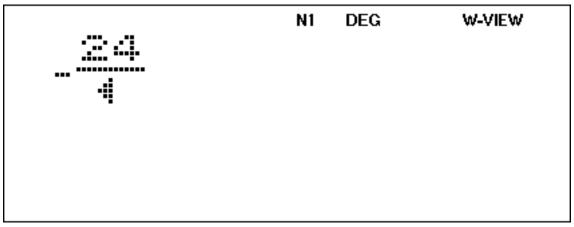
To find the factor pairs go to table mode by pressing  

Then enter your "c" or constant value (e.g. -24) by pressing 

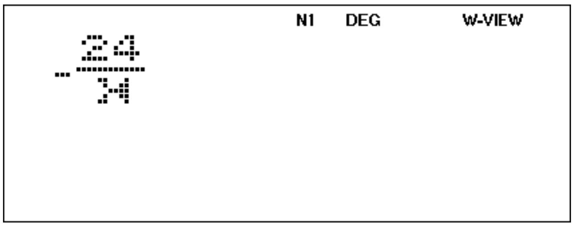
 



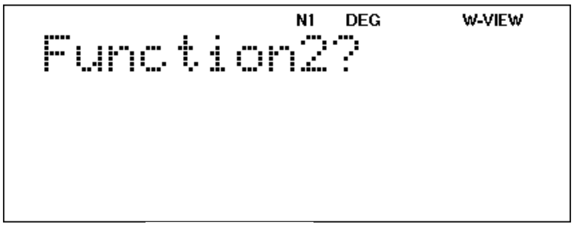
Then press  to put it at the top of a fraction.



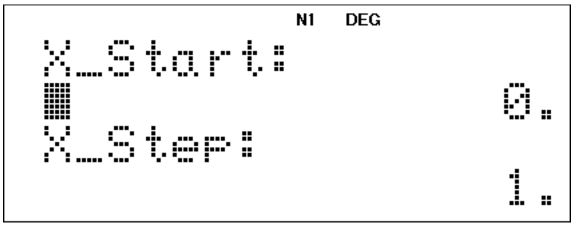
At the bottom of the fraction enter an X by pressing  twice.



Press the  button to go to the second function:

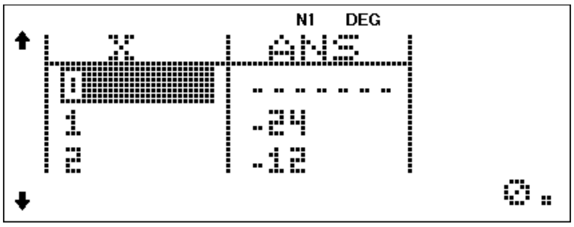


Press the  button to skip function 2.



Press the  button twice to leave the start at 0 and the steps as 1.

Now you will see a table with different factor pairs. Use your  and  arrow keys to scroll through the table.



Look at your factor pairs and add each set together, e.g. 1 + -24 = -23. -23 is not our middle term so we look at the next factor pair 2 and -12. When we add these together, we get -10, which is also not our middle term. Our next pair is 3 and -8 which gives us -5. We have found our factor pairs. We put these into our brackets, and remember to keep the sign:

$$(x+3)(x-8)$$

Remember, if the sign in front of your "c" or constant value is a "+" then the signs in both brackets are the same. Either both "+'s" or both "-'s". We decide this by checking the sign in front of the "b" or coefficient of x. If the sign in front of your "c" value is a negative, it means that your signs are different, and you need to check that you keep the correct signs for each factor.

If the the value of your "a" is not 1 or -1, then you need to follow the airplane method*. (You can use another method but I like this one because it uses similar steps to the one we've used above.)

Here is our example: $2x^2 - x - 6$.

Step 1: We write down our brackets and we put 2x into both brackets.

(Yes, we will sort out this extra 2 in a bit, just trust me for now.)

So, we have: $(2x+?)(2x-?)$

Step 2: Now we multiply our first and last term constants together (i.e. the "a" or "2" and the "c" or "-6". This gives us -12.

This part gives us our airplane wings.

$$2x^2 - x - 6.$$

Step 3: Now we look for the factors of -12 that give us -x.

This is the propeller.

$$2x^2 - x - 6.$$

We can use the table mode method that we used in the previous example to find these factors. The factors are -4 and +3.

Step 4: We put these factors into our brackets:

$(2x + 3)(2x - 4)$

Step 5: We can see that there is a common factor of 2 in our second bracket. So, we divide this bracket by 2. This is our landing gear:

$$(2x + 3)\frac{(2x - 4)}{2}$$

This also allows us to get rid of the extra 2 that we started with.

*The airplane method was discovered at this link:

http://squarerootofnegativeoneteachmath.blogspot.com/2011/12/airplane-method-for-factoring.html

Step 6: This gives us the final answer of:

$$(2x+3)(x-2)$$

c) If there are four terms, it means that you need to group terms that have the same variable(s) together and then take a common factor out of each group.

e.g. $3a^2+3ab+2a+2b$

As you can see this is already grouped together. The 3's are next to each other and the 2's are next to each other.

Take a common factor from the first two terms (here this would be 3a) and a common factor from the next two terms (2)

$$3a(a+b)+2(a+b)$$

Now you can see that the brackets for each group are the same, so we can take these brackets out as a common factor and put the original common factors into a bracket together behind the "new" common factor bracket.

$$(a+b)(3a+2).$$

Activity 6

1. Factorise the following by taking out a common factor:

a) $2a^2+4ab-8a^3b^2$

b) $5c^6d^6-30c^4d^2+85c^5d^3$

c) $72ab^2c+48bc^2-36ac$

d) $9xy+3x^2y-12x$

2. Factorise the following cubes and squares:

a) x^2-y^2

b) $2x^2+2y^2$

c) $8x^3-y^3$

d) $125+343g^3$

e) $54a-16a^4$

f) $16c^2-25d^2$

g) $216b^3+8a^3$

h) x^3y-y^4

i) $49e^3-64f^2e$

j) $50x^4-800y^4$

3. Factorise the following trinomials:

a) $x^2 - x - 30$
b) $x^2 + 9x + 20$
c) $x^2 + 13x + 42$
d) $x^2 - 7x + 6$
e) $x^2 + x - 12$
f) $x^2 - 2\frac{1}{5}x + \frac{2}{5}$
g) $3x^2 - 7x - 6$
h) $2x^2 - 11x + 5$
i) $4x^2 - 19x - 5$
j) $6x^2 + 7x - 5$
k) $3x^2 - 14x + 8$
l) $3x^2 - 5x + 2$
m) $2x^2 - 11x - 21$
n) $2x^2 - 27x + 81$
o) $2x^2 + 3x - 5$
p) $4x^2 + 20x + 25$

4. Factorise the following by grouping:

a) $8ab - 12a^2b - 20b^2 + 30ab^2$
b) $c^3 + 18d^2 - 6cd - 3c^2d$
c) $-12ab^2 - 18a^4b^2 + 20a^2b + 30a^5b$
d) $2e^2f^3 - 6e^2f - 3ef^3 + 9ef$
e) $16g^3 - 64g^2h - gh + 4h^2$
f) $2j^3 - 12k^3 + 3kj - 8j^2k^2$
g) $3m^3 - 14n^4 + 21mn - 2m^2n^3$
h) $q^3 - 3p^2q - 8q^2p + 24p^3$
i) $7r^3 - 56r^2t + 10rt - 80t^2$
j) $v - 15v^2w^2 + 5v^3w - 3w$

5. Factorise the following: (use any of the above methods)

a) $2x^2 - 49x - 25$
b) $x^2 - 6x + 9$
c) $80x^3 - 500xy^2$
d) $6x^3 - 48y^3$
e) $13a^3 - 30b^3 - 78ab + 5a^2b^2$
f) $4x^2 - 11x + 6$
g) $3x^2 + 6x - 24$
h) $\frac{1}{25}x^2 - 16y^4$
i) $6x^2 + 5x - 4$
j) $a^3b^6 + 125c^3$
k) $10x^2 - 3x - 18$
l) $\frac{1}{3}x^2 + 1\frac{2}{3}x + 2$
m) $8a^3 - 42b^3 - 6a^2b^2 + 56ab$
n) $0{,}01x^2 + y^2$
o) $2x^2 - \frac{1}{2}x - \frac{1}{4}$
p) $6x^2 + 43x + 65$
q) $2x^2 - 9x - 110$
r) $-81x^4 - 24xy^3$
s) $a^3 - 7ab + 112b^3 - 16a^2b^2$
t) $x^4 - 18x^2 + 32$
u) $3x^2 + 7x - 40$
v) $4x^2 - 5x - 6$
w) $16x^4 - y^4$
x) $x^6 + y^6$

Algebraic Fractions

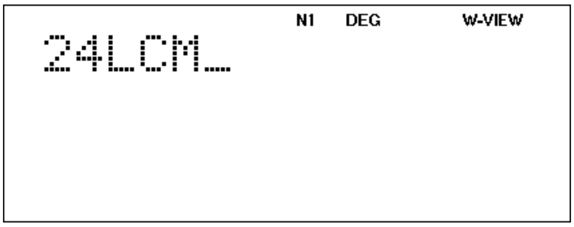
Please don't skip this section just because the heading says algebraic fractions. Fractions are really pretty straight forward if you remember the basics.

