SHARP

Investigation Memo: Exponents

Grade 9 Maths

Law 1

1. Complete the following table using your calculator:

   The first line has been done as an example:

a^m	a^n	$a^m \times a^n$	Prime factors	a^{m+n}
2^2	2^3	32	$32 = 2^5$	$2^{2+3} = 32$
3^3	3^3	729	$729 = 3^6$	$3^{3+3} = 729$
2^4	2^5	512	$512 = 2^9$	$2^{4+5} = 512$
3^4	3^5	$19\,683$	$19\,683 = 3^9$	$3^{4+5} = 19\,683$
5^3	5^7	$9\,765\,625$	$9\,765\,625 = 5^{10}$	$5^{3+7} = 9\,765\,625$
a^2	a^4	$aa \times aaaa$	$aaaaaa = a^6$	$a^{2+4} = a^6$
2^x	2^y	$2^x \times 2^y$	2^{x+y}	2^{x+y}

2. Can you give a general rule for what is happening in the table above?

   a) State this rule in words:

   When you multiply with the same bases you add the exponents together.

   b) $a^m \times a^n = a^{m+n}$

3. Can you think of any examples that disprove this rule?

   There are no examples that disprove this rule.

Law 2

1. Complete the following table using your calculator:

   The first line has been done as an example:

a^m	a^n	$a^m \div a^n$	Prime factors	a^{m-n}
2^5	2^3	4	$4 = 2^2$	$2^{5-3} = 4$
3^3	3^3	1	1 = 1 or 3^0	$3^{3-3} = 1$
2^6	2^5	2	2 = 2	$2^{6-5} = 2$
3^{14}	3^5	19 683	$19683 = 3^9$	$3^{14-5} = 19\,683$
5^7	5^3	625	$625 = 5^4$	$5^{7-3} = 625$
a^6	a^4	$aaaaaa \div aaaa$	$aa = a^2$	$a^{6-4} = a^2$
2^x	2^y	$2^x \div 2^y$	$2^x \div 2^y = 2^{x-y}$	2^{x-y}

2. Can you give a general rule for what is happening in the table above?

   a) State this rule in words.

   When you divide power that have the same bases, you subtract the divisor's exponent from the dividend's exponent.

   b) $a^m \div a^n = a^{m-n}$

3. Can you think of any examples that disprove this rule?

   There are no examples that disprove this rule.

Law 3

1. Complete the following table using your calculator:

   The first line has been done as an example:

a^m	n	$(a^m)^n$	Prime factors	$a^{m\times n}$
2^2	3	$(2^2)^3 = 64$	$64 = 2^6$	$2^{2\times 3} = 64$
3^3	2	$(3^3)^2 = 729$	$729 = 3^6$	$3^{3\times 2} = 729$
2^4	4	$(2^4)^4 = 65\,536$	$65\,536 = 2^{16}$	$2^{4\times 4} = 65\,536$
3^4	5	$(3^4)^5 = 3\,486\,784\,401$	$3\,486\,784\,401 = 3^{20}$	$3^{4\times 5} = 3\,486\,784\,401$
5^3	2	$(5^3)^2 = 15\,625$	$15\,625 = 5^6$	$5^{3\times 2} = 15\,625$
a^2	4	$(a^2)^4$	$aa \times aa \times aa \times aa = a^8$	$a^{2\times 4}$
2^x	y	$(2^x)^y$	2^{xy}	$2^{x\times y}$

2. Can you give a general rule for what is happening in the table above?

   a) state this rule in words.

   When you raise a power to another exponent you multiply the exponents together.

   b) $(a^m)^n = a^{m\times n} = a^{mn}$

3. Can you think of any examples that disprove this rule?

   There are no examples that disprove this rule.

4. Does this rule apply to every number in the brackets?

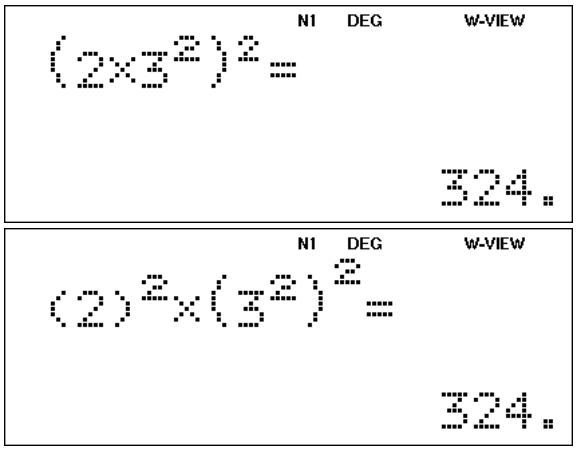
   Yes it does.

5. Is $(2 \times 3^2)^2$ the same as $(2)^2 \times (3^2)^2$?

   Yes it is:

   $(2 \times 3^2)^2 = 324$

   And $(2)^2 \times (3^2)^2 = 324$



6. What can we say about the above? Can you think of any examples that disprove this rule?

   Law 3 applies to more than one number or power in the bracket when they are multiplied. When powers that are multiplied in a bracket are raised to a further exponent, the exponent is applied to all of the powers in the bracket.

Law 4

1. What do you notice about the answer column?

   The answer column is always 1.

2. Is there any place in the answer column where this value changes?

   Only at zero, this answer is undefined.

3. Complete this table:

3^0	1
4^0	1
5^0	1
2^0	1
x^0	1

4. What is the general rule?

   Any base raised to the power of zero is 1.

5. What do you see?

   All the answers in the answer column are 1.

6. Do you think this proves the rule we discussed above?

   Yes, it does.

. Can you think of any examples that disprove this rule?

   No.

Law 5

1. Complete the table below.

X	ANS	Fraction
2	0.5	$\frac{1}{2}$
3	0.3333333333	$\frac{1}{3}$
5	0.2	$\frac{1}{5}$
10	0.1	$\frac{1}{10}$
20	0.05	$\frac{1}{20}$

2. What do you notice about the X and the fraction?

   The fraction's denominator is the value of x. In other words, the fraction is the inverse of X.

3. Complete the rule:

$$x^{-1} = \frac{1}{x}$$

4. Can you think of examples that disprove this rule?

   No, there are no examples.

5. Complete the table below.
The first row has been done as an example for you.

X	$\frac{1}{X^{-1}}$	ANS
2	$\frac{1}{2^{-1}}$	2
3	$\frac{1}{3^{-1}}$	3
5	$\frac{1}{5^{-1}}$	5
10	$\frac{1}{10^{-1}}$	10
20	$\frac{1}{20^{-1}}$	20

6. What do you notice about $\frac{1}{X^{-1}}$ and Ans?

The Ans the value of x is the denominator in $\frac{1}{X^{-1}}$. In other words, Ans is the inverse of $\frac{1}{X^{-1}}$

7. Complete the rule:

$$\frac{1}{a^{-1}} = a$$

8. Can you think of any of examples that disprove this rule?

No there are no examples that disprove this rule.