Introducing Reduction angle formulae

Grade 11 Trigonometry: Memo

1. Calculate the value of each of the following:

$Sin\ 30° = \frac{1}{2}$	$Cos\ 30° = \frac{\sqrt{3}}{2}$	$Tan\ 30° = \frac{\sqrt{3}}{3}$
$Sin\ 150° = \frac{1}{2}$	$Cos\ 150° = -\frac{\sqrt{3}}{2}$	$Tan\ 150° = -\frac{\sqrt{3}}{3}$
$Sin\ 210° = -\frac{1}{2}$	$Cos\ 210° = -\frac{\sqrt{3}}{2}$	$Tan\ 210° = \frac{\sqrt{3}}{3}$
$Sin\ 330° = -\frac{1}{2}$	$Cos\ 330° = \frac{\sqrt{3}}{2}$	$Tan\ 330° = -\frac{\sqrt{3}}{3}$

2. $\sin\theta =$

θ	$30°$	$45°$	$60°$	$75°$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0,97
$\sin(180-\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0,97
$\sin(180°+\theta)$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	- 0,97
$\sin(360°-\theta)$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	- 0,97
$\sin(360°+\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0,97

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$\cos\ \theta =$

$\boldsymbol{\theta}$	**30°**	**45°**	**60°**	**75°**
$\boldsymbol{\cos\theta}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0,26
$\boldsymbol{\cos\,(180-\theta)}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	- 0,26
$\boldsymbol{\cos\,(180°+\ \theta)}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	- 0,26
$\boldsymbol{\cos\,(360°-\theta)}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0,26
$\boldsymbol{\cos\,(360°+\ \theta)}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0,26

$\tan\theta =$

$\boldsymbol{\theta}$	**30°**	**45°**	**60°**	**75°**
$\boldsymbol{\tan\theta}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$
$\boldsymbol{\tan\,(180-\theta)}$	$-\frac{\sqrt{3}}{3}$	- 1	$-\sqrt{3}$	$-2-\sqrt{3}$
$\boldsymbol{\tan\,(180°+\ \theta)}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$
$\boldsymbol{\tan\,(360°-\theta)}$	$-\frac{\sqrt{3}}{3}$	- 1	$-\sqrt{3}$	$-2-\sqrt{3}$
$\boldsymbol{\tan\,(360°+\ \theta)}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$

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3.

θ	$30°$	$45°$	$60°$	$75°$
$Sin(-\theta)$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-0{,}97$
$Cos(-\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$0{,}26$
$Tan(-\theta)$	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$-2-\sqrt{3}$
$Sin(-180°+\theta)$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-0{,}97$
$Cos(-180°+\theta)$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$-0{,}26$
$Tan(-180°+\theta)$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$
$Sin(-180°-\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$0{,}97$
$Cos(-180°-\theta)$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$-0{,}26$
$Tan(-180°-\theta)$	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$-2-\sqrt{3}$

4. If you are adding or subtracting 360° from acute angle labelled as θ, the value of a trigonometric ratio does not change.

$$Sin(\theta \pm 360°) = +Sin\theta$$

$$Cos(\theta \pm 360°) = +Cos\theta$$

$$Tan(\theta \pm 360°) = +Tan\theta$$

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5. Answers:

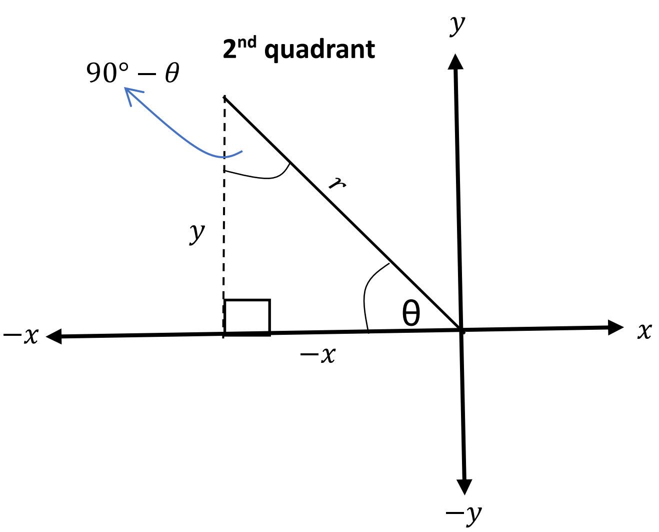
- What do you notice about answers?
  - We notice that if we compare one trigonometric ratio irrespective of the size of an angle, almost all answers are alike with different signs to show the values.
- What is the relationship between your answers?
  - One trigonometric ratio has the same answers with different signs.
- What causes the sign to change in some of the values?
  - The size of an angle, which means the change of the quadrant.
- Can you think of other cases like this?
  - Yes, trig ratio with the angle $270° \pm \theta$.
- What is your conclusion after you have engaged to this activity?