So, here are the basics:

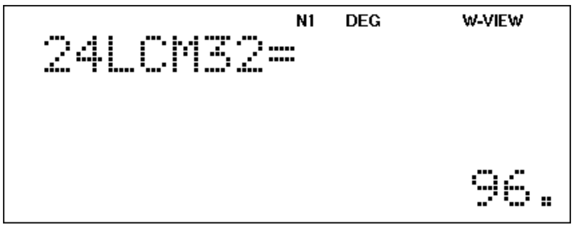
- Numerators are the numbers and variables at the top of the fraction, denominators are the numbers and variables at the bottom of the fraction.

  $$\frac{\boldsymbol{numerator}}{\boldsymbol{denominator}}$$

- The Golden rule: What you do to the top (e.g. multiply by two) you do to the bottom (also multiply by two) → fairs fair.
- Before you can add and/or subtract fractions, you need to make sure the denominators are the same, e.g. $\frac{1}{2}+\frac{1}{3}$ → the LCD (lowest common denominator) is 6. How did we get that? By multiplying the two denominators together (2 x 3 = 6). In the same way for $\frac{1}{a}+\frac{1}{b}$ the denominator is *ab* (because *a x b = ab).*
  - *You can find the Lowest common multiple (or denominator – same thing) of any set of numbers by using your Sharp EL-W535SA or EL-W506T.*
  - *E.g. find the LCM of 24 and 32.*
  - *Press* 

  - 

  - Then press 

  - 

  - So, the lowest common denominator or multiple of 24 and 32 is 96.

- You can never ever cancel over two terms (in other words - don't cancel over a plus or minus). If there are too many terms try factorising first.

  e.g. $\frac{x^2+4}{x}$ → you cannot cancel the x's because there are two terms on the top.

  e.g. $\frac{x^2+4x}{x}$ → first take out a common factor of x at the top → $\frac{x(x+4)}{x}$ (Now you have one term over one term - remember the definition of factorising - and so you **can** cancel. Your answer will now be $\frac{x+4}{1}$ $or\ x + 4$.
- Remember to try to simplify each fraction before you add, subtract, multiply or divide - this will save you lots of time later on.
- Your denominator can **NEVER** be zero (0). This will cause your entire sum to be undefined.
- Also remember that you can take out a negative 1 as a factor.
- When you multiply fractions, you multiply top with top and bottom with bottom.
- When you are dividing fractions remember to turn the dividing fraction up-side-down and then multiply, e.g.: $\frac{1}{a} \div \frac{b}{c} \rightarrow \frac{1}{a} \times \frac{c}{b} = \frac{c}{ab}$

Let's try some examples:

1. $\frac{x+1}{x^2-1} + \frac{x+2}{x^2+x-2}$

First factorise and simplify so that you can see all the possible factors that could go into your common denominator.

$= \frac{(x+1)}{(x-1)(x+1)} + \frac{(x+2)}{(x-1)(x+2)}$

You can see that from the first fraction the $(x + 1)$ in the numerator and the denominator cancel and the $(x + 2)$ in the numerator and the denominator in the second fraction cancel.

$= \frac{1}{x-1} + \frac{1}{x-1}$

We can see that the denominators are the same, so we can add the numerators together.

$= \frac{2}{x-1}$

2. $\frac{x^3+y^3}{2x+2y} \times \frac{4x+6y}{x^2-xy+y^2}$

$= \frac{(x+y)(x^2-xy+y^2)}{2(x+y)} \times \frac{2(2x+3y)}{(x^2-xy+y^2)}$

Always factorise first.

See if you can cross cancel → you can only cross cancel is there is a multiply sign between the two fractions.

$= \frac{(x+y)(x^2-xy+y^2)}{2(x+y)} \times \frac{2(2x+3y)}{(x^2-xy+y^2)}$

Do you see the three sets of "common" factors that you can take out?

$= \frac{1}{1} \times \frac{2x+3y}{1}$

Finally, we multiply top with top and bottom with bottom.

$= 2x + 3y$

3. $\frac{a^2+ab}{3ab} \div \frac{a^2-b^2}{6b}$

Always factorise first.

$= \frac{a(a+b)}{a(3b)} \div \frac{(a+b)(a-b)}{6b}$

Remember that you cannot cross cancel across a divide sign, so we first turn the dividing fraction up-side-down.

$= \frac{a(a+b)}{a(3b)} \times \frac{6b}{(a+b)(a-b)}$

Now we can cancel.

$= \frac{a(a+b)}{a(3b)} \times \frac{6b \ \ 2}{(a+b)(a-b)}$

Notice that 3 goes into 6 twice so there is a remainder of 2 at the top.

$= \frac{1}{1} \times \frac{2}{a-b}$

We can multiply

$= \frac{2}{a-b}$

4. $\frac{x^3-y^3}{x^2-y^2} + \frac{2x+y}{x-y}$

First factorise the expression.

$= \frac{(x-y)(x^2+xy+y^2)}{(x-y)(x+y)} + \frac{2x+y}{x-y}$

Cancel anything you can.

$= \frac{x^2+xy+y^2}{x+y} + \frac{2x+y}{x-y}$

Find the LCD – in this case $(x+y)(x-y)$ and multiply the numerators.

$= \frac{(x-y)(x^2+xy+y^2)+(x+y)(2x+y)}{(x-y)(x+y)}$

Multiply out and simplify – hint: The first term in the numerator is the factorised version of the original first term's numerator.

$= \frac{x^3-y^3+2x^2+xy+2xy+y^2}{(x-y)(x+y)}$

Add your like terms together

$= \frac{x^3+2x^2+3xy+y^2-y^3}{(x-y)(x+y)}$

You cannot factorise or simplify any further so your sum is complete. You can choose to leave your denominator in the factor brackets as we have done (and which most teachers prefer), or you can multiply them out.

Activity 7

Simplify the following fractions assume that all denominators do not equal zero:

a) $\frac{5x^2-20y^2}{5xy} \times \frac{2xy^2}{5x+10y}$

b) $\frac{a}{a-b} - \frac{b}{b-a}$

c) $\frac{p}{3p-6q} + \frac{2p-4q}{p^2-4q^2}$

d) $\frac{x^3-8y^3}{x^2-4y^2} \div \frac{x^2+2xy+4y^2}{6x+12y}$

e) $\frac{m+2n}{2mn} + \frac{m}{2m-4n} - \frac{3m+1}{2n-m}$

f) $\frac{x^2-4}{x^3-8} \times \frac{2-x}{x+2}$

g) $\frac{a+b}{a^2-b^2} \times \frac{6a^2b}{3a+6b} \div \frac{2ab}{a^3-b^3}$

h) $\frac{a}{a+b} + \frac{1}{b-a} \times \frac{a^2-b^2}{2a-3b}$

i) $\frac{x^2y}{3x^3+81y^3} \div \frac{x+2y}{3x^4-48y^4} \times \frac{x+3y}{x-2y}$

j) $\frac{3}{a+b} - \frac{2}{a^2-b^2} + \frac{5}{b-a}$

k) $\frac{-2(x+y)}{x^3-y^3} + \frac{1}{x-y} - \frac{6}{x^2+xy+y^2}$

l) $\frac{8x^3+125}{x^2+4x+4} \times \frac{-2-x}{x+5} \times \frac{4x+8}{2x+5}$

m) $\frac{v^2+vw}{v^2-w^2} \times \frac{w-v}{2v+4w} \div \frac{5v+8w}{4w+2v}$

n) $\frac{1}{a+b} + \frac{3}{a-b} - \frac{2a+b}{b^2-a^2}$

Solving Equations

What is the difference between an expression and an equation?

An expression does not have an equal sign in the middle - you can only simplify an expression, whereas an equation has an equal sign in the middle and you can solve for a variable, like x, because the one side is equal to the other side. You can manipulate an equation, but you cannot change an expression.

This is an expression: $2(x + y^2)$

And this is an equation: $x - 5 = 2x - 7$

Do you see that the expression can be simplified to $2x + 2y^2$ and then cannot be changed, or simplified anymore?

However - we can manipulate the equation so that we can actually find the value for x.

$x - 5 = 2x - 7$

$x - 5 - x = 2x - 7 - x$ What you do to one side you do to the other side.

$-5 + 7 = x - 7 + 7$

$2 = x$

Did you remember that whatever you do to one side you **HAVE** to do to the other side? (Fairs Fair - if your mom bought you an ice-cream but not your sister an ice-cream that wouldn't be fair - in the same way you can't do one thing on the one side and not do it on the other side because that wouldn't be very fair, would it? In fact, you would actually be changing the equation and your final answer would be wrong).

There are several different types of equations:

- Linear – when there are **NO** exponents in the equation, e.g. $2x + 4 = x - 5$
  - It is very easy to solve these kinds of equations, you simply take all of the $x's$ to one side and the numbers to the other side and then find the value for $1x$.
  - E.g. $2x + 4 = x - 5$ Take the x to the left-hand side by subtracting it from both sides.

    $2x + 4 - x = x - 5 - x$

    $x + 4 = -5$ Now take the 4 to the right-hand side.

    $x + 4 - 4 = -5 - 4$

    $x = -9$

- Quadratic – these are equations in the form $ax^2 + bx + c$ where *a, b* and *c* are constants (numbers without variables) or any question where the highest power of x is x^2.
  - To solve quadratic equations, you need to factorise the equation and make each factor equal to zero. Then you solve factor (which is linear) for x.
  - E.g. $x^2 + x = 30$ — First take everything over to the left-hand side so that you only have a zero on the right.

    $x^2 + x - 30 = 0$ — Now you can factorise.

    $(x + 6)(x - 5) = 0$ — We make each factor (the parts in brackets) equal to zero.

    $x + 6 = 0 \quad or \quad x - 5 = 0$ — Solve for each x individually.

    $x + 6 - 6 = 0 - 6$ OR $x - 5 + 5 = 0 + 5$

    $x = -6$ $\qquad$ $x = 5$

  - An easy way to remember that you need two answers is that there is a 2 on the x^2

- Simultaneous Equations – these are two equations that are happening at the same time on the same Cartesian plane. You have to figure out where the two graphs or equations will intersect. In other words, where are the two equations equal to each other? This means that the coordinates (or x and y values will be the same for both graphs / equations).
  - There are two methods for solving equations.
    - The first is substitution: you need to find either an x or y by itself and then substitute it into the other equation. Then you will only have one variable that you are solving for. Once you find the value of that variable, substitute it back into the first equation to find the other variable's value. (You can use this in any kind of simultaneous equations).
    - The second method is elimination: subtract the one equation from the other equation so that one of the variables is cancelled out. You may

need to multiply one of the equations first so that the one variable has the same coefficient. Then solve for the remaining variable. Once you have the answer you can substitute it back into one of the original equations to find the value of the other variable (use this method only if the two equations are of the same type – e.g. both are linear or both are quadratic).

- E.g. Solve for x and y if $y = 3x + 5 \text{ and } 2y = 7x - 3$
  - Method 1: *Substitution*
    - Get either x *or* y by itself (this has already been done).
    - Next substitute it into the other equation:

      $y = 3x + 5 \qquad \rightarrow 2(3x + 5) = 7x - 3$
    - Now solve that equation for x

      $6x + 10 = 7x - 3$

      $6x + 10 - 10 - 7x = 7x - 3 - 7x - 10$

      $-x = -13$ (Divide both sides by -1)

      $\therefore x = 13$
    - Now that you have x substitute it back into $y = 3x + 5$ to find the value of y.

      $y = 3(13) + 5$

      $y = 39 + 5$

      $y = 44$

  - Method 2: *Elimination*
    - In this example we will eliminate the y first. This means that we need to multiply the first equation (or equation 1) by two so that the $y's$ in both equations have the same constant.

      $2 \times (y = 3x + 5)$

      $2y = 6x + 10$ (Note that the entire equation is multiplied).
    - We then subtract the one equation from the other (it doesn't matter which order you put them in).

      $2y = 6x + 10$

      $-2y = -7x + 3$

      $0 = -x + 13$ Now we solve for x

      $+x = -x + 13 + x$

      $\therefore x = 13$

- Then we substitute it back into one of the original equations to find the value of y

  $\therefore y = 3(13) + 5$
  $\therefore y = 44$

- As you can see, both methods give the same solution, so use the method that you are most comfortable with.

- Word Problems – yes, that means story sums 😊 All you have to do is read the story very carefully and underline all the important information. Once you've done that put the information together – either in linear equations or in a table format. From there you can form the equations in order to solve for the variables. They can use linear, quadratic and simultaneous equations in word problems so pay special attention to how things are said.

- Literal equations – these are equations where you have to make one of the variables the subject of the formula (in other words, get the variable by itself and take all the other variables and other numbers to the other side).
  - E.g. Solve for r in terms of v, h and t

$V = \frac{rh}{t} + 3$ First take the 3 to the other side:

$V - 3 = \frac{rh}{t} + 3 - 3$

$V - 3 = \frac{rh}{t}$ Then take the t to the other side by multiplying both sides by t

$t(V - 3) = rh$ Then divide both side by h to get r by itself

$t(V - 3) \div h = \frac{rh}{h}$

$\frac{t(V-3)}{h} = r$ You now have r by itself so you have answered the question.

- Inequalities – these involve <, >, ≥ and ≤ signs. They work exactly like an equal sign with two exceptions:
    - When you divide or times by a negative number, your inequality sign flips over or is reversed.
    - You can never divide or multiply across the inequality sign by the variable you are looking for because you do not know the sign of the variable (whether its value is positive or negative).
  - When you are solving for an inequality you need to be able to draw a graph or representation of the inequality on a number line. Here are a couple of rules you need to remember:
    - < and > are represented by open dots. 
    - ≤ and ≥ are represented by closed dots. 
    - Real ($\mathbb{R}$) numbers are drawn with a solid line.
    - Integers ($\mathbb{Z}$) are drawn with closed dots between the two values of the solved inequality – do you remember why?
    - Whole numbers start from zero and are drawn with closed dots between the two values of the solved inequality
    - Natural numbers start from one and are drawn with closed dots between the two values of the solved inequality.
  - E.g. Solve for x if $x \in \mathbb{R}$ and represent the answer graphically:

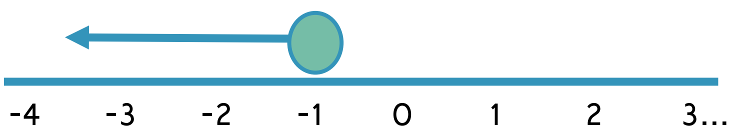
$-2x + 3 \geq 5$ — Take the constant (3) to the right-hand-side.

$-2x + 3 - 3 \geq 5 - 3$

$-2x \geq 2$

$-\frac{2x}{-2} \geq \frac{2}{-2}$ — Divide by -2 → Remember that because you are dividing by a negative number the inequality sign flips over.

$\therefore x \leq -1$



- Interval Notation – this is another way of writing inequalities. Sometimes you will be asked to give your answer in interval notation.
  - $<$ and $>$ are represented by (and)
  - $\leq$ and $\geq$ are represented by [and]
  - Put the bracket first and then the smallest number. Next write ";" and then the biggest number followed by the second bracket.
  - E.g. $2 < x \leq 5 \quad \rightarrow x \in (2;5]$

Activity 8

1. Solve for x in the following linear equations:

a)	$2x - 1 = -9x + 6$	b)	$6x - 1 = 3x - 2$
c)	$\frac{1}{2}x - 10 = 4x + 4$	d)	$-5x + 1 = 4x - 8$
e)	$3x - 5 = x + 13$	f)	$-2x - 5 = 7x + 2$
g)	$-\frac{1}{2}x - \frac{1}{6} = 5x + 1\frac{5}{6}$	h)	$3(x + 5) = 3x + 9$
i)	$x - 2 = 4(2x + 3)$	j)	$x + 7 = 3(2x + 4)$

2. Solve the following quadratic equations for x:

a)	$x^2 + 7x + 10 = 0$	b)	$x^2 + 12x + 36 = 0$
c)	$x^2 + 7x - 44 = 0$	d)	$x^2 - 5x + 4 = 0$
e)	$2x^2 - 13x + 15 = 0$	f)	$x^2 - 16 = 0$
g)	$2x^2 - 8 = 0$	h)	$3x^2 - 5x + 2 = 0$
i)	$5x^2 - 42x - 27 = 0$	j)	$3x^2 = 16 - 8x$

3. Solve the following simultaneous equations for x and y:

a)	$y = 2x - 3$	and	$2y = 5x - 8$
b)	$3y = -4x + 1$	and	$2y = x - 4$
c)	$3y = x$	and	$y = 2x + 5$
d)	$y + x = 1$	and	$5y = -x + 3$
e)	$-2y = 5x + 6$	and	$y = 4x - 5$
f)	$y = -5x$	and	$2y = -2x - 1$

g) $5y = 2x + 8$ and $2y = x + 4$

h) $y = 4x + 5$ and $3x - y = 2$

i) $y = 2x - 4$ and $x - 8y = 27$

j) $6y - x = 0$ and $y = -5 + \frac{1}{3}x$

4. Solve the following word problems:

a) Sarah is 20 years younger than her dad. In 5 years'-time she will be half of her dad's age. How old are Sarah and her dad now?

b) Joe cycles to school and back every day. It takes him twice as long on the way to school as on the way home. If Joe lives 10km from the school, determine how long it takes Joe to go to school and back home if his average speed for the entire trip is 25 km/h.

c) Bob goes to the local café and buys 3 cokes and a packet of chips for R27. The next time he goes to the café he buys 2 cokes and 3 packets of chips and it costs him R25. Work out how much a coke costs and how much a packet of chips costs.

d) Dan wants to work out how much fencing he needs for his farm. He knows that the ground is a rectangular shape and that the breadth is equal to twice the length plus 1km. Determine the length of the rectangle and hence determine the length of fencing Dan needs if the area of the farm is 300km^2.

e) George knows that the perimeter of his fence is 100m around his rectangular piece of garden (assume no gaps), and that the breadth is $x + 5m$. Determine the length of the garden given that the area is 400m^2.

f) Given the sketch of a wall below.

Determine the value(s) of x if the surface area of the wall is 33m^2.

g) Anne knows that an ice-cream cone can hold 47,1239cm^3 of ice-cream. If the formula for the volume of a cone is $v = \frac{1}{3}\pi r^2 h$ and Anne knows the height of the cone is 5cm, determine how wide her scoop of ice-cream can be.

5. For each of the following find the given variable in terms of the other variables:

a) Find r if $v = \pi r^2$ b) Find t if $s = \frac{d}{t}$

c) Find i if $A = P(1+i)^n$ d) find r if $v = \frac{4}{3}\pi r^3$

e) Find h if $v = \pi r^2 h$ f) Find m if $y = mx + c$

g) Find d if $T = a + (n-1)d$ h) Find x if $y = ax^2 + 3$

6. Determine the values for x and represent the answers both graphically and in interval notation.

a) $3x + 4 \leq 5x - 8$ b) $-2x + 5 > 6$

c) $2x + 5 \geq 5$ d) $x - 6 < 4$

e) $-4x + 1 \leq 2$ f) $5 - 6x \geq 2x$

g) $x - 6 > 9x - 2$ h) $-x + 3 \leq 2$

i) $x - 8 < 2x + 5$ j) $-2x + 3 > 10$

Answers for the Activities

Activity 1

1.