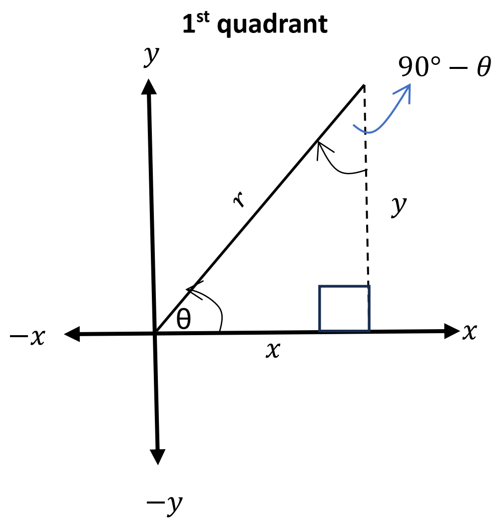
$Sin(180° - \theta) = +Sin\ \theta$	$\tan(180° + \theta) = +\tan\theta$	$\cos(360° - \theta) = +\cos\theta$
$Cos(180° - \theta) = -Cos\ \theta$	$\sin(180° + \theta) = -\sin\theta$	$\tan(360° - \theta) = -\tan\theta$
$Tan(180° - \theta) = -Tan\ \theta$	$\cos(180° + \theta) = -\cos\theta$	$\sin(360° - \theta) = -\sin\theta$
Negative Angles:		
$\sin(-180° - \theta) = +\sin\theta$	$\tan(-180° + \theta) = +\tan\theta$	$\cos(-\theta) = +\cos\theta$
$\cos(-180° - \theta) = -\cos\theta$	$\sin(-180° + \theta) = -\sin\theta$	$\tan(-\theta) = -\tan\theta$
$\tan(-180° - \theta) = -\tan\theta$	$\cos(-180° + \theta) = -\cos\theta$	$\sin(-\theta) = -\sin\theta$

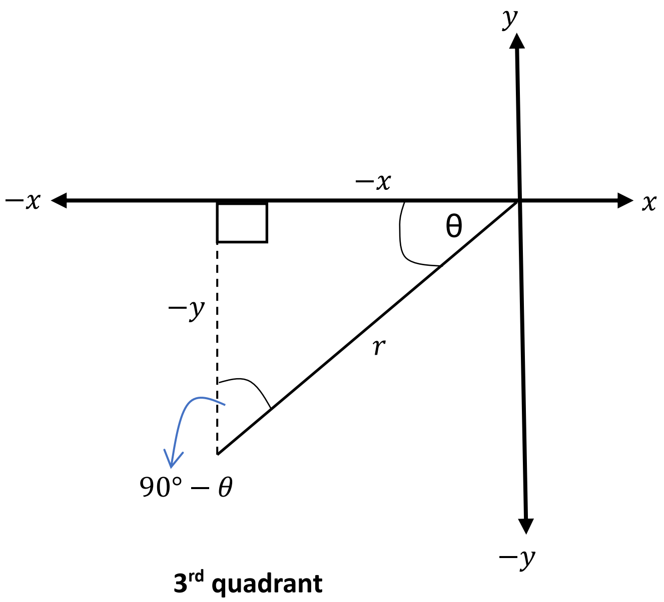
6. Follow these instructions carefully in your workbook:

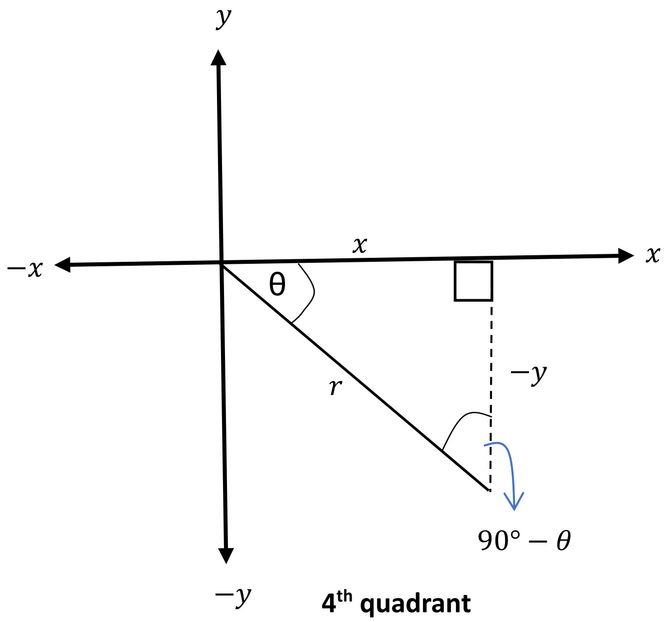
- Draw four different Cartesian Planes
- In each Cartesian Plane draw a terminal arm that lies in one quadrant. E.g., 1st Cartesian Plane, terminal arm lies in the first quadrant, 2nd Cartesian Plane, terminal arm lies in the second quadrant and so on…

- Construct a line from any point of your terminal arm to the x-axis and the line must be perpendicular to the x-axis. Do this on each Cartesian Plane.
- Label all angles between terminal arm and x-axis θ
- Label all sides using y-axis and x-axis accordingly.
- Remember to label terminal arm r
- Determine all third interior angles in terms of θ









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Now, complete the following table:

Trig function	1st quadrant	2nd quadrant	3rd quadrant	4th quadrant
$Sin\ \theta =$	$\frac{Opp}{Hyp} = \frac{y}{r}$	$\frac{Opp}{Hyp} = \frac{y}{r}$	$\frac{Opp}{Hyp} = \frac{-y}{r}$	$\frac{Opp}{Hyp} = \frac{-y}{r}$
$Sin(90° - \theta) =$	$\frac{Opp}{Hyp} = \frac{x}{r}$	$\frac{Opp}{Hyp} = \frac{-x}{r}$	$\frac{Opp}{Hyp} = \frac{-x}{r}$	$\frac{Opp}{Hyp} = \frac{x}{r}$
$Cos\ \theta =$	$\frac{Adj}{Hyp} = \frac{x}{r}$	$\frac{Adj}{Hyp} = \frac{-x}{r}$	$\frac{Adj}{Hyp} = \frac{-x}{r}$	$\frac{Adj}{Hyp} = \frac{x}{r}$
$Cos(90° - \theta) =$	$\frac{Adj}{Hyp} = \frac{y}{r}$	$\frac{Adj}{Hyp} = \frac{y}{r}$	$\frac{Adj}{Hyp} = \frac{-y}{r}$	$\frac{Adj}{Hyp} = \frac{-y}{r}$

- Through observing the findings recorded on the table above, what conclusion you can make?

$Sin(90° - \theta) = Cos\ \theta$ in all quadrants and $Cos(90° - \theta) = Sin\ \theta$ in all quadrants as well.

7. How could you prove $\text{Sin}(90° + \theta) = \text{Cos}\ \theta$ and $\text{Cos}(90° + \theta) = -\text{Sin}\ \theta$?

You can use your SHARP Calculator.

Go to table mode and type your 1st function $Sin(90° + x)$. Note on your calculator, you use x as a variable to use in the place of θ. In the 2nd function you type $Cos\ x$, then you choose 15 as your steps. After the table has displayed you can compare the two answers to show that they have the same value.