Number	Real	Non-Real	Rational	Irrational	Whole	Natural	Integer
0	😊		😊		😊		😊
π	😊			😊			
-1	😊		😊				😊
-1,45	😊		😊				
$\sqrt{3}$	😊			😊			
$\sqrt{-8}$		😊					
$\frac{13}{16}$	😊		😊				
$(\sqrt{-5})^2$		😊					

2. Given the equation: $0 = (x+3)(2x-5)(x^2+6)$

Solve for x when x is

a) real → $x = -3 \quad or \quad x = \frac{5}{2}$

b) non-real → $x = \pm\sqrt{-6}$

c) an integer → $x = -3$

Activity 2

1. a) 0,135 → 0,14 b) 2,369 → 2,37

c) 5,895 → 5,90 d) 4,351 → 4,35

2. a) up b) up

c) down d) down

e) up

Activity 3

1. Before we start, we should do a "square" number line first:

1 4 9 16 25 36 49 64 81 100

a) between 7 and 8 b) between -4 and -3

c) between -9 and -8 d) between 3 and 4

e) between -7 and -6 f) between -6 and -5

g) between 9 and 10 h) between 2 and 3

2. Before we start, we should do a "cube" number line first:

1 8 27 64 125 216 343...

a) between 1 and 2 b) between -4 and -3

c) between 4 and 5 d) between -6 and -5

Activity 4

1\. a) $\frac{c^4d^2e}{f^2d^{-2}} \times \frac{(e^3f)^{-1}}{\sqrt[3]{d^{-3}c^6}}$

$= \frac{c^4d^2ed^2}{f^2} \times \frac{e^{-3}f^{-1}}{d^{-1}c^2}$

$= \frac{c^4d^4e}{f^2} \times \frac{d}{c^2e^3f}$

b) $\frac{1}{a^{-2}} \times \left(\frac{b}{a^3}\right)^{-1} \div \frac{b^2}{a^2}$

$= \frac{a^2}{1} \times \frac{a^3}{b} \times \frac{a^2}{b^2}$

$= \frac{a^7}{b^3}$

c) $\frac{x^{-\frac{1}{4}}y^3}{z^3} \times \frac{(yz)^{-3}}{\sqrt[4]{x^5}}$

$= \frac{y^3}{x^{\frac{1}{4}}z^3} \times \frac{1}{x^{\frac{5}{4}}y^3z^3}$

$= \frac{y^3}{x^{\frac{6}{4}}y^3z^6}$

$= \frac{1}{x^{\frac{3}{2}}z^6}$

d) $\frac{18^x.24^{x+1}}{36^{2x-1}}$

$= \frac{(3^2.2)^x.(2^33)^{x+1}}{(3^22^2)^{2x-1}}$

$= \frac{3^{2x}2^x2^{3x+3}3^{x+1}}{3^{4x-2}2^{4x-2}}$

$= 3^{2x+x+1-(4x-2)}2^{x+3x+3-(4x-2)}$

$= 3^{3x+1-4x+2}2^{4x+3-4x+2}$

$= 3^{3-x}2^5$

$= 32.3^{3-x}$

e) $\frac{21^{x+1}.63^{x-1}}{9^x.49^{2x}}$

$= \frac{(3.7)^{x+1}.(3^2.7)^{x-1}}{(3^2)^x.(7^2)^{2x}}$

$= \frac{3^{x+1}.7^{x+1}.3^{2x-2}.7^{x-1}}{3^{2x}.7^{4x}}$

$= 3^{x+1+2x-2-(2x)}7^{x+1+x-1-(4x)}$

$= 3^{3x-1-2x}.7^{2x-4x}$

$= 3^{x-1}.7^{-2x}$

$= \frac{3^{x-1}}{7^{2x}}$

f) $\frac{a^{-1}b^6}{c^{-6}d^4} \times \frac{ab^4c^5}{d^{-7}} \div \frac{(a^3b^5)^2}{c^{-1}d^3}$

$= \frac{b^6c^6}{ad^4} \times \frac{ab^4c^5d^7}{1} \div \frac{a^6b^{10}c}{d^3}$

$= \frac{ab^{10}c^{11}d^7}{ad^4} \times \frac{d^3}{a^6b^{10}c}$

$= \frac{ab^{10}c^{11}d^{10}}{a^7b^{10}cd^4}$

$= \frac{c^{10}d^6}{a^6}$

g) $\frac{(ef^{-1})^4}{g^{-2}h^3} \times \frac{e^6f^4g^2}{h} \times \left(\frac{e}{f^{-2}}\right)^{-1}$

$= \frac{e^4f^{-4}}{g^{-2}h^3} \times \frac{e^6f^4g^2}{h} \times \frac{f^{-2}}{e}$

$= \frac{e^4g^2}{f^4h^3} \times \frac{e^6f^4g^2}{h} \times \frac{1}{ef^2}$

$= \frac{e^{10}f^4g^4}{ef^6h^4}$

$= \frac{e^9g^4}{f^2h^4}$

h) $\left(\frac{1}{x} + \frac{x}{y}\right)^{-2}$

$= \left(\frac{y+x^2}{xy}\right)^{-2}$ Add the two

$= \left(\frac{xy}{y+x^2}\right)^2$

$= \frac{x^2y^2}{(x^2+y)^2}$ Simplify

$= \frac{x^2y^2}{x^4+2x^2y+y^2}$

i) $\left(\left(\frac{x}{y}\right)^{-2} + \left(\frac{x}{y} - \frac{y}{x}\right)^{-3}\right)^{0}$

$= 1$

Anything to the power of

Zero is 1. We don't need to

Simplify anything else.

j) $\frac{7^{x+1} \times 2^{2x-3}}{14^{x-5}}$

$= \frac{7^{x+1}2^{2x-3}}{(2.7)^{x-5}}$

$= \frac{7^{x+1}.2^{2x-3}}{2^{x-5}.7^{x-5}}$

$= 7^{x+1-(x-5)}.2^{2x-3-(x-5)}$

$= 7^{x+1-x+5}.2^{2x-3-x+5}$

$= 7^{6}.2^{x+2}$

Don't need to simplify further.

2. a) $2^{x} - \frac{2^{x+1}}{5} = 0{,}3$

$5\left(2^{x} - \frac{2^{x+1}}{5}\right) = 0{,}3 \times 5$

$5.2^{x} - 2^{x+1} = 1\frac{1}{2}$

$5.2^{x} - 2^{x}.2 = 1\frac{1}{2}$

$2^{x}(5-2) = 1\frac{1}{2}$

$2^{x}(3) = \frac{3}{2}$

$2^{x} = \frac{1}{2}$ or 2^{-1}

$\therefore x = -1$

b) $2^{x+1} - \frac{2^{x+1}}{3} = 21\frac{1}{3}$

$3\left(2^{x}2 - \frac{2^{x}.2}{3}\right) = 3 \times 21\frac{1}{3}$

$3.2^{x}2 - 2^{x}.2 = 64$

$2^{x}(6-2) = 64$

$2^{x}(4) = 64$

$2^{x} = 16$

$2^{x} = 2^{4}$

$\therefore x = 4$

c) $10x^{3} + 50 = 1\,300$

$(10x^{3} + 50) \div 10 = 1\,300 \div 10$

$x^{3} + 5 = 130$

$x^{3} = 125$

$x = \sqrt[3]{125}$

$\therefore x = 5$

d) $\frac{4x^{4}}{3} - 5 = 103$

$4x^{4} - 15 = 309$

$4x^{4} = 324$

$x^{4} = 81$

$x = \sqrt[4]{81}$

$\therefore x = \pm 3$

e) $6^{2\frac{1}{2}} = x$

$x = 88.18$

f) $x = 4^{1\frac{1}{3}}$

$x = 6.35$

g) $4^{x+2} - 4^{x} = \frac{15}{16}$

$4^{x}.4^{2} - 4^{x} = \frac{15}{16}$

$4^{x}(4^{2} - 1) = \frac{15}{16}$

$4^{x}(15) = \frac{15}{16}$

$4^{x} = \frac{1}{16}$

$4^{x} = 4^{-2}$

$\therefore x = -2$

h) $4^{x} - \frac{4^{x}}{2} = 1$

$4^{x}\left(1 - \frac{1}{2}\right) = 1$

$4^{x}\left(\frac{1}{2}\right) = 1$

$4^{x} = 2$

$2^{2x} = 2^{1}$

$\therefore 2x = 1$

$\therefore x = \frac{1}{2}$

i) $5^x + 3.5^x = 4$

$5^x(1+3) = 4$

$5^x(4) = 4$

$5^x = 1$

$5^x = 5^0$

$\therefore x = 0$

Anything to the power of zero

Is 1.

j) $-\frac{1}{5}x^3 + \frac{3}{5} = -1$

$5\left(-\frac{1}{5}x^3 + \frac{3}{5}\right) = -1 \times 5$

$-1x^3 + 3 = -5$

$x^3 - 3 = 5$

$x^3 = 8$

$x = \sqrt[3]{8}$

$\therefore x = 2$

Activity 5

Multiply out and simplify the following expressions

a) $(3x-5)(x^2+4x-6)$

$= 3x^3 + 12x^2 - 18x - 5x^2 - 20x + 30$

$= 3x^3 + 7x^2 - 38x + 30$

b) $\left(\frac{1}{2}y + 2\right)(4y^2 - 6y + 3)$

$= 2y^3 - 3y^2 + \frac{3}{2}y + 8y^2 - 12y + 6$

$= 2y^3 + 5y^2 - 10\frac{1}{2}y + 6$

c) $\left(\frac{1}{4}x - 1\right)\left(-x^2 - \frac{1}{3}x + 4\right)$

$= -\frac{1}{4}x^3 - \frac{1}{12}x^2 + x + x^2 + \frac{1}{3}x - 4$

$= -\frac{1}{4}x^3 + \frac{11}{12}x^2 + 1\frac{1}{3}x \quad - 4$

d) $(6x+4y)(2x^2+4xy+3)$

$= 12x^3 + 24x^2y + 18x + 8x^2y + 16xy^2 + 12y$

$= 12x^3 + 32x^2y + 18x + 16xy^2 + 12y$

e) $(-4x+3)\left(\frac{1}{5}x^2 + 11x - 6\right)$

$= -\frac{4}{5}x^3 - 44x^2 + 24x + \frac{3}{5}x^2 + 33x - 18$

$= -\frac{4}{5}x^3 - 43\frac{2}{5}x^2 + 57x - 18$

f) $(-3a-2b)(-7a^2+8ab-b^2)$

$= 21a^3 - 24a^2b + 3ab^2 + 14a^2b - 16ab^2 + 2b^3$

$= 21a^3 - 10a^2b - 13ab^2 + 2b^3$

g) $\left(\frac{3}{4}c^2 - 3d\right)\left(\frac{1}{2}c^2 + cd + 6d^2\right)$

$= \frac{3}{8}c^4 + \frac{3}{4}c^3d + 4\frac{1}{2}c^2d^2 - \frac{3}{2}c^2d - 3cd^2 - 18d^3$

h) $(5e + 2f)\left(e^2 - 8ef + \frac{1}{3}f^2\right)$

$= 5e^3 - 40e^2f + \frac{5}{3}ef^2 + 2e^2f - 16ef^2 + \frac{2}{3}f^3$

$= 5e^3 - 38e^2f - 14\frac{1}{3}ef^2 + \frac{2}{3}f^3$

i) $(-2g + 5h)(6g - 6gh + h)$

$= -12g^2 + 12g^2h - 2gh + 30gh - 30gh^2 + 5h^2$

$= -12g^2 + 12g^2h + 28gh - 30gh^2 + 5h^2$

j) $\left(8k - \frac{4}{5}m\right)\left(2k^3 + \frac{1}{4}k^2m - km\right)$

$= 16k^4 + 2k^3m - 8k^2m - \frac{8}{5}k^3m - \frac{1}{5}k^2m^2 + \frac{4}{5}km^2$

$= 16k^4 + \frac{2}{5}k^3m - 8k^2m - \frac{1}{5}k^2m^2 + \frac{4}{5}km^2$

Activity 6

1. Factorise the following by taking out a common factor:

a) $2a^2 + 4ab - 8a^3b^2$

$= 2a(a + 2b - 4a^2b^2)$

b) $5c^6d^6 - 30c^4d^2 + 85c^5d^3$

$= 5c^4d^2(c^2d^4 - 6 + 17cd)$

c) $72ab^2c + 48bc^2 - 36ac$

$= 12c(6ab^2 + 4bc - 3a)$

d) $9xy + 3x^2y - 12x$

$= 3x(3y + xy - 4)$

To factorise with exponents, look for variables that occur in every term and then take out the variable with the smallest exponent as the common factor.

2. Factorise the following cubes and squares:

a) $x^2 - y^2$

$= (x - y)(x + y)$

b) $2x^2 + 2y^2$

$= 2(x^2 + y^2)$

Can't factorise further as this is not a difference of squares.

c) $8x^3 - y^3$

$= (2x - y)(4x^2 + 2xy + y^2)$

d) $125 + 343g^3$

$= (5 + 7g)(25 - 35g + 49g^2)$

e) $54a - 16a^4$

$= 2a(27 - 8a^3)$
$= 2a(3 - 2a)(9 + 6a + 4a^2)$

f) $16c^2 - 25d^2$

$= (4c - 5d)(4c + 5d)$

g) $216b^3 + 8a^3$

$= 8(27b^3 + a^3)$
$= 8(3b + a)(9b^2 - 3ab + a^2)$

h) $x^3y - y^4$

$= y(x^3 - y^3)$
$= y(x - y)(x^2 + xy + y^2)$

i) $49e^3 - 64f^2e$

$= e(49e^2 - 64f^2)$
$= e(7e - 8f)(7e + 8f)$

j) $50x^4 - 800y^4$

$= 50(x^4 - 16y^4)$
$= 50(x^2 - 4y^2)(x^2 + 4y^2)$
$= 50(x - 2y)(x + 2y)(x^2 + 4y^2)$

3. Factorise the following trinomials:

a) $x^2 - x - 30$

$= (x - 6)(x + 5)$

b) $x^2 + 9x + 20$

$= (x + 4)(x + 5)$

c) $x^2 + 13x + 42$

$= (x + 6)(x + 7)$

d) $x^2 - 7x + 6$

$= (x - 6)(x - 1)$

e) $x^2 + x - 12$

$= (x + 4)(x - 3)$

f) $x^2 - 2\frac{1}{5}x + \frac{2}{5}$

$= x^2 - \frac{11}{5}x + \frac{2}{5}$
$= \frac{1}{5}(5x^2 - 11x + 2)$
$= \frac{1}{5}(5x - 1)(x - 2)$

g) $3x^2 - 7x - 6$

$= (3x + 2)(x - 3)$

h) $2x^2 - 11x + 5$

$= (2x - 1)(x - 5)$

i) $4x^2 - 19x - 5$

$= (4x + 1)(x - 5)$

j) $6x^2 + 7x - 5$

$= (2x - 1)(3x + 5)$

k) $3x^2 - 14x + 8$

$= (3x - 2)(x - 4)$

l) $3x^2 - 5x + 2$

$= (3x - 2)(x - 1)$

m) $2x^2 - 11x - 21$

$= (2x + 3)(x - 7)$

n) $2x^2 - 27x + 81$

$= (x - 9)(2x - 9)$

o) $2x^2 + 3x - 5$

$= (x - 1)(2x + 5)$

p) $4x^2 + 20x + 25$

$= (2x + 5)(2x + 5)$

4. Factorise the following by grouping:

a) $8ab - 12a^2b - 20b^2 + 30ab^2$

$= 4ab(2 - 3a) - 10b^2(2 - 3a)$
$= (2 - 3a)(4ab - 10b^2)$
$= b(2 - 3a)(4a - 10b)$

b) $c^3 + 18d^2 - 6cd - 3c^2d$

$= c^3 - 3c^2d + 18d^2 - 6cd$
$= c^2(c - 3d) + 6d(3d - c)$
$= c^2(c - 3d) - 6d(c - 3d)$
$= (c - 3d)(c^2 - 6d)$

c) $-12ab^2 - 18a^4b^2 + 20a^2b + 30a^5b$

$= 2ab[-6b - 9a^3b + 10a + 15a^4]$
$= 2ab[-3b(2 + 3a^3) + 5a(2 + 3a^3)]$
$= 2a[(2 + 3a^3)(-3b + 5a)]$
$= 2a(2 + 3a^3)(5a - 3b)$

d) $2e^2f^3 - 6e^2f - 3ef^3 + 9ef$

$= ef[2ef^2 - 6e - 3f^2 + 9]$
$= ef[2e(f^2 - 3) - 3(f^2 - 3)]$
$= ef(f^2 - 3)(2e - 3)$

e) $16g^3 - 64g^2h - gh + 4h^2$

$= 16g^2(g - 4h) - h(g - 4h)$
$= (g - 4h)(16g^2 - h)$

f) $2j^3 - 12k^3 + 3kj - 8j^2k^2$

$= 2j^3 + 3kj - 12k^3 - 8j^2k^2$
$= j(2j^2 + 3k) - 4k^2(3k + 2j^2)$
$= (2j^2 + 3k)(j - 4k^2)$

g) $3m^3 - 14n^4 + 21mn - 2m^2n^3$

$= 3m^3 - 2m^2n^3 + 21mn - 14n^4$
$= m^2(3m - 2n^3) + 7n(3m - 2n^3)$
$= (3m - 2n^3)(m^2 + 7n)$

h) $q^3 - 3p^2q - 8q^2p + 24p^3$

$= q(q^2 - 3p^2) - 8p(q^2 - 3p^2)$
$= (q^2 - 3p^2)(q - 8p)$

i) $7r^3 - 56r^2t + 10rt - 80t^2$

$= 7r^3 + 10rt - 56r^2t - 80t^2$
$= r(7r^2 + 10t) - 8t(7r^2 + 10t)$
$= (7r^2 + 10t)(r - 8t)$

j) $v - 15v^2w^2 + 5v^3w - 3w$

$= v - 3w + 5v^3w - 15v^2w^2$
$= 1(v - 3w) + 5v^2w(v - 3w)$
$= (v - 3w)(1 + 5v^2w)$

5. Factorise the following: (use any of the above methods)

a) $2x^2 - 49x - 25$

$= (2x + 1)(x - 25)$

b) $x^2 - 6x + 9$

$= (x - 3)(x - 3)$

c) $80x^3 - 500xy^2$

$= 20x(4x^2 - 25y^2)$
$= 20x(2x - 5y)(2x + 5y)$

d) $6x^3 - 48y^3$

$= 6(x^3 - 8y^3)$
$= 6(x - 2y)(x^2 + 2xy + 4y^2)$

e) $13a^3 - 30b^3 - 78ab + 5a^2b^2$

$= 13a^3 + 5a^2b^2 - 78ab - 30b^3$
$= a^2(13a + 5b^2) - 6b(13a + 5b^2)$
$= (13a + 5b^2)(a^2 - 6b)$

f) $4x^2 - 11x + 6$

$= (x - 2)(4x - 3)$

g) $3x^2 + 6x - 24$

$= 3(x^2 + 2x - 8)$

$= 3(x + 4)(x - 2)$

h) $\frac{1}{25}x^2 - 16y^4$

$= \left(\frac{1}{5}x - 4y^2\right)\left(\frac{1}{5}x + 4y^2\right)$

i) $6x^2 + 5x - 4$

$= (2x - 1)(3x + 4)$

j) $a^3b^6 + 125c^3$

$= (ab^2 + 5c)(a^2b^4 - 5ab^2c + 25c^2)$

k) $10x^2 - 3x - 18$

$= (2x - 3)(5x + 6)$

l) $\frac{1}{3}x^2 + 1\frac{2}{3}x + 2$

$= \frac{1}{3}(x^2 + 5x + 6)$
$= \frac{1}{3}(x + 3)(x + 2)$

m) $8a^3 - 42b^3 - 6a^2b^2 + 56ab$

$= 8a^3 - 6a^2b^2 + 56ab - 42b^3$

$= 2a^2(4a - 3b^2) + 14b(4a - 3b^2)$

$= (4a - 3b^2)(2a^2 + 14b)$

n) $0{,}01x^2 + y^2$

$= \frac{1}{100}x^2 + y^2$

Cannot factorise, no common factor

and no difference of squares.

o) $2x^2 - \frac{1}{2}x - \frac{1}{4}$

$= \frac{1}{4}(8x^2 - 2x - 1)$
$= \frac{1}{4}(4x + 1)(2x - 1)$

p) $6x^2 + 43x + 65$

$= (x + 5)(6x + 13)$

q) $2x^2 - 9x - 110$

$= (x - 10)(2x + 11)$

r) $-81x^4 - 24xy^3$

$= \ -3x(27x^3 + 8y^3)$
$= -3x(3x + 2y)(9x^2 - 6xy + 4y^2)$

s) $a^3 - 7ab + 112b^3 - 16a^2b^2$

$= a(a^2 - 7b) + 16b^2(7b - a^2)$
$= a(a^2 - 7b) - 16b^2(a^2 - 7b)$
$= (a^2 - 7b)(a - 16b^2)$

t) $x^4 - 18x^2 + 32$

$= (x^2 - 2)(x^2 - 16)$
$= (x^2 - 2)(x - 4)(x + 4)$

u) $3x^2 + 7x - 40$

$= (x + 5)(3x - 8)$

v) $4x^2 - 5x - 6$

$= (x - 2)(4x + 3)$

w) $16x^4 - y^4$

$= (4x^2 - y^2)(4x^2 + y^2)$
$= (2x - y)(2x + y)(4x^2 + y^2)$

x) $x^6 + y^6$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Activity 7

Simplify the following fractions assume that all denominators do not equal zero:

a) $\frac{5x^2-20y^2}{5xy} \times \frac{2xy^2}{5x+10y}$

$= \frac{5(x^2-4y^2)}{5xy} \times \frac{2xy^2}{5(x+2y)}$

$= \frac{(x-2y)(x+2y)}{xy} \times \frac{2xy^2}{5(x+2y)}$

$= \frac{x-2y}{1} \times \frac{2y}{5}$

$= \frac{2y(x-2y)}{5}$

$OR = \frac{2xy-4y^2}{5}$

b) $\frac{a}{a-b} - \frac{b}{b-a}$

$= \frac{a}{a-b} + \frac{b}{(a-b)}$

$= \frac{a+b}{a-b}$

Remember that you can take out a common factor of -1 to make the denominators the same 😊 Your negative in front of the fraction becomes positive.

c) $\frac{p}{3p-6q} + \frac{2p-4q}{p^2-4q^2}$

$= \frac{p}{3(p-2q)} + \frac{2(p-2q)}{(p-2q)(p+2q)}$

$= \frac{p\times(p+2q)+3\times2(p-2q)}{3(p-2q)(p+2q)}$

$= \frac{p^2+2pq+6p-12q}{3(p-2q)(p+2q)}$

d) $\frac{x^3-8y^3}{x^2-4y^2} \div \frac{x^2+2xy+4y^2}{6x+12y}$

$= \frac{(x-2y)(x^2+2xy+4y^2)}{(x-2y)(x+2y)} \div \frac{(x^2+2xy+4y^2)}{6(x+2y)}$

$= \frac{(x^2+2xy+4y^2)}{(x+2y)} \times \frac{6(x+2y)}{(x^2+2xy+4y^2)}$

$= 6$

e) $\frac{m+2n}{2mn}+\frac{m}{2m-4n}-\frac{3m+1}{2n-m}$

$=\frac{m+2n}{2mn}+\frac{m}{2(m-2n)}-\frac{3m+1}{-(m-2n)}$

$=\frac{(m+2n)(m-2n)+m(mn)+(3m+1)(2mn)}{2mn(m-2n)}$

$=\frac{m^2-4n^2+m^2n+6m^2n+2mn}{2mn(m-2n)}$

$=\frac{m^2-4n^2+7m^2n+2mn}{2mn\,(m-2n)}$

f) $\frac{x^2-4}{x^3-8}\times\frac{2-x}{x+2}$

$=\frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}\times\frac{-(x-2)}{(x+2)}$

$=\frac{1}{x^2+2x+4}\times\frac{2-x}{1}$

$=\frac{2-x}{x^2+2x+4}$

g) $\frac{a+b}{a^2-b^2}\times\frac{6a^2b}{3a+6b}\div\frac{2ab}{a^3-b^3}$

$=\frac{(a+b)}{(a+b)(a-b)}\times\frac{6a^2b}{3(a+2b)}\div\frac{2ab}{(a-b)(a^2+ab+b^2)}$

$=\frac{1}{(a-b)}\times\frac{2a^2b}{(a+2b)}\times\frac{(a-b)(a^2+ab+b^2)}{2ab}$

$=\frac{a(a^2+ab+b^2)}{(a+2b)}$

h) $\frac{a}{a+b}+\frac{1}{b-a}\times\frac{a^2-b^2}{2a-3b}$

$=\frac{a}{a+b}+\frac{1}{-(a-b)}\times\frac{(a-b)(a+b)}{(2a-3b)}$

$=\frac{a(a-b)-1(a+b)}{(a+b)(a-b)}\times\frac{(a-b)(a+b)}{(2a-3b)}$

$=\frac{a^2-ab-a-b}{(a+b)(a-b)}\times\frac{(a-b)(a+b)}{(2a-3b)}$

$=\frac{a^2-ab-a-b}{2a-3b}$

i) $\frac{x^2y}{3x^3+81y^3}\div\frac{x+2y}{3x^4-48y^4}\times\frac{x+3y}{x-2y}$

$=\frac{x^2y}{3(x^3+27y^3)}\times\frac{3(x^4-16y^4)}{(x+2y)}\times\frac{(x+3y)}{(x-2y)}$

$=\frac{x^2y}{(x+3y)(x^2-3xy+9y)}\times\frac{(x^2-4y)(x^2+4y)}{(x+2y)}\times\frac{(x+3y)}{(x-2y)}$

$=\frac{x^2y}{(x^2-3xy+9y)}\times\frac{(x-2y)(x+2y)(x^2+4y)}{(x+2y)}\times\frac{1}{(x-2y)}$

$=\frac{x^2y\,(x^2+4y)}{(x^2-3xy+9y)}$

j) $\frac{3}{a+b}-\frac{2}{a^2-b^2}+\frac{5}{b-a}$

$=\frac{3}{a+b}-\frac{2}{(a-b)(a+b)}+\frac{5}{-(a-b)}$

$=\frac{3(a-b)-2-5(a+b)}{(a+b)(a-b)}$

$=\frac{3a-3b-2-5a-5b}{(a+b)(a-b)}$

$=\frac{-2a-8b-2}{(a+b)(a-b)}$

$=\frac{-2(a+4b+1)}{(a+b)(a-b)}$

k) $\frac{-2(x+y)}{x^3-y^3}+\frac{1}{x-y}-\frac{6}{x^2+xy+y^2}$

$=\frac{-2(x+y)}{(x-y)(x^2+xy+y^2)}+\frac{1}{(x-y)}-\frac{6}{(x^2+xy+y^2)}$

$=\frac{-2(x+y)+1(x^2+xy+y^2)-6(x-y)}{(x-y)(x^2+xy+y^2)}$

$=\frac{-2x-2y+x^2+xy+y^2-6x+6y}{x^3-y^3}$

$=\frac{x^2-8x+xy+4y+y^2}{x^3-y^3}$

l) $\frac{8x^3+125}{x^2+4x+4}\times\frac{-2-x}{x+5}\times\frac{4x+8}{2x+5}$

$=\frac{(2x+5)(4x^2-10x+25)}{(x+2)(x+2)}\times\frac{-(x+2)}{(x+5)}\times\frac{4(x+2)}{(2x+5)}$

$=\frac{-4(4x^2-10x+25)}{(x+5)}$

m) $\frac{v^2+vw}{v^2-w^2}\times\frac{w-v}{2v+4w}\div\frac{5v+8w}{4w+2v}$

$=\frac{v(v+w)}{(v-w)(v+w)}\times\frac{-(v-w)}{2(v+2w)}\times\frac{2(2w+v)}{(5v+8w)}$

$=\frac{v}{(v-w)}\times\frac{-(v-w)}{(v+2w)}\times\frac{(2w+v)}{(5v+8w)}$

$=\frac{-v}{5v+8w}$

n) $\frac{1}{a+b}+\frac{3}{a-b}-\frac{2a+b}{b^2-a^2}$

$=\frac{1}{a+b}+\frac{3}{a-b}-\frac{2a+b}{-(a^2-b^2)}$

$=\frac{1}{a+b}+\frac{3}{a-b}+\frac{2a+b}{(a-b)(a+b)}$

$=\frac{1(a-b)+3(a+b)+(2a+b)}{(a-b)(a+b)}$

$=\frac{a-b+3a+3b+2a+b}{(a-b)(a+b)}$

$=\frac{6a+3b}{(a-b)(a+b)}$

$=\frac{3(2a+b)}{(a-b)(a+b)}$

Activity 8

1. a) $2x-1=-9x+6$

$2x+9x=6+1$

$11x=7$

$\therefore x=\frac{7}{11}$

b) $6x-1=3x-2$

$6x-3x=-2+1$

$3x=-1$

$\therefore x=-\frac{1}{3}$

c) $\frac{1}{2}x-10=4x+4$

$\frac{1}{2}x-4x=4+10$

$-3\frac{1}{2}x=14$

$\therefore x=-4$

d) $-5x+1=4x-8$

$-5x-4x=-8-1$

$-9x=-9$

$\therefore x=1$

e) $3x - 5 = x + 13$

$3x - x = 13 + 5$
$2x = 18$
$\therefore x = 9$

f) $-2x - 5 = 7x + 2$

$-2x - 7x = 2 + 5$
$-9x = 7$
$\therefore x = -\frac{7}{9}$

g) $-\frac{1}{2}x - \frac{1}{6} = 5x + 1\frac{5}{6}$

$-\frac{1}{2}x - 5x = 1\frac{5}{6} + \frac{1}{6}$
$-5\frac{1}{2}x = 2$
$\therefore x = -\frac{4}{11}$

h) $3(x + 5) = 3x + 9$

$3x + 15 = 3x + 9$

$3x - 3x = 9 - 15$

$0x = -6$

$\therefore x \text{ does not exist.}$

i) $x - 2 = 4(2x + 3)$

$x - 2 = 8x + 12$
$x - 8x = 12 + 2$
$-7x = 14$
$\therefore x = -2$

j) $x + 7 = 3(2x + 4)$

$x + 7 = 6x + 12$
$x - 6x = 12 - 7$
$-5x = 5$
$\therefore x = -1$

2. a) $x^2 + 7x + 10 = 0$

$(x + 5)(x + 2) = 0$
$x + 5 = 0 \quad or \quad x + 2 = 0$
$\therefore x = -5 \quad or \quad x = -2$

b) $x^2 + 12x + 36 = 0$

$(x + 6)(x + 6) = 0$
$x + 6 = 0$
$x = -6$
We only need to do this once

because the factors are the same.

c) $x^2 + 7x - 44 = 0$

$(x + 11)(x - 4) = 0$
$\therefore x = -11 \quad or \quad x = 4$

d) $x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$
$\therefore x = 4 \quad or \quad x = 1$

e) $2x^2 - 13x + 15 = 0$

$(x - 1)(2x - 15) = 0$
$\therefore x = 1 \quad or \quad 2x = 15$
$\therefore x = \frac{15}{2} \; or \; 7\frac{1}{2}$

f) $x^2 - 16 = 0$

$(x - 4)(x + 4) = 0$
$\therefore x = 4 \quad or \quad x = -4$

g) $2x^2 - 8 = 0$

$2(x^2 - 4) = 0$
$2(x - 2)(x + 2) = 0$
$\therefore x = 2 \quad or \quad x = -2$

h) $3x^2 - 5x + 2 = 0$

$(3x - 2)(x - 1) = 0$
$\therefore 3x = 2 \quad or \quad x = 1$
$\therefore x = \frac{2}{3}$

i) $5x^2 - 42x - 27 = 0$

$(5x+3)(x-9) = 0$

$\therefore 5x = -3 \quad or \quad x = 9$

$\therefore x = -\frac{3}{5}$

j) $3x^2 = 16 - 8x$

$3x^2 + 8x - 16 = 0$

$(x+4)(3x-4) = 0$

$\therefore x = -4 \quad or \quad 3x = 4$

$\therefore x = \frac{4}{3} \quad or \quad 1\frac{1}{3}$

3. a) $y = 2x - 3$ ...1 and

$2y = 5x - 8$ ...2

Substitute 1 into 2

$\therefore 2(2x-3) = 5x - 8$

$\therefore 4x - 6 = 5x - 8$

$\therefore 4x - 5x = -8 + 6$

$\therefore -x = -2$

$\therefore x = 2$

Substitute back into 1

$y = 2(2) - 3$

$\therefore y = 1$

∴ (2; 1)

b) $3y = -4x + 1$ ...1 and

$2y = x - 4$ ...2

Get y by itself in 2:

$y = \frac{1}{2}x - 2$ ...3

Substitute 3 into 1

$\therefore 3\left(\frac{1}{2}x - 2\right) = -4x + 1$

$\therefore \frac{3}{2}x - 6 = -4x + 1$

$\therefore \frac{3}{2}x + 4x = 1 + 6$

$\therefore 5\frac{1}{2}x = 7$

$\therefore x = \frac{14}{11}$

Substitute back into 3

$\therefore y = \frac{1}{2}\left(\frac{14}{11}\right) - 2$

$\therefore y = -1\frac{4}{11} \quad or \quad -\frac{15}{11}$ $\quad \therefore \left(\frac{14}{11}; -\frac{15}{11}\right)$

c) $3y = x$ ...1 and

$y = 2x + 5$ ...2

Substitute 2 into 1

$\therefore 3(2x+5) = x$

$\therefore 6x + 15 = x$

$\therefore 6x - x = -15$

$\therefore 5x = -15$

$\therefore x = -3$

Substitute back into 2

$\therefore y = 2(-3) + 5$

$\therefore y = -1$

∴ (-3; -1)

d) $y + x = 1$ ...1 and

$5y = -x + 3$ ...2

Get y by itself in 1

$\therefore y = 1 - x$ ...3

Substitute 3 into 2

$\therefore 5(1-x) = -x + 3$

$\therefore 5 - 5x = -x + 3$

$\therefore -5x + x = 3 - 5$

$\therefore -4x = -2$

$\therefore x = \frac{1}{2}$

Substitute back into 3

$\therefore y = 1 - \left(\frac{1}{2}\right) = \frac{1}{2}$ $\quad \therefore \left(\frac{1}{2}; \frac{1}{2}\right)$

e) $-2y = 5x + 6$ ...1 and
$y = 4x - 5$ ...2

Substitute 2 into 1

$\therefore -2(4x - 5) = 5x + 6$
$\therefore -8x + 10 = 5x + 6$
$\therefore -8x - 5x = 6 - 10$
$\therefore -13x = -4$
$\therefore x = \frac{4}{13}$

Substitute back into 2

$\therefore y = 4\left(\frac{4}{13}\right) - 5$
$\therefore y = -3\frac{10}{13}$

$\therefore \left(\frac{4}{13}; -3\frac{10}{13}\right)$

f) $y = -5x$ ...1 and
$2y = -2x - 1$ ...2

Substitute 1 into 2

$\therefore 2(-5x) = -2x - 1$
$\therefore -10x = -2x - 1$
$\therefore -10x + 2x = -1$
$\therefore -8x = -1$
$\therefore x = \frac{1}{8}$

Substitute back into 1

$\therefore y = -5\left(\frac{1}{8}\right)$
$\therefore y = -\frac{5}{8}$

$\therefore \left(\frac{1}{8}; -\frac{5}{8}\right)$

g) $5y = 2x + 8$ ...1 and
$2y = x + 4$ ...2

Simplify 2 and get y by itself:

$\therefore y = \frac{1}{2}x + 2$ ...3

Now substitute 3 into 1

$\therefore 5\left(\frac{1}{2}x + 2\right) = 2x + 8$
$\therefore \frac{5}{2}x + 10 = 2x + 8$
$\therefore \frac{5}{2}x - 2x = 8 - 10$
$\therefore \frac{1}{2}x = -2$
$\therefore x = -4$

Substitute back into 3

$\therefore y = \frac{1}{2}(-4) + 2$
$\therefore y = 0$

$\therefore$ (-4; 0)

h) $y = 4x + 5$ ...1 and
$3x - y = 2$ ...2

Substitute 1 into 2

$\therefore 3x - (4x + 5) = 2$
$\therefore 3x - 4x - 5 = 2$
$\therefore -x = 2 + 5$
$\therefore -x = 7$
$\therefore x = -7$

Substitute back into 1

$\therefore y = 4(-7) + 5$
$\therefore y = -23$

$\therefore$ (-7; -23)

i) $y = 2x - 4$ ...1 and

$x - 8y = 27$ ...2

Substitute 1 into 2

$\therefore x - 8(2x - 4) = 27$
$\therefore x - 16x + 32 = 27$
$\therefore -15x = 27 - 32$
$\therefore -15x = -5$
$\therefore x = \frac{1}{3}$

Substitute back into 1

$\therefore y = 2\left(\frac{1}{3}\right) - 4$
$\therefore y = -3\frac{1}{3}$

$\therefore \left(\frac{1}{3}; -3\frac{1}{3}\right)$

j) $6y - x = 0$ ...1 and

$y = -5 + \frac{1}{3}x$ ...2

Substitute 2 into 1

$\therefore 6\left(-5 + \frac{1}{3}x\right) - x = 0$
$\therefore -30 + 2x - x = 0$
$\therefore x = 30$

Substitute back into 2

$\therefore y = -5 + \frac{1}{3}(30)$
$\therefore y = 5$

$\therefore$ (30; 5)

4. a)

	Now	In 5 years-time
Sarah	$x - 20$	$x - 20 + 5$
Dad	x	$x + 5$

$\therefore x - 20 + 5 = \frac{1}{2}(x + 5)$
$\therefore x - 15 = \frac{1}{2}x + \frac{5}{2}$
$\therefore x - \frac{1}{2}x = \frac{5}{2} + 15$
$\therefore \frac{1}{2}x = 17\frac{1}{2}$
$\therefore x = 35$

Dad is 35 so

Sarah is now 35 – 20 = 15 years old.

b)

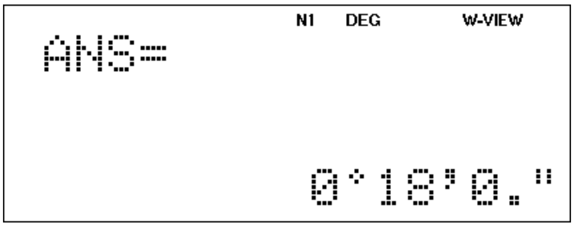
	Speed	Distance	Time
There	$\frac{10}{2x}$	10km	$2x$
Back	$\frac{10}{x}$	10km	x

$$\frac{1}{2}\left(\frac{10}{2x}+\frac{10}{x}\right)=25$$
$$\therefore \frac{1}{2}\left(\frac{5}{x}+\frac{10}{x}\right)=25$$
$$\therefore \frac{15}{2x}=25$$
$$\therefore 15=50x$$
$$\therefore \frac{15}{50}=x$$
$$\therefore x=\frac{3}{10}$$

Or 18 minutes from school

And 36 minutes to school

In order to find your time once you have got your answer (here $\frac{3}{10}$) press 2ndF ↔DEG D°M'S *and then you will get:*



Which means 0 hours, 18 minutes and 0 seconds.

c) Let $coke = x$ and $chips = y$

$\therefore 3x + y = 27$ ...1 and $2x + 3y = 25$ ...2

Simplify 1 and get y by itself:

$\therefore y = 27 - 3x$ ...3

Substitute 3 into 2

$$\therefore 2x + 3(27 - 3x) = 25$$
$$\therefore 2x + 81 - 9x = 25$$
$$\therefore -7x = 25 - 81$$
$$\therefore -7x = -56$$
$$\therefore x = 8$$

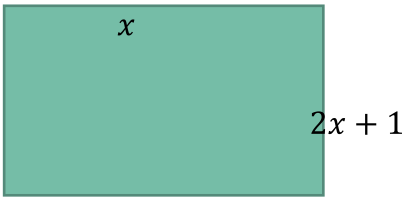
Substitute back into 3

$$\therefore y = 27 - 3(8)$$
$$\therefore y = 3$$

Don't forget to end your answer by writing a sentence that shows that you understood and answered the question.

$\therefore$ A coke costs R8.00 and a packet of chips cost R3.00

d)



$\therefore x(2x + 1) = 300$

$\therefore 2x^2 + x - 300 = 0$
$\therefore (2x + 25)(x - 12) = 0$
$\therefore 2x + 25 = 0$ OR $\therefore x - 12 = 0$

$\therefore 2x = -25$ $\therefore x = 12$
$\therefore x = -\frac{25}{2}$ or $-12\frac{1}{2}$

Length cannot be negative so $x = 12$

$\therefore P = 2l + 2b$
$\therefore P = 2(12) + 2(2(12) + 1)$
$\therefore P = 24 + 50$
$\therefore P = 74\ km$ $\therefore$ Dan needs 74km of fencing for his farm.

e) $P = 100 = 2b + 2l$

$\therefore 100 = 2(x + 5) + 2l$
$\therefore 100 = 2x + 10 + 2l$
$\therefore 100 - 2x - 10 = 2l$
$\therefore 90 - 2x = 2l$
$\therefore l = 45 - x$

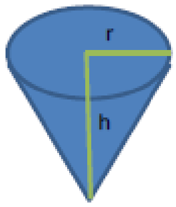
$\therefore Area = 400 = l \times b$
$\therefore 400 = (45 - x)(x + 5)$
$\therefore 400 = 45x + 225 - x^2 - 5x$
$\therefore 400 = 40x + 225 - x^2$
$\therefore 400 - 40x - 225 + x^2 = 0$
$\therefore x^2 - 40x + 175 = 0$
$\therefore (x - 5)(x - 35) = 0$
$\therefore x = 5$ or $x = 35$

$\therefore$ The length is = 45 – 5 =40m OR the length is = 45 – 35 = 10m

f) $SA = 33 = (x + 3)(x - 5)$

$\therefore 33 = x^2 - 5x + 3x - 15$
$\therefore 0 = x^2 - 2x - 15 - 33$
$\therefore 0 = x^2 - 2x - 48$
$\therefore 0 = (x + 6)(x - 8)$
$\therefore x = -6$ or $x = 8$

Length cannot be negative so $x = 8$.

g) 

$v = 47{,}1239 = \frac{1}{3}\,\pi\, r^2 h$

$\therefore 47{,}1239 = \frac{1}{3}\,\pi\, r^2\,(5)$

$\therefore 141{,}3717 = 5\pi\, r^2$

$\therefore 141{,}3717\ \div 5\pi = r^2$

$\therefore 9{,}0000\ldots. = r^2$

$\therefore r = \sqrt{9}$

$\therefore r = \pm 3$

(Length cannot be negative so the radius is 3cm)

∴ Anne's ice-cream scoop can be 6cm wide.

5. For each of the following find the given variable in terms of the other variables:

a) Find r if $v = \pi r^2$

$\therefore \frac{v}{\pi} = r^2$

$\therefore r = \pm\sqrt{\frac{v}{\pi}}$

b) Find t if $s = \frac{d}{t}$

$\therefore st = d$

$\therefore t = \frac{d}{s}$

c) Find i if $A = P(1+i)^n$

$\therefore \frac{A}{P} = (1+i)^n$

$\therefore \sqrt[n]{\frac{A}{P}} = 1 + i$

$\therefore i = \sqrt[n]{\frac{A}{P}} - 1$

d) find r if $v = \frac{4}{3}\pi r^3$

$\therefore v \div \frac{4}{3}\pi = r^3$

$\therefore \frac{3v}{4\pi} = r^3$

$\therefore r = \sqrt[3]{\frac{3v}{4\pi}}$

e) Find h if $v = \pi r^2 h$

$\therefore v \div \pi r^2 = h$

$\therefore h = \frac{v}{\pi r^2}$

f) Find m if $y = mx + c$

$\therefore y - c = mx$

$\therefore \frac{1}{m}(y - c) = x$

g) Find d if $T = a + (n-1)d$

$\therefore T - a = (n-1)d$

$\therefore \frac{(T-a)}{(n-1)} = d$

h) Find x if $y = ax^2 + 3$

$\therefore y - 3 = ax^2$

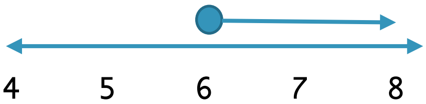
$\therefore \frac{1}{a}(y - 3) = x^2$

$\therefore x = \pm\sqrt{\frac{1}{a}(y-3)}$

6.

a) $3x + 4 \leq 5x - 8$

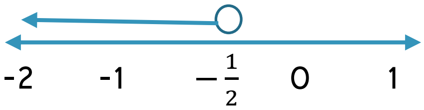
$\therefore 3x - 5x \leq -8 - 4$
$\therefore -2x \leq -12$
$\therefore x \geq 6$



$\therefore x \in [6;\ \infty)$

b) $-2x + 5 > 6$

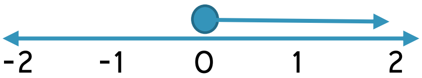
$\therefore -2x > 6 - 5$
$\therefore -2x > 1$
$\therefore x < -\frac{1}{2}$



$\therefore x \in \left(-\infty;\ -\frac{1}{2}\right)$

c) $2x + 5 \geq 5$

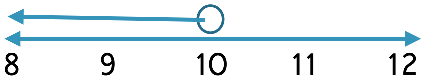
$\therefore 2x \geq 5 - 5$
$\therefore 2x \geq 0$



$\therefore x \in [0;\ \infty)$

d) $x - 6 < 4$

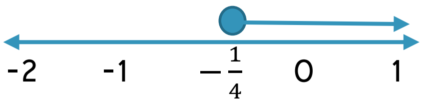
$\therefore x < 4 + 6$
$\therefore x < 10$



$\therefore x \in (-\infty; 10)$

e) $-4x + 1 \leq 2$

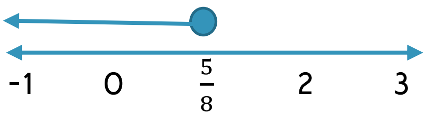
$\therefore -4x \leq 2 - 1$
$\therefore -4x \leq 1$
$\therefore x \leq -\frac{1}{4}$



$\therefore x \in \left[-\frac{1}{4};\ \infty\right)$

f) $5 - 6x \geq 2x$

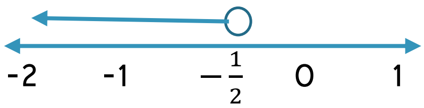
$\therefore -6x - 2x \geq -5$
$\therefore -8x \geq -5$
$\therefore x \leq \frac{5}{8}$



$\therefore x \in \left(-\infty; \frac{5}{8}\right]$

g) $x - 6 > 9x - 2$

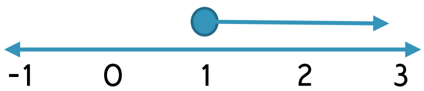
$\therefore x - 9x > -2 + 6$
$\therefore -8x > 4$
$\therefore x < -\frac{1}{2}$



$\therefore x \in \left(-\infty;\ -\frac{1}{2}\right)$

h) $-x + 3 \leq 2$

$\therefore -x \leq 2 - 3$
$\therefore -x \leq -1$
$\therefore x \geq 1$



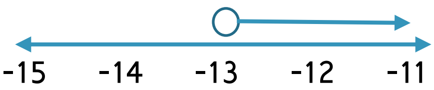
$\therefore x \in [1;\ \infty)$

i) $x - 8 < 2x + 5$

$\therefore x - 2x < 5 + 8$

$\therefore -x < 13$

$\therefore x > -13$



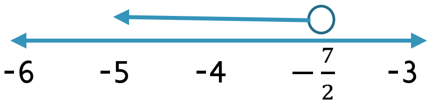
$\therefore x \in (-13;\ \infty)$

j) $-2x + 3 > 10$

$\therefore -2x > 10 - 3$

$\therefore -2x > 7$

$\therefore x < -\frac{7}{2}$



$\therefore x \in \left(-\infty;\ -\frac{7}{2}\right)$